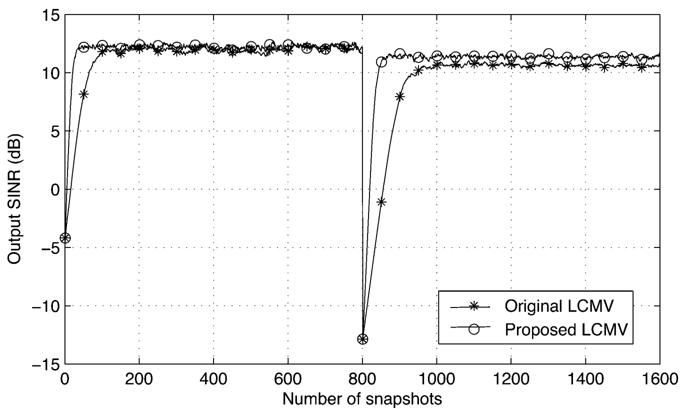

Fig. 8. Output SINR versus number of snapshots. $M = 8, L = 4$ at first 800 snapshots, $L = 5$ after that.

the number of variables in the update equation has been reduced effectively by half, which leads to a higher convergence rate. Moreover, since the imposed structure is derived from a beamformer based on maximizing its output SINR or minimizing its MSE, the proposed algorithms have also achieved a higher steady state output SINR value, given the same stepsize. Simulation results verified the effectiveness of the proposed method.

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Multiple-Antenna Multicasting Using Channel Orthogonalization and Local Refinement

Ahmed Abdelkader, Alex B. Gershman, and Nicholas D. Sidiropoulos

***Abstract*—The problem of single-group multiple-antenna multicasting is considered, where common information should be sent to a large number of users. The multiple-antenna transmitter is assumed to accurately know the instantaneous downlink channel state information (CSI) for all users, and our objective is to design the beamformer complex weights to minimize the total transmitted power subject to individual user quality-of-service (QoS) constraints. In this correspondence, a channel orthogonalization and local refinement based approach is developed to solve this problem in an approximate way. The proposed techniques are shown via computer simulations and real data processing to offer an attractive performance-to-complexity tradeoff as compared to the state-of-the-art multiple-antenna multicasting algorithms.**

***Index Terms*—Evolved multimedia broadcast/multicast service (E-MBMS), multicasting, orthogonalization, transmit beamforming.**

I. Introduction

In contrast to traditional broadcasting systems where the transmission power is radiated isotropically or using a fixed beampattern to

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cover a certain service area, some future digital broadcasting/multicasting services (e.g., in the context of E-MBMS[1] of 3 GPP[2]/UMTS-LTE[3]) will be based on subscription. Therefore, it is reasonable to assume for such systems that the user channel state information (CSI) is available at the transmitter [1]. In the latter case, advanced algorithms have been developed that exploit the CSI knowledge to substantially improve performance over the aforementioned traditional broadcasting techniques [1]–[6].

In this correspondence, we consider the *single-group multicasting problem* where a multiple-antenna transmitter sends *common* information to a large number of single-antenna users. The transmitter is assumed to have knowledge of the instantaneous downlink CSI for each user. Such a single-group multicasting problem has been originally considered in [1], where the following formulation has been addressed: Obtain the transmit beamformer weight vector that minimizes the total transmitted power subject to individual quality-of-service (QoS) constraints, one for each user. It has been proven in [1] that this problem is nonconvex and generally NP-hard. However, the authors of [1] have developed an approach to approximately solve this problem using the semidefinite relaxation (SDR) technique. Unfortunately, because of the SDR step, the approach of [1] is computationally quite demanding.

Another promising approach that applies to the single-group multicasting problem has been proposed in [3]. The iterative algorithm of [3] is very efficient from the computational point of view, yet it may fail to converge and its performance is very sensitive with respect to initialization [5], [6]. To improve the performance of the latter approach, the authors of [5] proposed its enhancement using i) the average signal-to-noise ratio (SNR) beamformer of Lopez [2] as initialization; and ii) a step-size damping strategy that was empirically optimized. In [5], the so-obtained technique is referred to as the dLLI (damped Lozano with Lopez Initialization) algorithm.

