Coordinated Beamforming with Limited Feedback in the MIMO Broadcast Channel

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***Abstract*—In this paper, we propose a new joint optimization of linear transmit beamforming and receive combining vectors for the multiple-input multiple-output (MIMO) broadcast channel. We consider the transmission of a single information stream to two users with two or more receive antennas. Unlike past work in which iterative computation is required to design the beamformers, we derive specific formulations for the transmit beamformers for two active users via a power iteration and a generalized eigen analysis. To enable practical implementation, a new limited feedback algorithm is proposed that exploits the structure of the algorithm to avoid full channel quantization. The feedback overhead of the proposed algorithm is independent of the number of receive antennas. Monte Carlo simulations are used to evaluate the bit error rate and the sum rate performances of the proposed algorithm. Simulation results show that the proposed method performs close to the sum capacity of the MIMO broadcast channel even with limited feedback.**

***Index Terms*—MIMO systems, broadcast channels, interference suppression.**

I. INTRODUCTION

THE CAPACITY region for the multiple-input multiple-output (MIMO) broadcast channel has recently been established [1], [2]. These results promise large spectral efficiencies, even in networks with single antenna receivers. The MIMO broadcast channel achieves high capacity on the downlink by coordinating the transmissions to multiple users. It is well known that dirty paper coding (DPC) achieves the capacity region for the MIMO broadcast channel [2]. Practical near-capacity dirty paper codes, however, do not yet exist and thus DPC is difficult to implement in practice [1]–[3]. Several search-based nonlinear precoding techniques have been proposed to enhance link quality and to approach the sum capacity, but these methods require high complexity at the base station (BS) [4]–[7].

There has been considerable interest in linear beamforming techniques that avoid the non-linear DPC-like processing [8]–[13]. Linear processing solutions, such as zero-forcing beamforming (ZFBF) or channel inversion at the transmitter, are easier to implement [14], [15]. ZFBF in [14], [15], however, was proposed for only one receive antenna per user, i.e., the number of transmit antennas must be greater than or equal to the total number of receive antennas in the network (the dimensionality constraint). Further, this technique suffers from a power enhancement. A related strategy is block diagonalization (BD) [8], [9] that is applicable for situations with multiple antennas and multiple data streams intended for each user. BD enforces a zero interference property at each user but requires that the number of receive antennas is equal to the number of data streams. It is possible to improve ZFBF and BD through transmit antenna selection or eigenmode selection [16] when additional transmit antennas are available, or through receive antenna selection [12] when extra receive antennas are available. In both cases, though, the transmitter and the receivers are not jointly optimized.

Coordinated beamforming algorithms work similarly to BD but allow fewer streams than the number of receive antennas by jointly optimizing the transmit beamforming and receive combining vectors [10]–[12], [17]. These approaches perform close to the sum capacity but require an iterative computation for the transmit beamformers and the receive combining vectors. Moreover, the convergence of these iterative algorithms in [10]–[12], [17] cannot be guaranteed. The authors in [18] proposed a coordinated interference-aware beamforming technique for the MIMO broadcast channel. This technique also has the disadvantage that each user is required to know the channel information and noise variance of other users to estimate the received symbols. The author in [19] proposed a generalized zero-forcing optimized beamforming solution. The solution, however, was only valid for two transmit antenna systems.[1]

Previous work on linear beamforming [8], [10]–[13], [18], [19] has assumed perfect CSI at the transmitter. One way to achieve this is limited feedback from the receiver to the transmitter. The impact of limited feedback on the performance of multiuser MIMO channels has been presented in [20]–[23]. In [20], the performance degradation due to quantized channel information was analyzed for ZFBF when a single antenna is employed at the receiver. In [21], [22], the authors combined linear beamforming with a user selection algorithm to improve performance via multiuser diversity. These strategies were also valid only for a single receive antenna. More recently, [23] proposed antenna combining techniques using multiple receive antennas at each user under the assumption that the number of receive antennas are less than the number of transmit antennas.

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[1]We will show that the solution in [19] is a subset of the proposed solution in Section V.

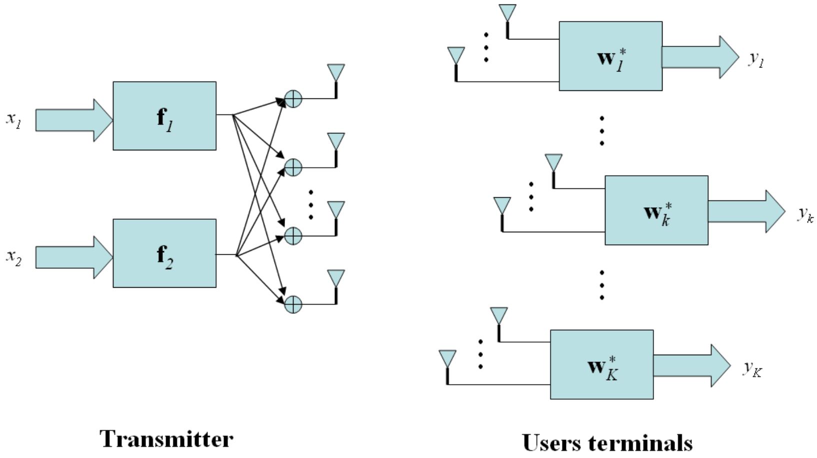

Fig. 1. MIMO broadcast channel model.

To the best of our knowledge, there is no linear multiuser MIMO strategy, based on limited feedback, proposed in the literature that does not have the dimensionality constraint on the antenna configuration.

In this paper, we propose non-iterative simple linear multiuser MIMO techniques where both transmitter and receiver are equipped with multiple antennas. We assume that two users are served through multiple antennas. The presence of control channel overhead in practical systems makes it reasonable to consider two users for simultaneous transmission. A scenario of more than two users will be considered in our future work. The main contributions of this paper are as follows.

- *Convergence of iterative coordinated beamforming*: We investigate the impact of the initial transmit beamformer for the iterative coordinated beamforming on the convergence of the algorithm. We discuss, using a power iteration, why the iterative coordinated beamforming algorithms in [10]–[13] sometimes take a very long time to converge or, at times, do not converge. We show that this convergence issue can be avoided using the proposed non-iterative coordinated beamforming algorithm.
- *Non-iterative algorithm for transmit beamformers*: We propose a new non-iterative solution for the transmit beamformers and the receive combining vectors used in coordinated beamforming where two users are served with two or more transmit antennas at the transmitter [10]–[13]. In this paper, we show that the beamformer computed through a generalized eigen analysis is the sufficient and necessary solution for removing inter-user interference for two transmit antenna systems. Further, we also show that this solution is also valid for more than two transmit antenna systems, i.e., no inter-user interference with more than two transmit antennas. The proposed solution avoids the iterative solutions of prior work [10]–[13]. Simulation results show that the proposed solution is better than the iterative coordinated beamforming even with 50 iterations.
- *Limited feedback using uniform/non-uniform channel quantization*: To enable practical implementation, we propose a limited feedback solution that requires only quantized CSI from each user to be sent to the transmitter. By taking advantage of the structure of the receive combining operation, our limited feedback solution requires quantizing the entries of symmetric Hermitian matrices derived from the channel. Unlike prior work, it does not use Grassmannian codebooks or random vector quantization [20]–[22], [24] since the structure of our problem is quite different. Compared to the work in [20], [23], our approach does not require the number of receive antennas to be less than the number of transmit antennas. Another advantage of the proposed limited feedback method is that the feedback overhead of the proposed limited feedback method is independent of the number of receive antennas. Note that the proposed method quantizes the normalized channel magnitude as well as the direction while the algorithms in [20], [23] only quantize the channel shape. Through Monte Carlo simulations, we show that the proposed method performs close to the sum capacity of the MIMO broadcast channel with only limited feedback.

