Capacity of Ad-Hoc Networks with Node Cooperation

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Abstract— **We consider a cluster of closely-packed nodes that wish to communicate with another cluster of closely-packed nodes. The nodes within each cluster are separated by small distances, relative to the distance between the two clusters. We examine the effect of cooperation between nodes in the transmitting cluster, and/or cooperation between nodes in the receiving cluster. We find that cooperation within the transmitting cluster yields significant capacity improvements, while cooperation within the receiving cluster does not improve capacity significantly.**

I. Introduction

Sensor networks and ad-hoc networks are receiving more and more attention from the research community. In such networks, it is easy to envision a group of nodes that wish to communicate data to another distant group of nodes. For example, a group of nodes may sense a phenomenon and then wish to communicate their measurements to surrounding sensors which may also sense the phenomenon. Thus, it is feasible to consider a closely packed group of nodes that wish to transmit information to another group of nodes.

We consider a scenario where there are two independent transmitting nodes, and two independent receivers. Each transmitter wants to send a message to a different receiver. In information theory, this channel is classified as an interference channel [1, Ch. 14], and is one of the most fundamental open problems in multi-user information theory. We attack this problem from a different perspective and ask the following question: How much does allowing cooperation between the transmitters and/or cooperation between the receivers increase the set of achievable data rates? However, we do not allow this cooperation to occur for free and instead explicitly constrict cooperation to consist of transmitting messages between the two transmitters and/or transmitting messages between the two receivers. To capture the cost of cooperation, we place a sum power constraint on the total power transmitted in the system by all nodes.

The notion of cooperative communication has been considered in several recent works. Sendonaris et. al. [2] considered the rates achievable in a channel with two cooperative transmitters and a single receiver. Yazdi et. al. [3] is a more recent work on the same channel model. A channel with two cooperative transmitters (using low-complexity schemes such as amplify-and-forward) and two non-cooperative receivers was considered in terms of outage and diversity for fading channels (without transmitter channel state information) in [4]. Recent work by Host-Madsen [5] analyzed the same channel without fading, but with more complicated transmitter cooperation schemes involving dirty paper coding. The cooperative nature of these channels makes them closely related to the classical relay channel [6].

In this paper we consider the two transmitter, two receiver case from the capacity region perspective for the case of no fading, or slow fading with perfect channel state information at all transmitters and receivers. We are concerned solely with achievable rates, as opposed to outage and diversity as many of the works in this area have considered. For transmitter cooperation we use dirty paper coding, which has been shown to achieve the sum capacity of the multiple-antenna broadcast channel [7]. Our work differs from previous research in this area in that 1) we consider cooperation schemes that asymptotically (i.e. as the distance between nodes in a cluster decreases to zero) achieve the information theoretic upper bounds, yet are simple enough to facilitate numerical computation of the achievable rates and therefore give general insight about the underlying problem, and 2) we consider receiver cooperation in addition to transmitter cooperation, which, to the best of our knowledge, no previous work has considered in this setting.

For simplicity and to gain intuition, we consider the scenario where the channel between the two transmitters, the channel between the transmitters and the receivers, and the channel between the two receivers are orthogonal (i.e. on separate frequency bands or time slots). We are most interested in the scenario where the distance between the two transmitters is small and the distance between the two receivers is small relative to the distance between each transmitter-receiver pair. This allows high-rate communication between the two transmitters or between the two receivers using small amounts of power. We consider the rates achievable without cooperation versus rates achievable with transmitter-only cooperation, receiver-only cooperation, and transmitter and receiver cooperation. We compare these achievable rates to three different information theoretic upper bounds: 1) perfect transmitter cooperation (multiple-antenna broadcast channel [7–10]), 2) perfect receiver cooperation (multiple-antenna multiple-access channel [11]), and 3) perfect receiver cooperation and perfect transmitter cooperation (multiple-antenna point-to-point channel [11]).

The remainder of this paper is organized as follows: In Section II we describe the system model. In Sections V - VII we describe different cooperation schemes. In Section VIII we describe upper bounds to the rates achievable using cooperation. Finally, in Section IX we give some numerical results followed by a description of planned extensions of this work in Section X.