In this correspondence, we develop a new approach to approximately solve the single-group multiple-antenna multicasting problem using channel orthogonalization. The key idea of our approach is to orthogonalize selected user downlink channel vectors using QR decomposition to satisfy their QoS constraints in a simple way. Using channel orthogonalization in this context was originally proposed in [4], which also included a successive *orthogonal* refinement algorithm that is similar in spirit to [3]. The best algorithm in [4] is called *reduced-complexity combine-2* (RCC2) with successive orthogonal refinement (Algorithm 2 in [4]). The simulations in [4] suggest that this combination can meet or even slightly outperform the SDR algorithm in [1] at considerably reduced complexity. However, the comparison in [4] was made using only 100 randomization trials for SDR, whereas an order of magnitude higher is suggested in [1] for the given problem size. Increasing the number of randomizations in this small sample-size regime significantly improves the performance of SDR.

The approach in [4] has merits, but its performance may be limited by its choice of orthogonalization order and scaling in the successive refinement algorithm, as well as by its (RCC2) initialization. In contrast to [4], the approach we propose in this correspondence examines various orthogonalization orders in a pseudorandom way[4] and then chooses the best one based on the criterion of minimum transmitted power. This allows exploitation of multiuser selection diversity (see also [5]) when there are many users with statistically independent channel vectors to choose from. Moreover, our approach uses an improved (with respect to [4]) *nonorthogonal* successive local refinement technique, drawing from the dLLI algorithm of [5].

As it will be seen from our simulation and measured channel data processing results, in the practically most important case when the number of users is large, the proposed techniques offer a more attractive performance-to-complexity tradeoff compared to the methods of [1], [4] and [5]. In particular, our techniques will be shown to perform better than all these methods, at comparable or even substantially lower complexity than that of the SDR-based approach of [1].

[1]Evolved Multimedia Broadcast/Multicast Service

[2]Third Generation Partnership Project

[3]Universal Mobile Telecommunications System—Long Term Evolution

[4]Ordering has been commonly used in a number of communications applications, e.g., [7].

II. Problem Formulation

Consider a multicasting scenario where a transmitter with N antennas sends common data to M single-antenna users. The channels are assumed frequency-flat. The SNR of the ith user is given by $\gamma_i = |\mathbf{w}^H\mathbf{h}_i|^2/\sigma_i^2$ where the $N\times 1$ vectors $\mathbf{w}$ and $\mathbf{h}_i$ are the transmit beamformer weight vector and the downlink channel vector of the ith user, respectively, σ_i^2 is the variance of the ith user additive white noise, and $(\cdot)^H$ denotes the Hermitian transpose.

The beamformer weight vector can be obtained by minimizing the total transmitted power subject to the user QoS constraints (one constraint per user). The latter problem can be expressed as [1]:

$$\min_{\mathbf{w}} \|\mathbf{w}\|^2 \quad \text{s.t.} \quad |\mathbf{w}^H\tilde{\mathbf{h}}_i|^2 \geq 1 \qquad \text{for all} \quad i=1,\ldots,M \tag{1}$$

where $\tilde{\mathbf{h}}_i \triangleq \mathbf{h}_i/\sqrt{\gamma_{\min,i}{\sigma_i}^2}$ is the ith user's normalized downlink channel vector and $\gamma_{\min,i}$ is the minimum required SNR for the ith user. The value of $\gamma_{\min,i}$ is determined by the system QoS requirements [1].

In [1], an SDR-based technique has been proposed to approximately solve (1) at worst-case computational cost $\mathcal{O}((M+N^2)^{3.5})$. An alternative, computationally simpler approach to solve this problem is the dLLI algorithm that was formulated in [6] to solve the joint beamforming and admission control problem in multicast networks. The computational complexity of the latter algorithm is $\mathcal{O}(IMN)$, where I is a bound on the number of iterations that depends only on the initial step-size μ (I is kept small due to the use of an aggressive damping strategy [6]).

III. The Proposed Approach

As typically the number of users is larger than the number of transmit antennas ($M > N$), hereafter only this case will be considered. Let us choose N vectors from the set $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$ to generate N orthonormal vectors $\mathbf{q}_i$ ($i=1,\ldots,N$). As these vectors span the whole N-dimensional space, the desired weight vector $\mathbf{w}$ can be represented as a linear combination of them:

$$\mathbf{w} = \sum_{i=1}^{N} c_i\mathbf{q}_i \tag{2}$$

where $\mathbf{c} = [c_1,\ldots,c_N]^T$ is the vector of complex coefficients and $(\cdot)^T$ is the transpose. From orthonormality, it follows that

$$\|\mathbf{w}\|^2 = \|\mathbf{c}\|^2. \tag{3}$$

The key idea of our approach is to choose each component $c_i\mathbf{q}_i$ of $\mathbf{w}$ in (2) to satisfy the QoS constraints corresponding to the chosen subset of channel vectors. The remaining $(M-N)$ QoS constraints can be then satisfied by scaling the so-obtained vector $\mathbf{w}$ so that the most violated constraint is satisfied with equality.