This paper is organized as follows. In Section II, we introduce the system model for the linear multiuser MIMO systems, specialized to the case of one data stream for each user. In Section III, we discuss a low complexity iterative algorithm and present the proposed non-iterative beamforming algorithms, followed by a limited feedback method in Section IV. Performance evaluation and conclusion are given in Sections V and VI, respectively. Finally, we present the derivations for the converged transmit beamformers in the Appendix.[2]

II. System Model

Consider a multiuser MIMO system with N_t antennas at the transmitter and N_r receive antennas for each of K users as shown in Fig. 1. We assume that the channel is flat fading, which can be obtained in practice using multiple-input multiple-output orthogonal frequency division multiplexing

[2]Upper case and lower case boldfaces are used to denote matrices $\boldsymbol{A}$ and vectors $\boldsymbol{a}$, respectively. If $\boldsymbol{A}$ denotes a complex matrix, and $\boldsymbol{A}^T$, $\boldsymbol{A}^*$, $\boldsymbol{A}^{-1}$, and $\boldsymbol{A}^\dagger$ denote the transpose, conjugate transpose, inverse, and pseudo inverse of $\boldsymbol{A}$, respectively. $[\boldsymbol{A}]_k$ denotes the k-th column of matrix $\boldsymbol{A}$. $\|\boldsymbol{A}\|_F$ denotes the Frobenius norm of matrix $\boldsymbol{A}$. adj$(\boldsymbol{A})$ denotes the adjoint matrix of $\boldsymbol{A}$. $\mathbb{E}$ denotes expectation.

(MIMO-OFDM). For simulation and limited feedback codebook design purposes, we model the elements of each user's channel matrix as independent complex Gaussian random variables with zero mean and unit variance. The channel between the transmitter and the k-th user is represented by an $N_r \times N_t$ matrix $\boldsymbol{H}_k$. Let x_k denote the transmit symbol for the k-th user, and $\boldsymbol{n}_k$ be the additive white Gaussian noise vector of size $N_r \times 1$ observed at the receiver. Let $\boldsymbol{f}_k$ denote the unit-norm transmit beamformer and $\boldsymbol{w}_k$ denote the unit-norm receive combining vector for the k-th user. Then, the signal at the k-th user after receiver combining is given by

$$y_k = \boldsymbol{w}_k^* \boldsymbol{H}_k \boldsymbol{f}_k x_k + \boldsymbol{w}_k^* \boldsymbol{H}_k \sum_{l=1, l \neq k}^{K} \boldsymbol{f}_l x_l + \boldsymbol{w}_k^* \boldsymbol{n}_k. \quad (1)$$

Using the coordinated transmission strategies [10]–[12], the transmitter chooses the transmit beamforming and receive combining vectors such that zero multiuser interference is experienced at each receiver. This implies that the transmit beamforming vector is chosen in the null space of $\boldsymbol{w}_l^* \boldsymbol{H}_l$ $(\forall l \neq k)$, that is $\boldsymbol{w}_k^* \boldsymbol{H}_k \boldsymbol{f}_l = 0$ where $k = 1, \ldots, l-1, l+1, \ldots, K$. If chosen in this way, $\boldsymbol{f}_k$ will then cause zero interference to user l by completely removing the interference term in (1).

We restrict ourselves to one stream per user though it is possible to send more than one stream per user. Our results show that the throughput achieved is quite close to the sum capacity, so very little is lost by this restriction. In addition, this limitation is acceptable in real systems, where spatial division multiplexing access (SDMA) is used in conjunction with single user MIMO transmissions using adaptive switching [25]. While SDMA targets high cell throughput, which is optimal with infinite number of users in the network, spatial multiplexing to a single user targets high peak rates.

III. Coordinated Beamforming

In this section, we first focus on the case where the transmitter is equipped with two transmit antennas. This is in accordance with the mandatory BS antenna configurations of $N_t = 2$ listed in the IEEE 802.11n [26], IEEE 802.16m [27], 3GPP long term evolution (LTE) [28]. In this case, the maximum supportable number of users is two. We then extend the proposed algorithm to systems where $N_t > 2$ and the number of active users in the network is still fixed at two. The presence of control channel overhead in practical systems makes it reasonable to consider two users for simultaneous transmission. After that, we explain a detection method at the receiver.

A. Low Complexity Iterative Coordinated Beamforming Algorithm

In this section, we briefly describe a low complexity iterative coordinated beamforming algorithm [29]. We assume that a single stream is transmitted to each of two users. It is assumed that two users have already been selected among a larger number of users using a scheduling algorithm. The low complexity iterative coordinated beamforming algorithm does not require computing receive combining vectors at each iteration, since it expresses the updated transmit beamformers at each iteration directly from the expression of the effective channels, assuming that the receive combining vectors are maximal ratio combining matched filters, given by $\boldsymbol{w}_k = \boldsymbol{H}_k \boldsymbol{f}_k$ [29].[3] This is a reasonable design (but not necessarily the only one) since it achieves the sum rate very close to capacity under the zero interference constraint. In fact the achievable sum rate can be slightly enhanced in a low signal-to-noise-ratio (SNR) regime by using regularized channel inversion. This strategy, however, requires knowing all users' noise variances at the transmitter, which requires additional feedback. In this paper, we focus on obtaining zero inter-user interference that can be practically implemented.

The effective channel of the k-th user, which includes the effect of the receiver matched filter (or combining vector), is $\boldsymbol{f}_k^* \boldsymbol{H}_k^* \boldsymbol{H}_k$. The two transmit beamformers are initialized to some random vectors $\boldsymbol{f}_{k,1}$, where $k = 1, 2$. Then, the following two operations are repeated with increasing i (iteration index) untill a stopping criterion is met

$$\begin{aligned} \tilde{\boldsymbol{H}}_i &= \left[(\boldsymbol{f}_{1,i}^* \boldsymbol{H}_1^* \boldsymbol{H}_1)^T (\boldsymbol{f}_{2,i}^* \boldsymbol{H}_2^* \boldsymbol{H}_2)^T \right]^T, \\ \boldsymbol{F}_{i+1} &= \tilde{\boldsymbol{H}}_i^{-1} \end{aligned} \quad (2)$$

where $\boldsymbol{F}_{i+1}$=$[\boldsymbol{f}_{1,i+1}\ \boldsymbol{f}_{2,i+1}]$, and $\boldsymbol{f}_{k,i+1}$ is the transmit beamformer column-vector for the k-th user at the $(i+1)$-th iteration, without normalization. To avoid numerical overflow, $\boldsymbol{F}_i$ can be scaled by a constant at each iteration without affecting the outcome. The transmitter repeats this procedure until the change in $\boldsymbol{f}_{k,i}$ is sufficiently small i.e., $\|\boldsymbol{f}_{k,i} - \boldsymbol{f}_{k,i-1}\| < \epsilon$ where ϵ is an arbitrary small number. It was noted in [10] that although the iterative algorithm seemed to converge in most cases, it cannot be guaranteed. Theorem 1, which will be introduced in the next section, provides insights into the reason why the algorithm may not converge, or converges very slowly in some cases.