II. System Model

Consider a system with two transmitters and two receivers as shown in Fig. 1. We assume that the distance between each of the four transmitter-receiver pairs is the same, which is roughly true if the distance between the transmitter and receiver clusters is large. The channel gain amplitudes are normalized to one. Thus, the channels between each transmitter-receiver pair are the same, except for random phases, denoted by θ_i, which are

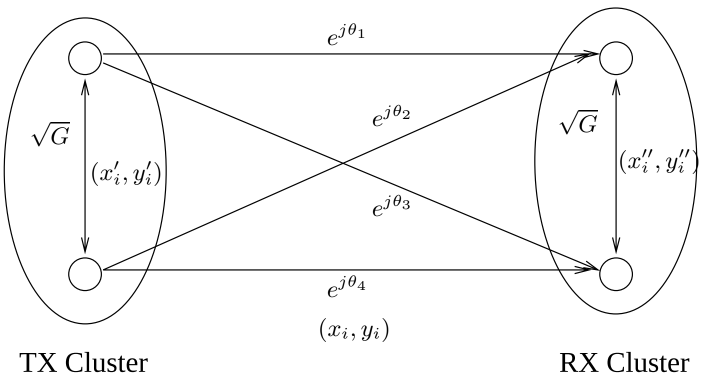

Fig. 1. System Model

assumed to be uniformly distributed in $[0, 2\pi]$.

There are three orthogonal communication channels: the channel between the transmitters and receivers, the channel between the two transmitters, and the channel between the two receivers. We first describe the channel connecting the transmitters and receivers. We let x_1 and x_2 denote the two transmit signals, and y_1 and y_2 denote the two corresponding received signals. Transmitter 1 wishes to communicate to receiver 1, and transmitter 2 wishes to communicate to receiver 2. In matrix form, the channel can be written as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{1}$$

where n_1 and n_2 are independent $N(0, 1)$ noises. As shown in Fig. 1, the channel gains $H_{i,j}$ are only phases: $H_{1,1} = e^{j\theta_1}$, $H_{1,2} = e^{j\theta_2}$, $H_{2,1} = e^{j\theta_3}$, and $H_{2,2} = e^{j\theta_4}$.

There is also an AWGN channel between the two transmitters, with channel gain equal to $\sqrt{G}$. If there is only distance based path-loss with $1/d^2$ attenuation, this corresponds to the scenario when the distance between the two clusters is $\sqrt{G}$ times larger than the distance between nodes in a cluster. For simplicity, we assume that the two transmitters can simultaneously transmit and receive on this channel[1] . We let x_1' denote the signal that transmitter 1 sends to transmitter 2, and we let y_2' denote the corresponding received signal at transmitter 1. The channels are then defined by $y_1' = \sqrt{G}x_2' + n_3$ and $y_2' = \sqrt{G}x_1' + n_4$, where n_3 and n_4 are independent unit-variance Gaussian noises. There is an analogous AWGN channel between the two receivers, also with channel gain equal to $\sqrt{G}$. If we let x_1'' denote the signal that receiver 1 transmits to receiver 2 on this channel, and we let y_2'' denote the corresponding received signal at receiver 2, then this channel is defined by $y_1'' = \sqrt{G}x_2'' + n_5$ and $y_2'' = \sqrt{G}x_1'' + n_6$, where n_5 and n_6 are independent unit-variance Gaussian noises. Here x_1'' and x_2'' are constrained to be functions of the previously received signals y_1 and y_2, respectively.

We assume that transmitter 1 has a message intended for receiver 1, and transmitter 2 has a message intended for receiver 2. We impose a total system power constraint of P on the total transmit energy, i.e. we require

$$E[x_1^2 + x_2^2 + x_1'^2 + x_2'^2 + x_1''^2 + x_2''^2] \leq P.$$

This power constraint is intended to capture the system-wide cost of transmitter and receiver cooperation. We assume that each of the three channels has a bandwidth of 1 Hz, and we let $N_0 = 1$. Though we work with the simplifying assumptions of equal amplitude channel gains, results generalize to the case of arbitrary channel gains.