A. A Technique Based on QR Decomposition Without Pivoting

Let us consider the $N \times M$ matrix $\mathbf{G} \triangleq [\tilde{\mathbf{h}}_1, \ldots, \tilde{\mathbf{h}}_M]$ whose columns are the vectors $\tilde{\mathbf{h}}_i$ $(i = 1, \ldots, M)$. Let the $N \times N$ matrix $\mathbf{H}$ be obtained by dropping any $(M - N)$ columns of $\mathbf{G}$ and possibly reordering the remaining N columns. Applying QR decomposition to $\mathbf{H}$, we obtain

$$\mathbf{H} = [\mathbf{q}_1, \ldots, \mathbf{q}_N] \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1N} \\ 0 & r_{22} & \cdots & r_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & r_{NN} \end{bmatrix} \triangleq \mathbf{QR} \tag{4}$$

where $r_{ii} > 0$ for all $i = 1, \ldots, N$.

Equation (2) can be rewritten as

$$\mathbf{w} = \mathbf{Qc}. \tag{5}$$

Using (4) and (5), and the property $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$, we have

$$\mathbf{w}^H\tilde{\mathbf{h}}_i = \mathbf{c}^H\mathbf{Q}^H\tilde{\mathbf{h}}_i = \mathbf{c}^H\mathbf{Q}^H\mathbf{Q}\mathbf{r}_l = \mathbf{c}^H\mathbf{r}_l \tag{6}$$

where $\mathbf{I}$ is the identity matrix and without any loss of generality it is assumed that $\tilde{\mathbf{h}}_i$ has been chosen as the lth column of $\mathbf{H}$ and $\mathbf{r}_l$ denotes the lth column of $\mathbf{R}$. Then, using (3) and (6), and keeping in (1) only the N QoS constraints that correspond to the columns of $\mathbf{H}$, the latter problem can be transformed to

$$\min_{\mathbf{c}} \|\mathbf{c}\|^2 \quad \text{s.t.} \quad |\mathbf{c}^H\mathbf{r}_i| \geq 1 \quad \text{for all} \quad i = 1, \ldots, N. \tag{7}$$

Although the problem (7) has the same mathematical form as (1), an important difference between these two problems is that the vectors $\mathbf{r}_i$ inherit the upper-triangular structure of the matrix $\mathbf{R}$. Also, as $N < M$, the number of constraints in (7) is less than in (1). These two facts make it possible to satisfy the constraints in (7) by computing the coefficients $c_i, i = 1, \ldots, N$ successively. In particular, from the first constraint $|\mathbf{c}^H\mathbf{r}_1| \geq 1$, we obtain that $|c_1 r_{11}| \geq 1$ and, hence, $|c_1| = 1/r_{11}$ is the minimum needed to satisfy the constraint. Note that the phase of c_1 can be chosen arbitrarily. Indeed, due to the successive way of computing the coefficients c_i $(i = 1, \ldots, N)$, any change of $\arg\{c_1\}$ will only cause a rotation of the computed weight vector; and, clearly, such a rotation will not alter the cost function. Therefore, without loss of generality, we can set $\arg\{c_1\} = 0$. That is, the first coefficient can be computed as

$$c_1 = 1/r_{11}. \tag{8}$$

From the kth constraint $|\mathbf{c}^H\mathbf{r}_k| = 1$ for any $k = 2, \ldots, N$, we have

$$\left| \sum_{i=1}^{k} c_i^* r_{ik} \right| \geq 1 \tag{9}$$

where $(\cdot)^*$ denotes the complex conjugate. Defining $\beta_k \triangleq \sum_{i=1}^{k-1} c_i^* r_{ik}$ for $k = 2, \ldots, N$, we can rewrite (9) as

$$|c_k^* r_{kk} + \beta_k| \geq 1. \tag{10}$$

Equation (10) illustrates the kth step of our proposed successive algorithm to compute the vector $\mathbf{c}$. In this step, all c_i for $i = 1, \ldots, k-1$ have already been computed (that is, the value of β_k is given), and c_k should be obtained from (10) so that the increase of the cost function $\|\mathbf{c}\|^2$ caused by c_k is minimum. Obviously, this is equivalent to selecting c_k that satisfies (10) and has the smallest absolute value. The solution is

$$c_k = \begin{cases} \frac{1-|\beta_k|}{r_{kk}} e^{-j\arg\{\beta_k\}}, & |\beta_k| < 1 \\ 0, & |\beta_k| \geq 1 \end{cases}. \tag{11}$$