B. Proposed Non-iterative Coordinated Beamforming Algorithms

The convergence of the iterative update algorithm in Section III-A cannot be guaranteed but it typically converges with a small ϵ in almost all trial cases with more than 20 iterations for two transmit antenna systems.[4] This procedure, however, may also affect the system's stability because iterative coordinated beamforming converges very slowly at times. Therefore, in this section, we propose non-iterative coordinated beamforming algorithms that have better ergodic achievable sum rate performance than iterative coordinated beamforming by avoiding the slow convergence. In Section III-B1, we derive a closed-form expression for the transmit beamformers for two antenna systems and extend this algorithm to more than two transmit antenna systems in Section III-B2. In this section, we show that the proposed beamformer design is the sufficient and necessary solution for two transmit antenna systems and is also valid for more than two antenna systems. Note that the number of users is two in both Sections III-B1 and III-B2.

[3]When $N_r = 1$, the iterative coordinated beamforming is the same as ZFBF, thus we assume that the number of receive antennas is more than one in this paper.

[4]This iteration number would be increased as the number of transmit antenna increases.

1) Two transmit antennas: Let us now assume that the transmitter has two transmit antennas, and there are two active users in the system, where each user has at least two receive antennas. In this case, since $\boldsymbol{w}_k = \boldsymbol{H}_k \boldsymbol{f}_k$, $k = 1$ or 2, the discrete-time received signals at users 1 and 2 are given by

$$y_1 = \boldsymbol{w}_1^* \boldsymbol{H}_1 \boldsymbol{f}_1 x_1 + \boldsymbol{w}_1^* \boldsymbol{H}_1 \boldsymbol{f}_2 x_2 + \boldsymbol{w}_1^* \boldsymbol{n}_1,$$
$$y_2 = \boldsymbol{w}_2^* \boldsymbol{H}_2 \boldsymbol{f}_2 x_2 + \boldsymbol{w}_2^* \boldsymbol{H}_2 \boldsymbol{f}_1 x_1 + \boldsymbol{w}_2^* \boldsymbol{n}_2,$$

where $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are the unit-norm vectors of size 2×1 [30]. Let us define the normalized matched channel by

$$\boldsymbol{R}_k \triangleq \frac{\boldsymbol{H}_k^* \boldsymbol{H}_k}{||\boldsymbol{H}_k||_F^2} = \begin{pmatrix} R_{k,11} & R_{k,12} \\ R_{k,21} & R_{k,22} \end{pmatrix}$$

where $||\boldsymbol{H}_k||_F^2$ is the squared Frobenius norm of the 2×2 complex matrix $\boldsymbol{H}_k$.

Theorem 1: The transmit beamformers of the converged iterative coordinated beamforming system are the generalized eigenvectors of the matched channel matrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$.

Proof: See Appendix. ∎

Theorem 1 means that if the iterative algorithm converges, then it converges to generalized eigenvectors of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ and we can avoid slow convergence cases by using the generalized eigenvectors directly. Thus once the transmitter knows the normalized matched channel matrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, the transmit beamformers can be computed using (18). Note that the proposed algorithm does not require any iterations.

Theorem 2: If $N_t = 2$, $N_r \geq 2$, and $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are both invertible, then the following claim holds. If (non-zero) transmit beamforming vectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ satisfy the zero inter-user interference conditions, i.e.,

$$\boldsymbol{f}_2^* \boldsymbol{R}_1 \boldsymbol{f}_1 = 0 \tag{3}$$
$$\boldsymbol{f}_2^* \boldsymbol{R}_2 \boldsymbol{f}_1 = 0, \tag{4}$$

then $\boldsymbol{f}_1, \boldsymbol{f}_2$ are the generalized eigenvectors of $(\boldsymbol{R}_1, \boldsymbol{R}_2)$ which means:

$$\boldsymbol{R}_1 \boldsymbol{f}_1 = \lambda_1 \boldsymbol{R}_2 \boldsymbol{f}_1 \tag{5}$$
$$\boldsymbol{R}_1 \boldsymbol{f}_2 = \lambda_2 \boldsymbol{R}_2 \boldsymbol{f}_2 \tag{6}$$

for some scalars λ_1 and λ_2.

Proof: Equation (3) implies that $\boldsymbol{R}_1 \boldsymbol{f}_1$ is in the null space of vector $\boldsymbol{f}_2$. Similarly, (4) implies that $\boldsymbol{R}_2 \boldsymbol{f}_1$ is in the null space of vector $\boldsymbol{f}_2$. Since $\boldsymbol{f}_2$ is a two-dimentional non-zero vector, its null space is one-dimensional. Therefore, $\boldsymbol{R}_1 \boldsymbol{f}_1$ and $\boldsymbol{R}_2 \boldsymbol{f}_1$ must be co-linear (the fact that $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are invertible means that $\boldsymbol{R}_1 \boldsymbol{f}_1$ and $\boldsymbol{R}_2 \boldsymbol{f}_1$ must be non-zero), i.e.,

$$\boldsymbol{R}_1 \boldsymbol{f}_1 = \lambda_1 \boldsymbol{R}_2 \boldsymbol{f}_1$$

which is the same as (5) for some constant λ_1. If we take the transpose of (3) and (4), and apply the same argument to $\boldsymbol{R}_1 \boldsymbol{f}_2$ and $\boldsymbol{R}_2 \boldsymbol{f}_2$, we get a condition similar to (6). Since $\boldsymbol{R}_1$, $\boldsymbol{R}_2$ are assumed to be invertible (because $N_r \geq N_t$), the pair $\boldsymbol{f}_1$, $\boldsymbol{f}_2$ being generalized eigenvectors of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ is equivalent to $\boldsymbol{f}_1$, $\boldsymbol{f}_2$ being eigenvectors of $\boldsymbol{R}_2^{-1} \boldsymbol{R}_1$ as well as of $\boldsymbol{R}_1^{-1} \boldsymbol{R}_2$. ∎

Theorem 2 means that for $N_t = 2$, any zero inter-user interference solution is a generalized eigenvector of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$.

Theorem 3: Any set of generalized eigenvectors of $(\boldsymbol{R}_1, \boldsymbol{R}_2)$ satisfy the zero inter-user interference condition.

Proof: See Theorem 4. This is a special case of Theorem 4 with $m = n = 2$, where m and n are the number of generalized eigenvectors. ∎

From Theorems 2 and 3, it is seen that the generalized eigenvectors of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are the sufficient and necessary solution that satisfy the zero inter-user interference constraint for $N_t = 2$ and $N_r \geq 2$.