III. Broadcast and Multiple-Access Channel Background

Throughout this work we discuss the broadcast and multiple-access channels implicitly contained in the two transmitter/two receiver channel. If the receivers are assumed to cooperate perfectly, the channel becomes a multiple-access channel (MAC) with two single-antenna transmitters and a two-antenna receiver. The channels of the transmitters are given by the *columns* of the matrix $\mathbf{H}$. In terms of Fig. 1, this corresponds to communicating from the left cluster to the right cluster, with perfectly cooperative nodes in the right cluster (RX cluster). If the transmitters are assumed to cooperate perfectly, the channel becomes a broadcast channel with two single-antenna receivers and a two-antenna transmitter. The channels of the two receivers are equal to the *rows* of the matrix $\mathbf{H}$. In terms of Fig. 1, this corresponds to communicating from the left cluster to the right cluster, with perfectly cooperative nodes in the left cluster (TX cluster).

In [8], it is shown that the broadcast channel is closely related to the *dual* multiple-access channel, which is the MAC where the two nodes in the receiving cluster are the single-antenna independent *transmitters* and the cooperative nodes in the transmitter cluster are the two-antenna *receiver*. In terms of Fig. 1, this corresponds to communicating from the right cluster to the left cluster (opposite the normal direction of communication), with perfectly cooperative nodes in the left cluster. The channels of the two transmitters are the transposes of the channels of the two receivers in the broadcast channel. Thus, the transmitter channels correspond to the transpose of the *rows* of $\mathbf{H}$. It is important to note that the MAC corresponding to perfect receiver cooperation is different from the dual MAC. However, in Section VI we show that the capacity regions of these multiple-access channels are the same.

IV. Non-Cooperative Transmission

Without cooperation on either the transmitter or receiver side, the channel is a Gaussian interference channel, for which the capacity region is in general not known. However, the channel we consider is a "strong" interference channel[2], for which the capacity region is known [12]. For this class of interference channels, the strong interference channel implies that each receiver can decode the transmitted messages of *both* transmitters. Thus, the capacity region is upper bounded by each receiver's multiple-access channel, and this bound is in fact tight. If transmitter 1 uses power P_1 and transmitter 2 uses power

[1]Though this is not practical, this is a common theoretical assumption. In future work we will investigate the effect of not allowing simultaneous transmit and receive on a single channel.

[2]A strong interference channel refers to the situation where the channel gain of the interference is as large as the channel gain of the desired signal.

$P_2 = P - P_1$, the multiple-access region is given by the pentagon described by $R_1 \leq \log(1 + P_1)$, $R_2 \leq \log(1 + P_2)$, and $R_1 + R_2 \leq \log(1 + P)$. Since there is a sum power constraint on the transmitters instead of individual power constraints, the non-cooperative capacity region is equal to the set of rates satisfying $R_1 + R_2 \leq \log(1 + P)$. It is easy to see that this set of rates is also achievable using TDMA. We will thus refer to the TDMA rate as a non-cooperative benchmark to compare our cooperative schemes against.

V. Transmitter Cooperation

In this section, we describe a transmitter cooperation scheme. If the transmitters were allowed to jointly encode their messages, the channel would be a multiple-antenna broadcast channel. For such a channel, the sum capacity can be achieved by using dirty paper coding [8]. Motivated by this, we consider a strategy where the two transmitters first exchange their intended messages (or codewords, since each transmitter is assumed to know the other transmitter's codebook) using some fraction of the total power P, and then *jointly* encode both messages using dirty paper coding (i.e. encode as if they were a joint transmitter) with the remaining power. Causality is not a problem for any of our cooperative schemes since we consider orthogonal channels for cooperation and we can offset communication by one block initially.

Assume power $\frac{P_t}{2}$ is used by each transmitter to send his intended message to the other transmitter. Then the intra-transmitter rate is equal to $R_t = \log(1 + \frac{P_t}{2}G)$. The remaining power $P - P_t$ is used to jointly encode using dirty-paper coding. We require that R_t is high enough to ensure that each transmitter *fully* knows the intended codeword of the other transmitter (i.e. R_t must be as large as the rate of the message of each user). Since each transmitter knows both messages after this exchange, each user can then perform standard dirty paper coding as if the two antennas were actually cooperative, but then only send the information on one of the two antennas. The sum rate achievable using joint dirty paper coding is equal to the sum-rate capacity of the dual multiple-access channel [8] with power $P - P_t$. Since each element of the channel matrix $\mathbf{H}$ has amplitude one, this is equal to:

$$R_{DPC} = \log \left| I + \frac{P - P_t}{2} \left(H_1^T H_1 + H_2^T H_2 \right) \right| \qquad (2)$$

where $H_i = [H_{i,1} \; H_{i,2}]$ is the row vector representing the received channel of Receiver i. For a given P_t, the achievable sum rate is $\min(2R_t, R_{DPC})$. Since R_t is an increasing function of P_t and R_{DPC} is a decreasing function of P_t, the optimum is achieved at the P_t for which $R_t = \frac{1}{2} R_{DPC}$.