Equations (8) and (11) describe the proposed technique to successively compute the coefficients c_k, $k = 1, \ldots, N$. After computing the whole coefficient vector $\mathbf{c}$ in this way, the associated weight vector can be found from (2). The remaining $(M - N)$ QoS constraints to be satisfied correspond to the dropped $(M - N)$ columns of $\mathbf{G}$. To satisfy the latter constraints, we check all of them and then rescale the resulting weight vector so that the most violated constraint is satisfied with equality.

Since the choice of the columns of $\mathbf{H}$ and their particular order in $\mathbf{H}$ can greatly affect the resulting performance of our technique, multiple candidate values of $\mathbf{w}$ are computed. These candidate weight vectors correspond to different choices of the dropped columns of $\mathbf{G}$ and different orders of the remaining columns of $\mathbf{H}$. Then, from these candidate weight vectors, the vector with the smallest norm (i.e., with the lowest total transmitted power) is finally chosen.

The process of finding the best (in terms of performance) ordered subset of N vectors out of the set of M channel vectors $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$ requires checking $M!/(M-N)!$ possibilities. Clearly, for large M and N this is prohibitive. Therefore, we propose to check $J \ll M!/(M-N)!$ random permutations where J is a design parameter that can be used to trade off between computational complexity and performance. As a result, there will be J candidate weight vectors $\mathbf{w}_{\text{cand},i}$ $(i = 1, \ldots, J)$ and the resulting dominant complexity of our algorithm is given by $\mathcal{O}(J(N^3 + MN))$. Therefore, for a reasonably low choice of J, the proposed technique represents a computationally attractive alternative to the SDR-based technique of [1].

B. A Technique Based on QR Decomposition With Pivoting

As the computational complexity of the QR decomposition based technique of Section III-A can be still quite high, let us consider a computationally more efficient *ad hoc* approach to choose the columns of $\mathbf{H}$ and their order, using the Gram-Schmidt procedure to orthogonalize the selected channel vectors.

We start by choosing any initial channel vector $\mathbf{f}_1$ from the set $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$. In what follows, we denote the N vectors chosen from this set at the N steps of the Gram–Schmidt procedure as $\mathbf{f}_i, i = 1, \ldots, N$, so that $\mathbf{H} = [\mathbf{f}_1, \ldots, \mathbf{f}_N]$. The way to select these vectors will be discussed in the sequel. The Gram–Schmidt orthogonalization procedure can be written as

$$\mathbf{b}_k = \mathbf{f}_k - \sum_{i=1}^{k-1} \left(\mathbf{q}_i^H\mathbf{f}_k\right) \mathbf{q}_i, \quad \mathbf{q}_k = \mathbf{b}_k / \|\mathbf{b}_k\| \tag{12}$$

for $k = 2, \ldots, N$ where $\mathbf{q}_1 = \mathbf{f}_1/\|\mathbf{f}_1\|$. In the kth step of this procedure, the intermediate weight vector can be obtained as

$$\mathbf{w}_k = \sum_{i=1}^{k} c_i\mathbf{q}_i \tag{13}$$

where the principle of computing the coefficients c_i is the same as in the QR decomposition based technique of the previous subsection. The key of our approach to select the channel vectors $\mathbf{f}_i$ from $\{\tilde{\mathbf{h}}_i\}_{i=1}^M$ can be described as follows. At the kth step $(k > 1)$ of the above Gram–Schmidt procedure, the vector for which it is most difficult to satisfy the corresponding QoS constraint is selected. In other words, the vector whose inner product with $\mathbf{w}_{k-1}$ has the smallest magnitude is chosen. As the component that is added to the weight vector in any step is orthogonal to all the channel vectors selected in the previous steps, it will not affect any of the previously satisfied constraints.

TABLE I
SUMMARY OF THE TECHNIQUE OF SECTION III-A WITH LOCAL REFINEMENT

1. Obtain the matrix $\mathbf{H}$ by randomly selecting and permuting N columns of $\mathbf{G}$.
2. Obtain the matrices $\mathbf{Q}$ and $\mathbf{R}$ using QR decomposition of $\mathbf{H}$.
3. Compute the candidate weight vector using (2), (8) and (11).
4. Locally refine this weight vector.
5. Rescale the refined vector so that the most violated from all the M constraints is satisfied with equality.
6. Repeat steps 1 to 5 J times to obtain $\{\tilde{\mathbf{w}}_i\}_{i=1}^{J}$.
7. Select from $\{\tilde{\mathbf{w}}_i\}_{i=1}^{J}$ the vector with the minimum norm to be the final solution.