2) More than two transmit antennas: In the previous section, we derived a closed-form expression for the transmit beamformer for the two transmit antenna case. Here we extend the solution to two user systems that have more than two transmit antennas. Under the zero interference constraint, we need to solve the optimization problem as follows:

$$\left\{\boldsymbol{f}_{1,opt}, \boldsymbol{f}_{2,opt}\right\} = \arg \max_{\boldsymbol{f}_1:||\boldsymbol{f}_1||=1, \boldsymbol{f}_2:|\boldsymbol{f}_2||=1} \left\{ \log_2\left(1 + |\boldsymbol{f}_1 \boldsymbol{R}_1 \boldsymbol{f}_1|\right) + \log_2\left(1 + |\boldsymbol{f}_2 \boldsymbol{R}_2 \boldsymbol{f}_2|\right) \right\}$$
$$s.t. |\boldsymbol{f}_1^* \boldsymbol{R}_1 \boldsymbol{f}_2| = |\boldsymbol{f}_2^* \boldsymbol{R}_2 \boldsymbol{f}_1| = 0. \tag{7}$$

where $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are the $N_t \times N_t$ normalized matched channel matrices and $\boldsymbol{f}_1$, $\boldsymbol{f}_2$ are the transmit beamformers of size $N_t \times 1$. Let us consider the case $N_t > 2$.

Theorem 4: If $\boldsymbol{t}_m$, $\boldsymbol{t}_n$ are generalized eigenvectors of $(\boldsymbol{R}_1, \boldsymbol{R}_2)$ and they correspond to distinct eigenvalues, then any $\boldsymbol{t}_m$, $\boldsymbol{t}_n$ satisfy the zero inter-user interference constraint (7), where $m, n = 1, 2, \cdots$, the number of generalized eigenvectors, $m \neq n$.

Proof: The conditions for the generalized eigenvectors are given by

$$\boldsymbol{R}_1 \boldsymbol{t}_m = \lambda_m \boldsymbol{R}_2 \boldsymbol{t}_m \tag{8}$$
$$\boldsymbol{R}_2 \boldsymbol{t}_n = \lambda_n \boldsymbol{R}_1 \boldsymbol{t}_n \tag{9}$$

for some scalars $\lambda_m \neq \lambda_n$. Therefore, using (8)

$$\boldsymbol{t}_n^* \boldsymbol{R}_1 \boldsymbol{t}_m = \lambda_m \boldsymbol{t}_n^* \boldsymbol{R}_2 \boldsymbol{t}_m$$

and using (9)

$$\boldsymbol{t}_n^* \boldsymbol{R}_2 \boldsymbol{t}_m = \boldsymbol{t}_m^* \boldsymbol{R}_2 \boldsymbol{t}_n = (1/\lambda_n) \boldsymbol{t}_m^* \boldsymbol{R}_1 \boldsymbol{t}_n.$$

This in turn implies that

$$\lambda_n \boldsymbol{t}_n^* \boldsymbol{R}_2 \boldsymbol{t}_m = \boldsymbol{t}_m^* \boldsymbol{R}_1 \boldsymbol{t}_n = \boldsymbol{t}_n^* \boldsymbol{R}_1 \boldsymbol{t}_m.$$

Therefore, we have

$$\lambda_m \boldsymbol{t}_n{}^* \boldsymbol{R}_2 \boldsymbol{t}_m = \lambda_n \boldsymbol{t}_n^* \boldsymbol{R}_2 \boldsymbol{t}_m$$

which implies $\boldsymbol{t}_n^* \boldsymbol{R}_2 \boldsymbol{t}_m = 0$ because $\lambda_m \neq \lambda_n$. The same argument shows that $\boldsymbol{t}_n^* \boldsymbol{R}_1 \boldsymbol{t}_m = 0$. ∎

From Theorem 4, it is clear that the generalized eigenvectors of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ satisfy the zero inter-user interference constraint (7). The authors recognize that this solution is not sum capacity optimal for arbitrary antenna configurations. The idea here is to use the proposed transmit beamformers to obtain zero inter-user interference even for more than two antennas. Note that this solution can be directly used where the transmitter has perfect CSI.

From these results, we can now compute the transmit beamformers and the receive combining vectors satisfying the zero inter-user interference criterion, for the case where the transmitter and the receiver have two or more antennas and the system supports two users. The procedure to compute the transmit beamformers and the combining vectors is as follows. First the transmitter computes the matched channel matrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ and finds all generalized eigenvectors of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$. Then, let $\boldsymbol{T}$ be the set of eigenvectors of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$. The eigenvector pair that maximizes the sum rate is selected as follows:

$$\{\boldsymbol{f}_1, \boldsymbol{f}_2\} = \arg \max_{\boldsymbol{t}_n, \boldsymbol{t}_m \in \{\boldsymbol{T}\}, \boldsymbol{t}_n \neq \boldsymbol{t}_m} \left\{ \log_2 \left(1 + \frac{P}{2\sigma^2} |\boldsymbol{H}_1 \boldsymbol{t}_n| \right) + \log_2 \left(1 + \frac{P}{2\sigma^2} |\boldsymbol{H}_2 \boldsymbol{t}_m| \right) \right\} \quad (10)$$

where P is the total power at the transmitter. Note that there are two eigenvectors when $N_t = 2$ so we need to compute (10) using only two beamformer pairs $\{\boldsymbol{f}_1, \boldsymbol{f}_2\} = [\{\boldsymbol{t}_1, \boldsymbol{t}_2\}$ or $\{\boldsymbol{t}_2, \boldsymbol{t}_1\}]$. Thus, the computational complexity for finding the beamformers is marginal. In general, $g(g-1)$ computations are required to the beamformer set where g is the number of generalized eigenvectors.[5]

C. Detection Method

The users can use receive combining vectors matched to the effective channel where the effect of the transmit beamformers is included using $\boldsymbol{w}_k = \boldsymbol{H}_k \boldsymbol{f}_k$. In a communication system where the downlink channel contains dedicated pilot symbols for channel estimation, each user can estimate their effective channel information $\boldsymbol{H}_k \boldsymbol{f}_k$.

Since 3GPP long term evolution (LTE) systems use only a common pilot channel, the same for all users [28], there is no way to estimate the effective channel gain at the receiver. Hence, the receiver cannot estimate the optimal post-processing receiver combining vector. To solve this problem, the authors in [29], [31], [32] proposed limited feedforward to inform the receiver of post-processing information. In our work, however, we restrict ourselves to the situation where the receiver can estimate the effective channel.

IV. Limited Feedback of the Matched Channel Matrix

We propose to quantize the sufficient CSI $\boldsymbol{R}_k$ at the receiver and send it to the transmitter through a limited feedback link so that the transmitter can compute the transmit beamformers. As the transmitter needs the complete channel matrix $\boldsymbol{R}_k$, and not just its subspace information as with Grassmannian beamforming [24], we use direct quantization exploiting the symmetry of $\boldsymbol{R}_k$. We propose uniform and non-uniform quantization codebooks. For the uniform quantization, the transmitter and the receivers do not need to save the distributions of elements or the pre-defined codebook since it is easily calculated once the feedback size is fixed. The non-uniform quantization, however, requires storing the distributions of elements but shows better performance when the channel statistics match the design assumptions. In this section, we omit the user index k for simplicity, though we still analyze two user systems.

[5]If $N_t \leq N_r$ then $n = N_t$, and if $N_t > N_r$, $n = \min(\text{rank}(\boldsymbol{R}_1, \boldsymbol{R}_2))$.