VI. Receiver Cooperation

In this section, we describe a method which allows the receivers to cooperate. Since the channels of each of the signals are equivalent except for the phase differences, the amount of information decodable at each of the receivers is the same (assuming that the transmitters send independent messages). Thus, there is no advantage gained if a receiver attempts to first decode the message intended for the other receiver and then pass it on to the other receiver. With perfect receiver cooperation, receiver 1 would get to see the received signal y_2 in addition to its own signal y_1. Thus, a logical method for cooperation is for each receiver to amplify-and-forward their received signal to the other receiver, which always results in some noise amplification.

Each receiver uses the fraction of power $\frac{P_r}{2}$ to amplify-and-forward its received signal to the other receiver. Since the transmitters do not cooperate in this mode, the signals x_1 and x_2 are independent and are chosen to be $N(0, \frac{P-P_r}{2})$. The expected received power at y_1 is given by $E[y_1^2] = E[x_1^2] + E[x_2^2] + E[n_1^2] = P - P_t + 1$. Thus, receiver 1 transmits

$$\sqrt{\frac{P_r/2}{P - P_r + 1}} y_1 = \sqrt{\frac{P_r/2}{P - P_r + 1}} (H_{1,1}x_1 + H_{2,1}x_2 + n_1)$$

The corresponding received signal at receiver 2 is given by $\sqrt{G \frac{P_r/2}{P - P_t + 1}} (h_{1,1}x_1 + h_{2,1}x_2 + n_1) + n$, where $n \sim N(0, 1)$. The aggregate signal at receiver 1 is then given by:

$$\tilde{y_1} = \begin{bmatrix} H_1 \\ \alpha H_2 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (3)$$

$$= F_1 x_1 + F_2 x_2 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (4)$$

where $\alpha = \frac{\sqrt{G \frac{P_r/2}{P - P_r + 1}}}{1 + \sqrt{G \frac{P_r/2}{P - P_r + 1}}}$, $F_1 = \begin{bmatrix} e^{j\theta_1} \\ \alpha e^{j\theta_3} \end{bmatrix}$, and $F_2 = \begin{bmatrix} e^{j\theta_2} \\ \alpha e^{j\theta_4} \end{bmatrix}$. Notice that $\tilde{y_1}$ differs from the pair (y_1, y_2) only due to the α factor, which is caused by noise amplification. By symmetry, the sum rate decodable at each receiver (using aggregate signals $\tilde{y_1}$ and $\tilde{y_2}$) are the same. The sum rate decodable at receiver 1 is given by:

$$R_{coop} = \log \left| I + \frac{P - P_r}{2} \left(F_1 F_1^T + F_2 F_2^T \right) \right| \qquad (5)$$

Since α is a function of P_r, this expression must be maximized over P_r to find the largest achievable rate. When the power gain G is very large (i.e. when the receivers are very close to each other), we get $\alpha \approx 1$ and we expect to come close to the MAC (fully cooperative receivers) upper bound.

Notice that the expression for the rate given in (5) is quite similar in form to the rate achievable using only transmitter cooperation. Though the expressions are not the same, it can be shown that the rates achievable using transmitter cooperation and using receiver cooperation are closely related:

Lemma 1: The transmitter cooperation scheme described in Section V achieves a rate at least as large as the amplify-and-forward receiver cooperation scheme.