TABLE II
SUMMARY OF THE TECHNIQUE OF SECTION III-B WITH LOCAL REFINEMENT

For $i = 1, \ldots, M$
1. Define $\mathcal{G} = \{\tilde{\mathbf{h}}_j\}_{j=1}^{M}$ and select $\mathbf{f}_1 = \tilde{\mathbf{h}}_i$.
2. $\mathcal{G} := \mathcal{G} \setminus \tilde{\mathbf{h}}_i$. Re-index all vectors in $\mathcal{G}$.
3. Compute $\mathbf{q}_1 = \mathbf{f}_1/\|\mathbf{f}_1\|$.
4. Compute c_1 using (8) and obtain $\mathbf{w}_1 = c_1\mathbf{q}_1$.
5. For $k = 2, \ldots, N$
   - 5a. For all current vectors in $\mathcal{G}$, compute $\alpha_j = |\mathbf{w}_{k-1}^H\tilde{\mathbf{h}}_j|, \ j = 1, \ldots, M-k+1$.
   - 5b. Select $\mathbf{f}_k = \tilde{\mathbf{h}}_l$ where α_l is the minimum value from $\{\alpha_j\}_{j=1}^{M-k+1}$ and $\tilde{\mathbf{h}}_l$ is the corresponding channel vector.
   - 5c. $\mathcal{G} := \mathcal{G} \setminus \tilde{\mathbf{h}}_l$. Re-index all vectors in $\mathcal{G}$.
   - 5d. Compute c_k using (11) and $\mathbf{q}_k$ using (12).
6. Compute the candidate weight vector $\tilde{\mathbf{w}}_i$ using (2).
7. Locally refine this weight vector.
8. Rescale the refined vector so that the most violated from all the M constraints is satisfied with equality.

9. Select from $\{\tilde{\mathbf{w}}_i\}_{i=1}^{M}$ the vector with the minimum norm to be the final solution.

Finally, (2) is used to compute the resulting $\mathbf{w}$. This vector is then rescaled to satisfy the most violated of the remaining $(M-N)$ constraints with equality.

Since the choice of initial channel vector is important, and the Gram–Schmidt procedure is relatively simple from a complexity point of view, we propose to repeat the process M times, where each time a new channel vector is chosen as the initial vector $\mathbf{f}_1$ for the Gram–Schmidt procedure. As a result, we end up with M candidate weight vectors $\mathbf{w}_{\mathrm{cand},i}$ $(i = 1, \ldots, M)$ and the one having the smallest norm is chosen as the final weight vector. The complexity of this technique is $\mathcal{O}(MN^3 + M^2N)$.

C. Local Refinement

To further improve the performance of the techniques developed in Subsections III-A and III-B, let us develop a local search based refinement step. The idea is to perform an unconstrained local search for any candidate weight vector $\mathbf{w}_{\mathrm{cand},i}$ used in these techniques.

For all values of i, the local refinement algorithm takes $\mathbf{w}_{\mathrm{cand},i}$ as an initial value and then searches for another vector $\tilde{\mathbf{w}}_i$ in its neighborhood that maximizes the worst user SNR. This can be achieved by finding the local minimum of

$$f(\tilde{\mathbf{w}}_i) = \frac{\|\tilde{\mathbf{w}}_i\|}{\min_k |\tilde{\mathbf{w}}_i^H\tilde{\mathbf{h}}_k|}.$$

The resulting vectors are then treated as the refined candidate weight vectors. Note that global maximization of the worst user SNR under a power constraint is also nonconvex, NP-hard, and closely related to our original problem [1]; but what we advocate here is only local refinement, which can be easily accomplished using a variety of standard methods. We will use the damped version of Lozano's algorithm [3], as proposed in [5]. Then, with I denoting a bound on the number of gradient iterations (as for dLLI, I is rather small, and only depends on the initial value of the step-size μ), the overall complexities of the techniques of Sections III-A and III-B combined with the local refinement step are $\mathcal{O}(J(N^3 + IMN))$ and $\mathcal{O}(M(N^3 + IMN))$, respectively.

These two algorithms with local refinement are summarized in Tables I and II. Note that their robust extensions to the case of imperfect CSI can be straightforwardly obtained using the results of [8], by a proper weight vector rescaling.