A. Uniform Quantization

We quantize the elements in the matched channel matrix $\boldsymbol{R}$ using scalar parameters α, β, and γ. Since $\boldsymbol{R}$ is a Hermitian matrix with unit Frobenius norm,

$$R_{11} = \alpha_1 \cdots R_{N_t-1 N_t-1} = \alpha_{N_t-1},\ R_{N_t N_t} = 1 - \sum_{n=1}^{N_t-1} \alpha_n,$$
$$R_{kl} = \beta_{kl} + j\gamma_{kl} \text{ and } R_{kl} = R^*_{lk} k, l = 1, \cdots, N_t, k \neq l$$

where $0 \leq \alpha_n \leq 1$ and $-0.5 \leq \beta_{kl}, \gamma_{kl} \leq 0.5$. By assuming that α_n, β_{kl}, and γ_{kl} are uniformly distributed in these intervals, the users can directly quantize the channel using a finite number of bits. We choose the following quantization method. Parameters α_n, β_{kl} and γ_{kl} are quantized using Q bits as shown below

$$\alpha_n \in \left[\frac{1}{2^{Q+1}}, \frac{1}{2^{Q+1}} + \frac{1}{2^Q}, \cdots, 1 - \frac{1}{2^{Q+1}} \right] \text{and}$$
$$\beta_{kl}, \gamma_{kl} \in \left[\frac{1}{2^{Q+1}} - \frac{1}{2}, \frac{1}{2^{Q+1}} + \frac{1}{2^Q} - \frac{1}{2}, \cdots, \frac{1}{2^{Q+1}} + \frac{1}{2} \right].$$

This approach is attractive in practice if the true channel distribution is unknown a priori [30].

B. Non-uniform Quantization

If we know the distribution of each of the elements in the normalized matched channel matrix $\boldsymbol{R}$, we can minimize the quantization error [33], [34]. Let us define

$$\boldsymbol{H} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r 1} & h_{N_r 2} & \cdots & h_{N_r N_t} \end{pmatrix}$$

and $\|\boldsymbol{H}\|_F^2 = \text{tr}(\boldsymbol{H}^* \boldsymbol{H}) = \sum_{p=1}^{N_r} \sum_{q=1}^{N_t} |h_{pq}|^2$, and $h_{pq} = h_{pq,r} + j h_{pq,i}$ (the subscripts r and i denote real and imaginary, respectively). For codebook design, we model the elements $(h_{11} \sim h_{N_r N_t})$ of each users channel matrix $\boldsymbol{H}$ as independent complex Gaussian random variables with zero mean and unit variance $\mathcal{CN}(0,1)$. Then the normalized matched channel can be written as

$$\boldsymbol{R} = \frac{\boldsymbol{H}^* \boldsymbol{H}}{\|\boldsymbol{H}\|_F^2} = \begin{pmatrix} R_{11} & R_{12} & \cdots & R_{1N_t} \\ R_{21} & R_{22} & \cdots & R_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N_t 1} & R_{N_t 2} & \cdots & R_{N_t N_t} \end{pmatrix}. \quad (11)$$

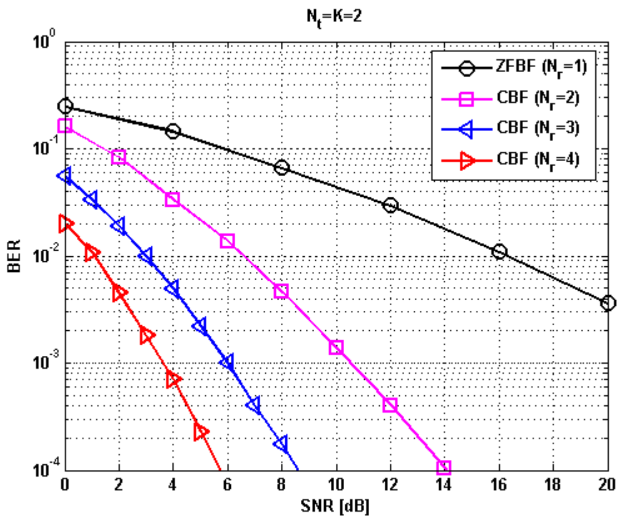

Fig. 2. BER vs. SNR for zero-forcing beamforming (ZFBF) and the proposed coordinated beamforming (CBF).

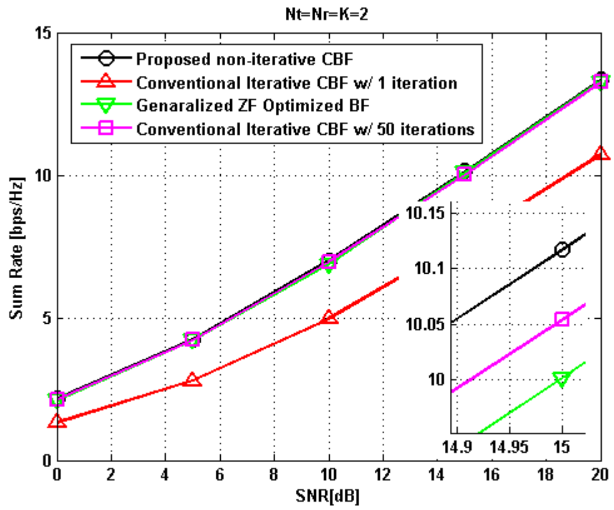

Fig. 3. Achievable sum rate comparison of the proposed closed-form CBF, iterative CBF with various iterations, and the generalized zero-forcing optimized beamforming [19].

For example, for $N_t = 2$ case,

$$\begin{aligned}
R_{11} &= \frac{h_{11,r}^2 + h_{11,i}^2 + h_{21,r}^2 + h_{21,i}^2}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)} \\
R_{12} &= \frac{(h_{11,r} - jh_{11,i})(h_{12,r} + jh_{12,i})}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)} \\
&\quad + \frac{(h_{21,r} - jh_{21,i})(h_{22,r} + jh_{22,i})}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)} \\
R_{21} &= \frac{(h_{11,r} + jh_{11,i})(h_{12,r} - jh_{12,i})}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)} \\
&\quad + \frac{(h_{21,r} + jh_{21,i})(h_{22,r} - jh_{22,i})}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)} \\
R_{22} &= \frac{h_{12,r}^2 + h_{12,i}^2 + h_{22,r}^2 + h_{22,i}^2}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)}.
\end{aligned}$$

For R_{ii} $(i = 1, \cdots, N_t)$, let $A = \sum_{l=1}^{N_r} |h_{li}|^2$ and $B = \sum_{p=1}^{N_r} \sum_{q=1}^{N_t} |h_{pq}|^2 - A$. Then A and B have chi-sqaure distributions with $2N_r$ degrees of freedom ($\chi_{2N_r}^2$) and with $2(N_t-1)N_r$ degrees of freedom ($\chi_{2(N_t-1)N_r}^2$), respectively. Before we proceed to describe the distribution of $R_{11}, \cdots, R_{N_tN_t}$, we present some preliminary observations.

Lemma 5: The chi-square distribution $X \sim \chi_\nu^2$ is a special case of gamma distribution, in that $X \sim \Gamma(\frac{\nu}{2}, 2)$.

Lemma 6: If X and Y are independent random variables, such that $X \sim \Gamma(p, \theta)$ and $Y \sim \Gamma(q, \theta)$, then $\frac{X}{X+Y}$ has a beta distribution with parameters p and q.

Lemmas 5 and 6 are results from [35].

Theorem 7: R_{ii}, $(i = 1, \cdots, N_t)$ has a beta distribution with parameters $(N_r, (N_t-1)N_r)$.