Proof: Consider the expression for RX-only cooperation given in (5). For a fixed P_r and any $\alpha < 1$, the rate given in (5) is less than the expression in (5) evaluated with $\alpha = 1$ because if α were equal to 1, the receiver could scale the received signal on the second antenna by α and add Gaussian noise to get a signal statistically equivalent to the actual received signal when using the receiver cooperation scheme. Furthermore, it can be shown by direct computation that the expression for receiver cooperation rate with $\alpha = 1$ is equal to the achievable rate using transmitter cooperation given in (2) with $P_t = P_r$. ■

VII. Transmitter Cooperation & Receiver Cooperation

In this section we describe a scheme in which the two transmitters cooperate by exchanging their intended message and then cooperatively signal using dirty paper coding, and the two receivers cooperate by amplifying-and-forwarding. We let P_t denote the power used to exchange messages between the transmitter. The corresponding rate is $R_t = \log(1+\frac{P_t}{2})$. We again require that each transmitter completely knows the intended message of the other transmitter. Once the transmitters exchange messages, we encode using dirty paper coding (similar to the transmitter cooperation). However, in this case, each user has two receive antennas, where the second antenna is the signal received via the amplify-and-forward channel from the other receiver. Power P_r is used to perform amplify-and-forward between the two receivers. This leaves power $P - P_t - P_r$ to jointly transmit data using dirty paper coding.

Because cooperative dirty paper encoding is performed at the two transmitters, x_1 and x_2 are correlated with covariance matrix Σ_x. The expected received power at y_1 is then equal to $1 + H_2 \Sigma_x H_2^T$. As in the case with only amplify-and-forward, the resultant composite signal at receiver 1 is given by

$$\tilde{y_1} = \begin{bmatrix} H_1 \\ \beta H_2 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{6}$$

where $\beta = \frac{\sqrt{G \frac{P_r/2}{H_2 \Sigma_x H_2^T + 1}}}{1+\sqrt{G \frac{P_r/2}{H_2 \Sigma_x H_2^T + 1}}}$, For fixed β and P_r and P_t, the sum rate achievable from the cooperative transmitters to the receivers (with composite channels $\tilde{y_1}$ and $\tilde{y_2}$) is equal to the sum capacity of the dual multiple-access channel. In the dual multiple-access channel, the composite receivers are the two-antenna transmitters and the cooperative transmitters become the two-antenna receiver. Since each transmitter has two antennas, we are not able to invoke symmetry to find the sum capacity of the dual multiple-access channel as before. Thus, the sum capacity must be characterized in terms of a maximization:

$$R_{coop} = \max_{Tr(Q_1+Q_2) \le P} \log \left| I + \tilde{H_1}^T Q_1 \tilde{H_1} + \tilde{H_2}^T Q_2 \tilde{H_2} \right|$$

where the maximization is over covariance matrices Q_1 and Q_2, with $\tilde{H_1} \triangleq \begin{bmatrix} H_1 \\ \beta H_2 \end{bmatrix}$, and $\tilde{H_2} \triangleq \begin{bmatrix} \beta H_1 \\ H_2 \end{bmatrix}$.

The maximizing covariances can be found using convex optimization techniques. Given Q_1 and Q_2, the sum rate achieving covariance matrix for the downlink (i.e. Σ_x) can be found[3] [8].

For fixed P_t and P_r, the achievable sum rate is $\min(2R_t, R_{coop})$. By the same reasoning used for transmitter-only cooperation, for a fixed P_r, the optimal choice of P_t yields $2R_t = R_{coop}$. However, it is necessary to directly maximize the achievable rates over all choices of P_r. When $P_r = 0$ this strategy is identical to the transmitter-only cooperation scheme, and thus this scheme performs at least as well as the transmitter cooperation scheme. Since transmitter cooperation yields higher rates than receiver cooperation, there is a full ordering on the achievable rates of the three different schemes for any channel.

Finally notice that as G becomes very large, the scaling term β can be made close to one. Thus in the limit (i.e. $\beta \to 1$), the composite channels of both receivers become equal to $[y_1\ y_2]^T$. Since both received channels are the same, the broadcast channel capacity region is equal to the point-to-point capacity from the cooperative transmitter to either of the receivers, i.e. the point-to-point MIMO capacity of the original channel.

VIII. Upper Bounds

There are three information theoretic upper bounds that bound the rates achievable with transmitter cooperation only, receiver cooperation only, and transmitter and receiver cooperation. Note that for all three bounds, we only use a bandwidth of 1 Hz, i.e. the channel set aside for communication between the two clusters, and thus and are not using the two channels set aside for cooperation (this point is discussed in some more detail in Section X).