IV. SIMULATION AND REAL DATA PROCESSING RESULTS

In all our examples, the acronyms QR-dL and GS-dL stand for the proposed QR decomposition (without pivoting) based algorithm of Section III-A and the Gram-Schmidt orthogonalization (QR decomposition with pivoting) based technique of Section III-B, respectively, both using damped Lozano's (dL) local refinement step. The QR-dL and GS-dL techniques are compared with the dLLI technique of [5], the RCC2 algorithm with successive orthogonal refinement of [4] (referred to as RCC2-SOR), the SDR-based approach of [1], and the same SDR-based technique combined with the dL local refinement. The latter technique is referred to as SDR-dL. Plain QR without dL, GS without dL, and RCC2 without SOR are also included in two examples to illustrate the relative importance of initialization versus local refinement. The choice of the initial step-size μ and the stopping threshold in the dL technique were empirically optimized to achieve fast convergence and good performance. To optimize the parameters of the SDR-based approach, we have followed the guidelines of [1] where three different randomization procedures have been used in parallel, with 1000 randomizations for each. The number of iterations in the successive orthogonal refinement part of the RCC2-SOR technique was equal to the number of randomizations used in the SDR-based technique $(= 3000)$. For the QR-dL technique, $J = 200$ has been taken. This value of J corresponds to nearly equal computational complexities (measured in terms of MATLAB run times) of the SDR, SDR-dL and QR-dL methods. Note that the run time of the GS-dL technique is substantially smaller than that of the QR-dL, SDR and SDR-dL techniques.

A. Simulations

Throughout our simulations, a Rayleigh fading channel with i.i.d. circularly symmetric unit-variance channel coefficients is assumed. We

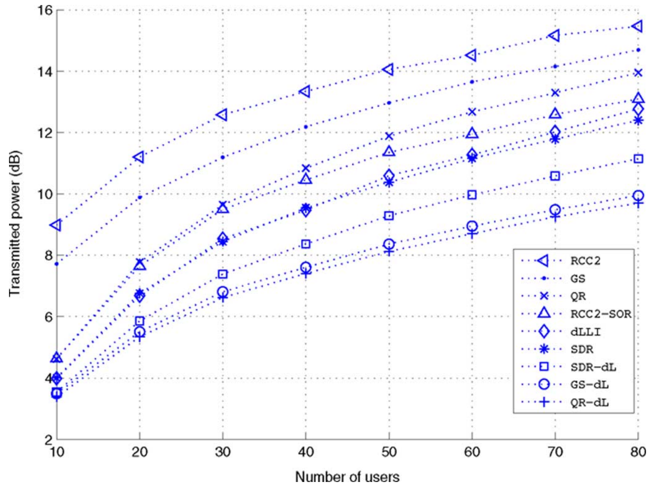

Fig. 1. Total transmitted power versus number of users; first simulation example.

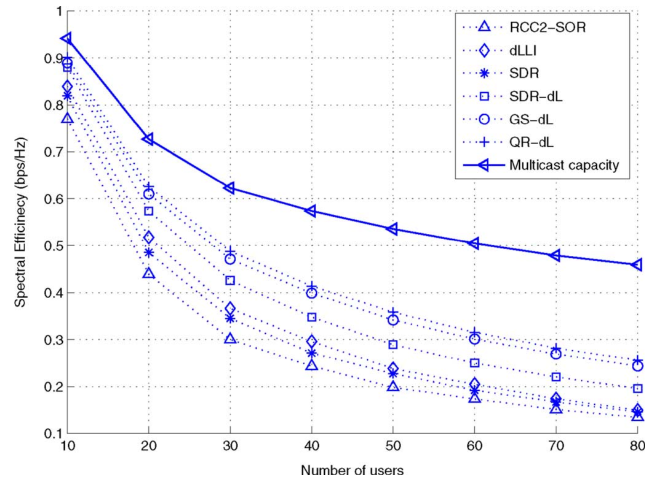

Fig. 2. Achievable multicast rates versus number of users; second simulation example.