Proof: $R_{ii} = \frac{A}{A+B}$, where A and B are random variables with gamma distributions $\Gamma(N_r, 2)$, $\Gamma((N_t-1)N_r, 2)$. From Lemma 6, R_{ii} has a beta distribution with parameters $(N_r, (N_t-1)N_r)$. ■

Therefore, the probability density function (pdf) and cumulative density function (cdf) of R_{ii} are given by

$$\begin{aligned}
f_{R_{ii}}(x) &= \frac{\Gamma(N_r + (N_t-1)N_r)}{\Gamma(N_r)\Gamma((N_t-1)N_r))} x^{N_r-1}(1-x)^{((N_t-1)N_r-1)} \\
F_{R_{ii}}(x) &= \frac{B(x; N_r, (N_t-1)N_r)}{B(N_r, (N_t-1)N_r)}
\end{aligned} \tag{12}$$

where $B(x; a, b)$ is the incomplete beta function, i.e., $B(x; a, b) = \int_0^x t^{a-1}(1-t)^{b-1} dt$.

Theorem 8: The real and imaginary parts of R_{pq} have a beta distribution with a shift by $1/2$.

Proof: For R_{pq}, we consider only the real part of R_{pq} since imaginary part also has the same distribution. As an example, then the real and imaginary parts of R_{12} are given by

$$\begin{aligned}
Re(R_{12}) &= \frac{h_{11,r}h_{12,r} + h_{21,r}h_{22,r} - h_{11,i}h_{12,i} - h_{21,i}h_{22,i}}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)} \\
Im(R_{12}) &= \frac{h_{11,r}h_{12,i} + h_{11,i}h_{12,r} + h_{21,i}h_{22,r} + h_{21,r}h_{22,i}}{\sum_{p=1}^2 \sum_{q=1}^2 (h_{pq,r}^2 + h_{pq,i}^2)}
\end{aligned}$$

where, $Re(\cdot)$ and $Im(\cdot)$ are the real and imaginary part of a complex value. Let

$$\begin{aligned}
T =& (h_{11,r} + h_{12,i})^2/T_1 + (h_{11,i} + h_{12,r})^2/T_1 \\
&+ (h_{21,i} + h_{22,r})^2(h_{21,r} + h_{22,i})^2/T_1 \\
=& \frac{\sum_{p=1}^2 \sum_{q=1}^2 h_{pq,r}^2 + h_{pq,i}^2}{2\sum_{p=1}^2 \sum_{q=1}^2 h_{pq,r}^2 + h_{pq,i}^2} \\
&+ \frac{2h_{11,r}h_{12,i} + 2h_{11,i}h_{12,r} + 2h_{21,i}h_{22,r} + 2h_{21,r}h_{22,i}}{2\sum_{p=1}^2 \sum_{q=1}^2 h_{pq,r}^2 + h_{pq,i}^2} \\
=& \frac{1}{2} + Im(R_{12})
\end{aligned}$$

where $T_1 = (h_{11,r} + h_{12,i})^2 + (h_{11,i} + h_{12,r})^2 + (h_{21,i} + h_{22,r})^2(h_{21,r} + h_{22,i})^2 + (h_{11,r} - h_{12,i})^2 + (h_{11,i} - h_{12,r})^2 + (h_{21,i} - h_{22,r})^2(h_{21,r} - h_{22,i})^2$. Since $T = \frac{C}{C+D}$ where, $C = (h_{11,r} + h_{12,i})^2 + (h_{11,i} + h_{12,r})^2 + (h_{21,i} + h_{22,r})^2(h_{21,r} + h_{22,i})^2$ and $D = (h_{11,r} - h_{12,i})^2 + (h_{11,i} - h_{12,r})^2 + (h_{21,i} -$

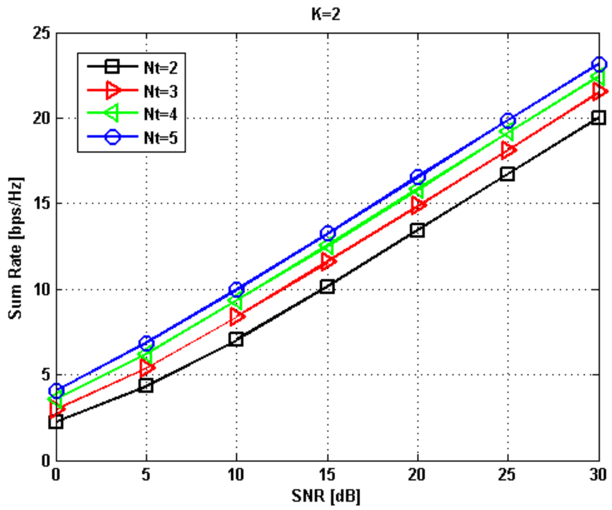

Fig. 4. Sum rates vs. SNR with various N_t parameters. Note that full multiplexing gain cannot be achieved since two users are supported regardless of the number of transmit antennas.

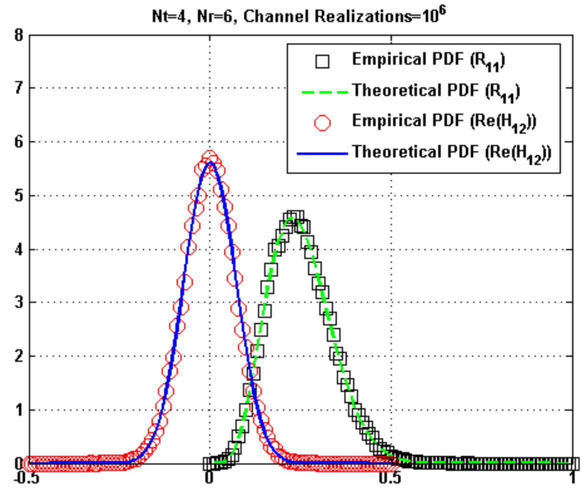

Fig. 5. An example of channel distribution of R_{11} and the real part of R_{12} where $N_t = 4$ and $N_r = 6$.

$h_{22,r})^2(h_{21,r} - h_{22,i})^2$, we can use Lemma 6. Thus, T follows a beta distribution. ■

Since all diagonal and off-diagonal elements in $\boldsymbol{R}$ have the beta distribution with different parameters, we need to generate only two codebooks in this case. Suppose that N_b-bit scalar codewords denoted by $C = \{c_1, c_2, \cdots, c_{2^{N_b}}\}$ are used for the channel quantization. For α, we find the codebook for the diagonal entries that satisfies the following condition:

$$\int_{c_{i-1}}^{c_i} f_{R_{11}}(x)dx = \frac{1}{2^{N_b}+1}, \tag{13}$$

where $i = 1, 2, \cdots, 2^{N_b}$. Since the CDF $F_{R_{11}}$ in (12) is a regularized beta function $I(x; N_r, (N_t - 1)N_r)$, we can rewrite (13) as follows:

$$I(c_i, N_r, (N_t-1)N_r) - I(c_{i-1}, N_r, (N_t-1)N_r) = \frac{1}{2^{N_b}+1},$$

where

$$I(x; N_r, (N_t-1)N_r) = \sum_{j=N_r}^{N_tN_r-1} \frac{(N_tN_r-1)!}{j!(N_tN_r-1-j)!} x^j (1-x)^{N_tN_r-1-j}$$

and c_0=0. For β and γ, we use the similar method to construct the codebook after shifting the mean by $\frac{1}{2}$.