First, consider the scenario where only the transmitters attempt to cooperate. The capacity of the channel where the transmitters are allowed to perfectly cooperate (without use of any power), but the receivers are not allowed to cooperate is an upper bound to the rates achievable using only transmitter cooperation. This is not a general upper bound on our system, but is a bound when only transmitter side cooperation is allowed. Since the receivers must decode their messages independently, the channel becomes a two transmit antenna, two receiver (single receive antenna each) broadcast channel with transmit power constraint P. The sum capacity of this channel is known [7–10], but the full capacity region is not known. However, an achievable region (referred to as the "dirty-paper region") for this channel is known. The sum capacity of the broadcast channel is equal to the sum capacity of the dual multiple-access channel, given by:

$$R_{DPC} = \log \left| I + \frac{P}{2} \left(H_1^T H_1 + H_2^T H_2 \right) \right| \tag{7}$$

due to the symmetry of the channel.

Next consider the scenario where only the receivers attempt to cooperate. In this scenario, an upper bound is reached by allowing the receivers to perfectly cooperate. The channel then becomes a two transmitter (single antenna each), two receive antenna multiple-access channel, for which the capacity region is known. Due to the symmetry of the channels, the sum capacity of the multiple-access channel is given by:

$$R_{coop} = \log \left| I + \frac{P - P_r}{2} \left(F_1 F_1^T + F_2 F_2^T \right) \right| \tag{8}$$

where $F_1 = \begin{bmatrix} e^{j\theta_1} \\ e^{j\theta_3} \end{bmatrix}$ and $F_2 = \begin{bmatrix} e^{j\theta_2} \\ e^{j\theta_4} \end{bmatrix}$. As noted in Lemma 1, it can be shown by direct computation that equations (7) and (8) are equal. Furthermore, the dirty paper achievable region corresponding to transmitter-only cooperation is equal to the multiple-access capacity region which bounds receiver-only cooperation.

[3] The sum rate was maximized assuming a fixed value of β, but interestingly, the choice of Σ_x in fact determines the value of β. Thus, we initially assume that Σ_x is a scaled version of the identity when determining β. We then maximize the sum capacity of the broadcast channel assuming this β. After finding the corresponding Σ_x, we re-calculate the value of β. This procedure can be repeated, but we empirically found this to yield a negligible increase in rate.