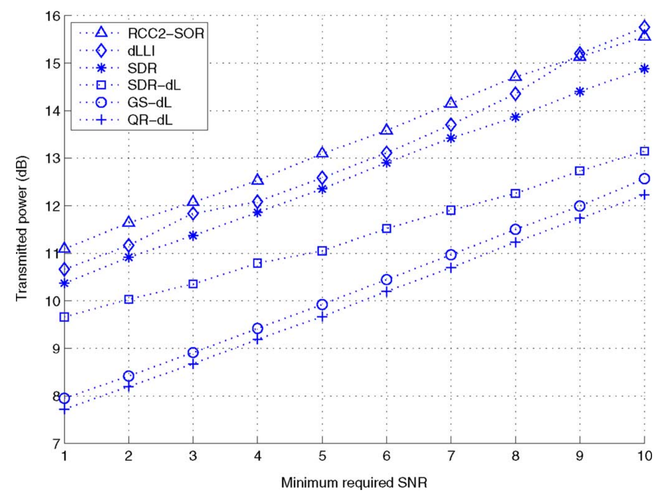

Fig. 3. Total transmitted power versus minimum required SNR; third simulation example.

also assume that $\sigma_i^2 = \sigma^2 = 1$ and $\gamma_{\min,i} = \gamma_{\min}$ for all $i = 1, \ldots, M$. All our results are averaged over 1000 Monte Carlo runs.

In the first example, we assume that $\gamma_{\min} = 5$ dB. Fig. 1 displays the average transmitted power required by the methods tested versus the number of users M. It can be clearly seen from the figure that the QR-dL and the GS-dL techniques perform substantially better than the SDR, SDR-dL, RCC2-SOR, and dLLI techniques. These performance improvements become more pronounced when increasing the value of M. For example, when $M = 80$ the improvement in terms of transmitted power is more than 1.2 dB as compared to the SDR-dL technique, and more than 2.5 dB as compared to the dLLI approach. To appreciate the difference it is instructive to "reverse the axes": for the same transmit power, QR-dL and GS-dL serve 80 users whereas SDR-dL serves 60, and dLLI and SDR about 45, on average. It can be observed from Fig. 1 that QR-dL is slightly better in performance than GS-dL. This is, however, compensated by a lower computational complexity of GS-dL with respect to QR-dL. Fig. 1 includes results for plain QR without dL, GS without dL, and RCC2 without SOR to illustrate the relative importance of initialization versus local refinement.

The complexity of RCC2-SOR mainly comes from the refinement step (SOR): the dominant complexity term is MNR, where R is the number of SOR iterations. The performance of RCC2-SOR quickly saturates as R increases (after about $R = 300$ in our experiments). When the parameters are chosen to approximately equalize the complexities of RCC2-SOR and GS-dL, the latter is better in terms of performance—e.g., by 3 dB for $M = 80$ users in the Rayleigh scenario of Fig. 1. But RCC2-SOR with moderate R and dLLI are both simpler than GS-dL for large M.

In our second example, we illustrate the achievable rates of the different beamformers for fixed transmit power. Fig. 2 shows these rates versus the number of users M. Also, the multicast capacity is shown in the figure as an upper bound on the achievable rate. According to [10] and [1], the achievable rate and multicast capacity can be computed as $R = \log_2(1 + \min_i(|\mathbf{h}_i^H \mathbf{w}_{\text{fin}}|^2/\sigma_i^2))$ and $C = \log_2(1 + \min_i(\mathbf{h}_i^H \mathbf{X}_{\text{opt}} \mathbf{h}_i/\sigma_i^2))$, respectively, where $\mathbf{X}_{\text{opt}}$ is the max-min optimal transmit covariance matrix, and both $\mathbf{X}_{\text{opt}}$ and $\mathbf{w}_{\text{fin}}$ are normalized to satisfy the transmit power constraint $\text{trace}\{\mathbf{X}_{\text{opt}}\} = \|\mathbf{w}_{\text{fin}}\|^2 = P$ ($P = 1$ for Fig. 2). Note that determining rate and capacity requires solving the power-constrained *max-min* problem, instead of (1); but the two are related via a simple scaling transformation, see [1].

It can be observed from this figure that the proposed QR-dL and GS-dL techniques have better achievable rates that the other multicasting algorithms tested. As before, QR-dL performs slightly better than GS-dL. Another interesting observation is that the multicast capacity can be seen as a relatively loose upper bound on the achievable rate. This can be explained by the fact that achieving capacity generally requires a higher rank transmit covariance matrix $\mathbf{X}_{\text{opt}}$. In the latter case, substantially more complicated encoding and decoding schemes have to be used instead of simple beamforming and single-input single-output encoding-decoding [1].

In our third example, we take $N = 4$, $M = 80$, and the minimum required SNR is varied. All the other parameters are the same as in the first example. Fig. 3 shows the transmitted powers versus the minimum required SNR. As in the first example, we can observe from this figure than the proposed GS-dL and QR-dL techniques perform better in terms of transmitted power than the other techniques tested. Also, as before, the QR-dL beamformer has a slightly better performance than the GS-dL one.