Upon receiving the codebook indices of R_{11}, $Re(R_{12})$, and $Im(R_{12})$ from the receiver over a control channel, the transmitter can estimate $\boldsymbol{R}$ and compute the transmit beamformers before transmitting the data. In this paper, we use the same quantization level N_b for all elements for simplicity. Vector quantization could be used to optimize the feedback overhead but we leave this issue for future research.

In [23], it was proposed that each user quantizes its effective channel after multiplication with the pre-combining vector that produces the lowest quantization error, using a random codebook. This approach, however, has some drawbacks. A search over the codebooks for the different number of receive antennas is required to find the best quantization. Computational complexity increases as the number of receive antennas increases. In the proposed solution, the feedback overhead remains the same regardless of the number of receive antennas, since $\boldsymbol{R}$ is always a Hermitian matrix of size $N_t \times N_t$. The total feedback overhead per user is $(N_t^2 - 1)Q\ bits$. Note that the proposed method quantizes channel magnitude as well as direction while the algorithm in [23] quantizes only channel directions.

V. Performance Evaluation

With perfect CSI at the transmitter, there is no inter-user interference thanks to the zero-interference constraint. This is, however, not possible in the limited feedback system since the transmitter computes the transmit beamformers based on the quantized CSI through a low rate limited feedback channel. Therefore, we use the achievable sum rate given by

$$R_{sum} = \sum_{k,l=1}^{2} \log_2 \left(1 + \frac{\frac{P}{K\sigma^2}|\boldsymbol{w}_k^* \boldsymbol{H}_k \boldsymbol{f}_k|^2}{\frac{P}{K\sigma^2}|\boldsymbol{w}_k^* \boldsymbol{H}_k \boldsymbol{f}_l|^2 + 1}\right),$$

where, $k \neq l$. Note that the transmit beamformers $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are obtained through the estimated channel matrices while $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ are the perfect channel matrices. We assume equal power allocation for numerical results. In this section, we compare the achievable sum rates of the proposed algorithm with best single user closed loop MIMO (Best SU CL MIMO) [36], iterative coordinated beamforming [10], [12], ZFBF with finite rate feedback (with receive antenna combining), and with the sum capacity achieved by DPC [2]. For simulation purposes, we assume that the channel undergoes uncorrelated Rayleigh block fading, with perfect channel estimation at the receiver, no feedback delay and no feedback errors. In this paper, we ignore temporal channel correlation in wireless channels. Future work will consider temporally correlated channels.

In contrast to [20], the proposed algorithm utilizes multiple receive antennas so we expect to achieve comparatively more diversity gain. In Fig. 2, we compare the diversity gain of ZFBF with one receive antenna, and the proposed CBF with different number of receive antennas at the receiver. QPSK

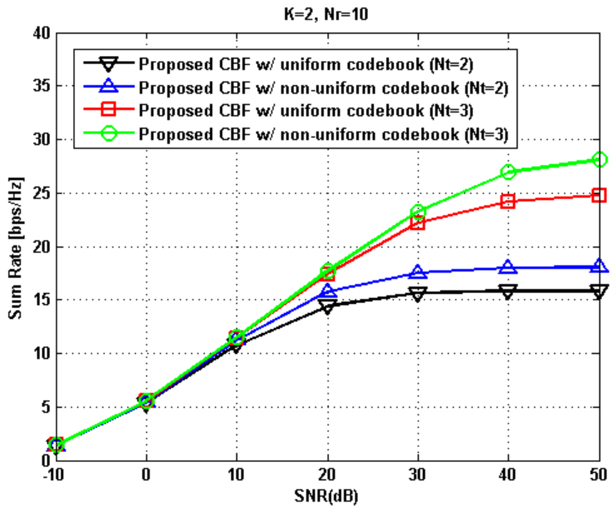

Fig. 6. Sum rate comparisons between uniform and nonuniform channel quantization. 3 bits and 6 bits per each parameter are used for channel quantization for $N_t = 2$ and $N_t = 3$, respectively.

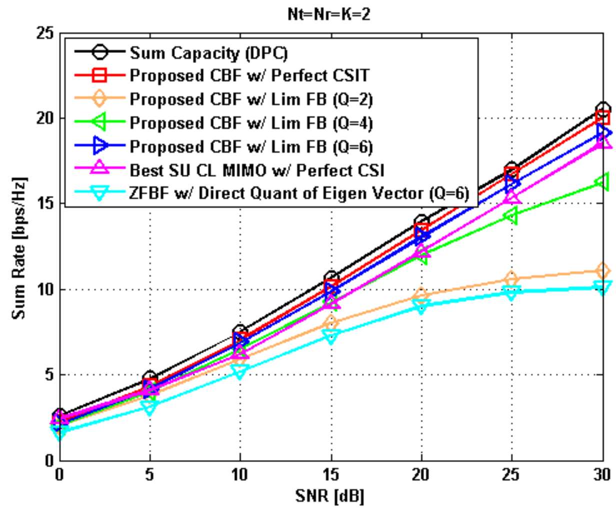

Fig. 7. Sum rates vs. SNR where 2 transmit antennas at the transmitter, 2 receive antennas at the receiver, and 2 users in the network.

is used for this simulation. It is seen that ZFBF achieves a diversity order of one while the proposed algorithm achieves a diversity order of N_r. Even with two receive antennas, at BER$= 10^{-2}$, we obtain a 10 dB gain in SNR.

In Fig. 3, we compare the proposed non-iterative algorithm with the iterative coordinated beamforming algorithms [10], [12], [31] and the generalized zero-forcing optimized beamforming algorithm [19]. Indeed, the solution in [19] is a subset of the proposed solution since the author's final formulation in [19] is the principle eigenvectors of $\boldsymbol{R}_2^{-1}\boldsymbol{R}_1$ and $\boldsymbol{R}_1^{-1}\boldsymbol{R}_2$ even though the author in [19] used a different method to get the final expression. In this paper, we consider all generalized eigenvectors to find the transmit beamformers. As we proved in Section III, all generalized eigenvectors satisfy the zero inter-user interference. Moreover, the solution in [19] works only for two transmit antenna system while our solutions works for two or more antenna systems. In the simulation, we use 1 and 50 iterations for the iterative coordinated algorithm. Note that the proposed algorithm does not include any iterative procedure. It is seen from Fig. 3 that the proposed method yields a better achievable sum rate performance than the iterative coordinated beamforming. In fact, the proposed algorithm outperforms the iterative coordinated beamforming even when 50 iterations are used. Note that the required number of iterations increases as the number of transmit antennas increases. The proposed algorithm also shows better sum rate performance than the generalized zero-forcing optimized beamforming algorithm [19], which works only for two transmit antenna systems while the proposed algorithm works for two or more transmit antenna systems.

Fig. 4 illustrates the achievable sum rates of the proposed method with different numbers of transmit antennas. We can see that the achievable sum rate increases continuously as the number of transmit antennas increases. Since we restrict ourselves to one stream per user, full multiplexing gain cannot be achieved.

So far, we have compared the proposed solution with [10], [12], [31] with perfect CSI. Now, we consider limited feedback. In Fig. 5, we compare the derived theoretical PDFs of R_{11} and the real part of R_{12} in (11) with the empirical PDFs generated by many channel realizations for $N_t = 4$ and $N_r = 6$ as an example. In the simulation, we generated 10^6 random channels to validate the derived PDFs with the empirical PDFs. Fig. 6 shows the achievable sum rates for the proposed coordinated beamforming with uniform/non-uniform codebooks. As shown in Fig. 6, the non-uniform codebook outperforms the uniform codebook by about 2 bps/Hz at SNR$=$ 30 dB where $N_t = 2$ and $Q = 3$ bits. Note that we used $Q = 6$ bits for $N_t = 3$ case so the total feedback overhead of this case is larger than that of $N_t = 2$ case.