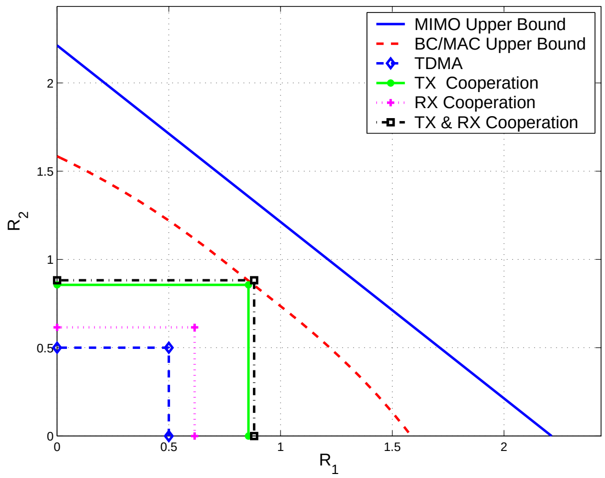

Fig. 2. Upper bounds and achievable rates for SNR = 0 dB

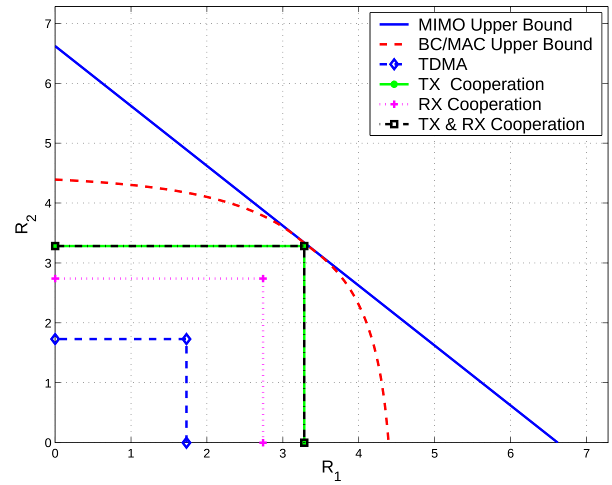

Fig. 3. Upper bounds and achievable rates for SNR = 10 dB

A true upper bound to our system is reached by allowing perfect cooperation at the transmitters *and* at the receivers. The channel then becomes a 2×2 MIMO channel, whose capacity is given by water-filling the eigenvalues of the channel matrix $\mathbf{H}$ [11]. Interestingly, Theorem 3 of [7] shows that the difference between the MIMO point-to-point capacity and the sum capacity of the BC goes to zero as the SNR P goes to infinity. Thus, at high SNR we expect cooperation at either the TX or at the RX cluster to be sufficient to come close to the MIMO upper bound.

IX. Numerical Results

In Fig. 2, the upper bounds and achievable rates are plotted for a random channel chosen with an SNR of 0 dB and for $G = 100$. If we assume a path-loss exponent of 2, this corresponds to a physical scenario where the distance between the nodes in the clusters is 10 times less than the distance between the two clusters. The rates achievable with TX cooperation and with TX & RX cooperation are virtually identical, and both come extremely close to the broadcast channel upper bound. There, is however, a sizable gap between the BC/MAC upper bound and the MIMO upper bound. The TX & RX cooperation scheme will approach the MIMO upper bound, but for larger values of G. The rates achievable with RX cooperation do exceed the non-cooperative rates achievable with TDMA, but they are considerably smaller than the TX cooperation rates.

In Fig. 3, the bounds and rates are plotted for a channel with 10 dB and $G = 100$. As expected, the gap between the MIMO upper bound and the BC/MAC upper bound becomes much smaller. We again see that the TX cooperation scheme and the TX & RX cooperation schemes come quite close to the capacity upper bound, but the RX cooperation scheme performs quite poorly.

In Fig. 4, the average achievable sum rates using the different cooperation schemes are plotted versus G for an average SNR of 0 dB. To compute these results, a large sample of channels were instantiated (i.e. different random phases) and the achievable rates were calculated for different values of G, and then an average was taken over the instantiations. Notice that the three upper bounds are independent of G because they assume perfect cooperation. Since the SNR is only 0 dB, there is a significant gap between the MIMO upper bound and the MAC/BC upper bound. As G increases (i.e. as the nodes within each cluster move closer to each other), the achievable rates approach the upper bounds. As discussed before, TX & RX cooperation always performs better than TX cooperation, which always outperforms RX cooperation. However, it is most interesting to note that TX & RX cooperation and TX cooperation are virtually identical for $G \leq 20$ dB. Upon closer examination, one finds that the optimum TX & RX scheme for such values of G is achieved by only using transmitter cooperation, i.e. not having the receivers use any power for amplify-and-forward. For $G > 20$ dB, a gap does open up between the TX & RX scheme and the TX scheme. Interestingly, this gap appears at the point where the TX cooperation scheme achieves the the BC upper bound. Thus, up to the BC upper bound it seems that is not worthwhile to do both TX & RX cooperation, but beyond this point (i.e. for larger values of G) it becomes worthwhile to cooperate in both clusters.

In Fig. 5 the same plot is provided for an SNR of 10 dB. The same general trends are noticed in this graph, but notice that the difference between the MIMO upper bound and the BC/MAC upper bound is quite small. Again, the TX/RX scheme is virtually identical to the TX cooperation until the point where the TX cooperation scheme comes very close to the BC upper bound. Beyond this point, the TX/RX scheme outperforms the TX scheme and approaches the MIMO upper bound.

In Figures 4 and 5, there is a significant gap between the rates achievable using TDMA and the rates achievable using TX cooperation, even at relatively small values of G (i.e. 10 dB). Thus, there is in fact a significant advantage to performing cooperation in either or both of the clusters. Another general trend seen in all plots is the poor performance of the RX cooperation scheme relative to the TX cooperation scheme. For all but very small values of G, the RX cooperation scheme performs much poorer than the TX cooperation scheme. Transmitter cooperation allows for joint encoding (similar to coherent combining) of the two messages, while receiver cooperation only provides an additional scaled antenna output, where the scaling is proportional to G. For large enough G, however, the simply amplify-