B. Measured Indoor Data

To further compare the performance of the proposed and existing multicasting methods, we used measured channel data collected in the 902–928 MHz ISM band by the iCORE HCDC Lab, University of Alberta in Edmonton [9]. The raw data and associated documentation files were downloaded from http://www.ece.ualberta.ca/~mimo/. Channel selection and preprocessing have been performed as detailed in [5]. The specific data set that we used here corresponds to the *indoor* scenario in [5], and it is available from the authors upon request.

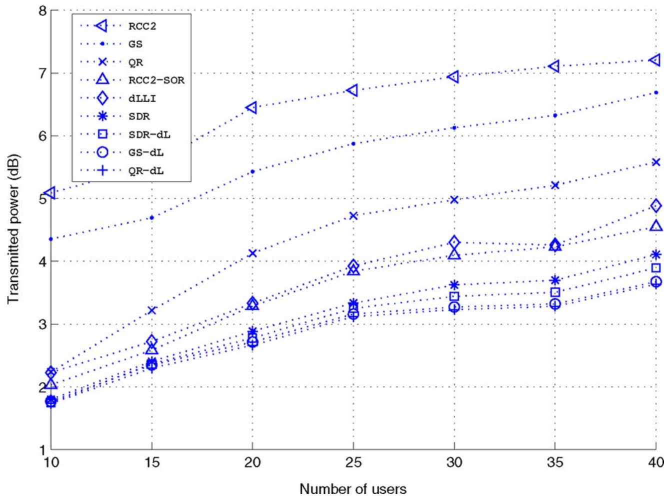

Fig. 4. Total transmitted power versus number of users; measured channel data.

There are $N = 4$ transmit antennas, and $M = 12$ user channels, measured every 3 seconds for a total of 30 temporal snapshots. In order to test with a large number of users, we randomly selected and concatenated 4 out of 30 snapshots (there are 27405 possible combinations), and averaged the results over 1000 such draws. Fig. 4 shows the transmitted power versus the number of users M. The required minimum SNR has been set to 0 dB.

It can be seen that in this figure, the QR-dL and GS-dL techniques have quite similar performance. Both of them outperform the other methods tested. These performance improvements become more significant with increasing M.

Summarizing, the results of our simulations and measured data processing clearly demonstrate an improved performance of the proposed QR-dL and GS-dL techniques with respect to the state-of-the-art multicasting methods such as the SDR, dLLI and RCC2-SOR algorithms. These improvements become especially pronounced in the large number of users case.

V. Conclusion

Two methods have been developed to approximately solve the problem of single-group multiple-antenna multicasting. The proposed techniques use channel orthogonalization and a subsequent local refinement algorithm to further improve the beamformer weight vector. Our methods have been shown via computer simulations and measured channel data processing to offer an improved performance in terms of power and spectral efficiency (and an attractive performance-to-complexity tradeoff) as compared to the current state-of-the-art multiple-antenna multicasting techniques.

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An Impulse Response Function for Evaluation of UWB SAR Imaging

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***Abstract*—Based on analysis of a point target imaged by different synthetic aperture radar (SAR) systems, the commonly used impulse response function in SAR Imaging (IRF-SAR)—a two-dimensional (2-D) sinc function—is shown to be inappropriate for ultrawideband-ultrawidebeam (UWB) SAR systems utilizing a large fractional signal bandwidth and a wide antenna beamwidth. As a consequence, the applications of the 2-D sinc function such as image quality measurements and spatial resolution estimations are limited to narrowband-narrowbeam (NB) SAR systems exploiting a small fractional signal bandwidth and a narrow antenna beamwidth. In this paper, a more general IRF-SAR, which aims at UWB SAR systems, is derived with an assumption of flat two-dimensional (2-D) Fourier transform (FT) of a SAR image and called IRF-USAR. However, the derived IRF-USAR is also valid for NB SAR systems.**

***Index Terms*—Impulse response function in SAR imaging (IRF-SAR), impulse response function in UWB SAR imaging (IRF-USAR), synthetic aperture radar (SAR), Sinc, ultrawideband-ultrawidebeam (UWB).**

I. Introduction

Synthetic aperture radar (SAR) is widely used as ground-imaging radars. SAR has its own surface illuminating capability that allows it to work in hazy weather (fog, rain, etc.) and even in the night. During such unfavorable weather condition, most other remote sensing sys-

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