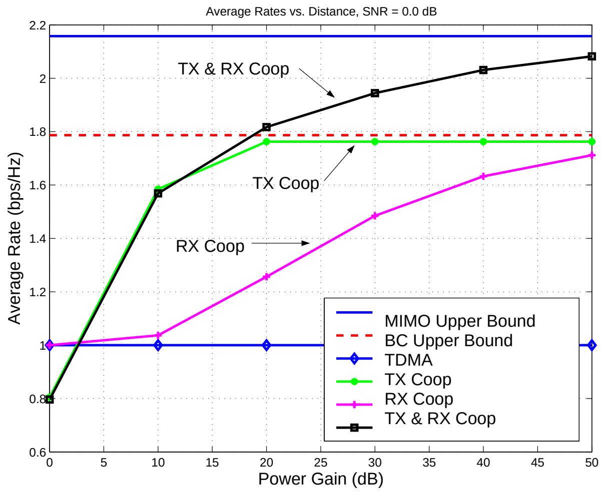

Fig. 4. Plot of rate vs. gain for SNR = 0 dB

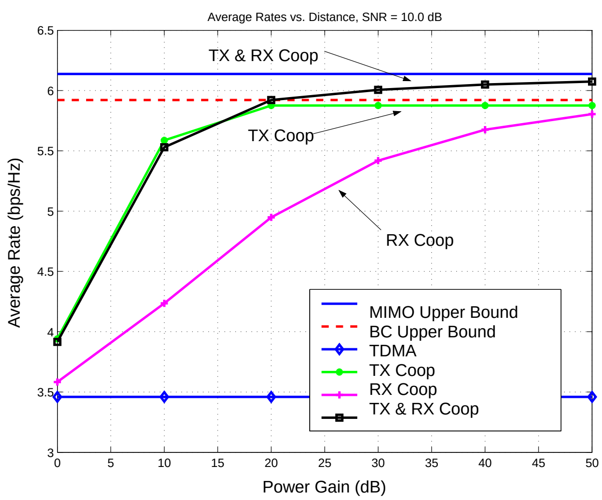

Fig. 5. Plot of rate vs. gain for SNR = 10 dB

and-forward operation performed at the receivers is sufficient to achieve the MAC/BC upper bound.

X. Extensions

We anticipate the final paper will include the following extensions:

- More than 2 users: We plan on considering clusters with more than 2 users. In this scenario the channels used for cooperation on both the transmit and receive side will each be multiple-access channels. The potential gains for cooperation appear to be much larger, but it also seems to be more difficult to cooperate with more than two users.
- Bandwidth/Time Tradeoff: So far we have assumed that separate frequency bands of equal bandwidth were allocated for the three modes of communication. This is quite restrictive, and it may be more useful to allocate unequal bandwidth/time to different channels. This will also affect the upper bounds, since the upper bounds will use the entire system bandwidth (i.e. 3 Hz in our model) for TX-RX communication.
- Relationship to Relay Channel: We have only considered the scenario where the three different modes of communication occur in orthogonal channels. This is in not necessarily a requirement of such a system, and we could allow all communication to occur simultaneously in frequency and time.
- Transmission of information to both receivers: We have considered the situation where each transmitter wishes to communicate with only one of the two receivers. In a more general setting, each transmitter may wish to communicate different information to the two receivers, or may wish to communicate the same information to both receivers.
- Different transmitter and receiver cooperation schemes: We have considered a single scheme for each of the three modes of cooperation. There are, however, many other ways of allowing the nodes to cooperate. For example, the transmitters can cooperate by using an amplify-and-forward technique. On the receiver side, it may also be possible to do some partial decoding of the received message and conveying this information to the other receiver. This strategy appears to be more useful in a less symmetric channel, i.e. when one receiver has a much larger channel gain than the other receiver.

XI. Conclusions

In this work we have quantified the benefits of transmitter and/or receiver cooperation in sensor/ad-hoc network-type settings. We found that transmitter cooperation or transmitter and receiver cooperation can lead to significant performance improvements in terms of increased data rates. On the other hand, receiver cooperation without transmitter cooperation does not appear to be very beneficial. Though the model we have worked with in this paper is quite simple, this appears to only be the beginning of a promising line of research examining the benefits of node cooperation.

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