Fig. 7 shows the sum rates of the proposed non-iterative coordinated beamforming with limited feedback (using non-uniform quantization), the sum capacity, and the ZFBF with receive antenna combining, when the number of users in the network is the same as the number of transmit antennas, i.e., $K = 2$. In this case, no scheduling algorithm is needed. Here it is seen that the proposed method yields good sum rate performance results in spite of no multiuser diversity gain. This contrasts with opportunistic beamforming methods and unitary codebook-based precoding methods [22] that fail in this situation. To illustrate the effect of limited feedback, we use three different feedback sizes $Q = 2$, $Q = 4$ and $Q = 6$. The same codebook size Q per parameter is used for simplicity thus total $3Q$ bits are required per user. We observe that full multiplexing gain cannot be maintained with $Q = 2$ at high SNR, due to the residual inter-user interference caused by quantization errors. The proposed method, however, shows better gain than the ZFBF with the same amount of limited feedback.

We also compare the sum rates achievable with many users at SNR$=$ 10 dB with the maximum sum rate scheduling. To achieve multiuser diversity gain, additional feedback information $\|\boldsymbol{H}_k\|_F^2$ is necessary. This type of feedback is usually available to support modulation and rate adaptation in the standards. Hence, we assume the transmitter knows all users' channel quality information for this simulation. For numerical results, we consider two user selection algorithms, i.e., a full

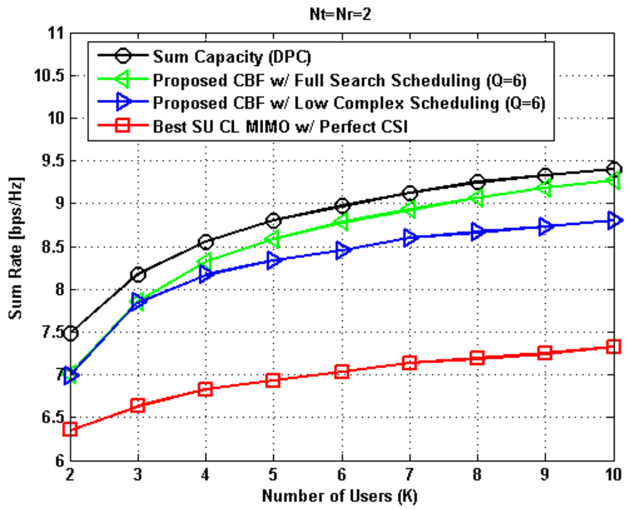

Fig. 8. Sum rates vs. SNR where 2 transmit antennas at the transmitter, 2 receive antennas at the receiver, and K users in the network ($SNR = 10$ dB).

search method which requires very high computational complexity and a low complexity greedy user selection algorithm. The low complexity greedy user selection algorithm we used for plotting Fig. 8 is summarized in Table I. After choosing one user with the highest channel norm, the transmitter finds two candidate users (i. with the largest Chordal distance with the selected first user and ii. with the second highest channel norm). After comparing the achievable sum rates with these two candidates, the transmitter finally selects the second user for CBF. Fig. 8 illustrates the sum rates as a function of the number of users. As the number of users increases, the proposed algorithm offers an increasing gain over the best single-user closed-loop MIMO capacity even with constant feedback rate ($Q = 6$).

VI. Conclusion

This paper presented downlink multiuser MIMO algorithms tailored for practical implementation. In particular, for the downlink channel where the transmitter is equipped with multiple transmit antennas, the proposed system supports transmission of one stream to each of two users simultaneously. For two or more antenna systems, we proposed a closed-form expression for the transmit beamformers, avoiding the need for iterative computation using the power iteration and generalized eigen analysis. We also extended the proposed algorithm to systems with more than two transmit antenna. To enable practical implementation, we proposed a new limited feedback algorithm that exploits the structure of the algorithm. The feedback overhead of the proposed algorithm is independent of the number of receive antennas. Future work will investigate more general cases like the more than two user scenario.

Acknowledgment

The authors are grateful to K.-K. Wong for providing simulation results of [19].

TABLE I
A Low Complexity Greedy User Scheduling Algorithm.

1. Find a user $s_{\hat{k}}$ such that
   $s_{\hat{k}} = \arg\max_{k \in K} \|\boldsymbol{H}_k\|_F^2$
   $\boldsymbol{R}_{\hat{k}} = \boldsymbol{H}_{\hat{k}}^* \boldsymbol{H}_{\hat{k}} / \|\boldsymbol{H}_{\hat{k}}\|_F^2$.
2. Find a user maximizing the Chordal distance with the selected user $s_{\hat{k}}$.
   $s_c = \arg\max_{k \in K, k \neq \hat{k}} \frac{1}{\sqrt{2}} \|\boldsymbol{R}_{\hat{k}} - \boldsymbol{R}_k\|_F^2$
   Set $S_{\texttt{active,cd}} = \{s_{\hat{k}}, s_c\}$
3. Find a user with the second largest channel norm.
   $s_n = \arg\max_{k \in K, k \neq \hat{k}} \|\boldsymbol{H}_k\|_F^2$
   Set $S_{\texttt{active,no}} = \{s_{\hat{k}}, s_n\}$
4. Achievable sum rate comparison.
   if $\mathcal{R}_{S_{\text{active,cd}}} > \mathcal{R}_{S_{\text{active,no}}}$
   $S_{\texttt{final}} = \{s_{\hat{k}}, s_c\}$
   else
   $S_{\texttt{final}} = \{s_{\hat{k}}, s_n\}$
   end

Appendix A
Proof of Theorem 1

Let us rewrite (2) as

$$\begin{bmatrix} \boldsymbol{f}_{1,i+1} & \boldsymbol{f}_{2,i+1} \end{bmatrix} = \left[[(\boldsymbol{f}_{1,i}^* \boldsymbol{R}_1)^T (\boldsymbol{f}_{2,i}^* \boldsymbol{R}_2)^T]^T \right]^{-1} \quad (14)$$

where i is the iteration index. We solve (14) by using the adjoint matrix rather than the matrix inverse. We obtain an expression for the recursion and then take the limit over an infinite number of iterations. Define the following matrices:

$$\boldsymbol{E}_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \boldsymbol{E}_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},$$
$$\boldsymbol{E}_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \boldsymbol{E}_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Thus, $\boldsymbol{E}_{pq}\boldsymbol{A}\boldsymbol{E}_{mn}$ is a matrix where the only non-zero coefficient a_{qm} is in position (p, n). In particular,

$$\boldsymbol{E}_{pq}\boldsymbol{E}_{mn} = \begin{cases} \boldsymbol{E}_{pn}, & q = m \\ 0, & q \neq n. \end{cases}$$

So, adj$(\boldsymbol{A})$ can be expressed as

$$\begin{aligned} \text{adj}(\boldsymbol{A}) &= \boldsymbol{E}_{12}\boldsymbol{A}\boldsymbol{E}_{21} - \boldsymbol{E}_{11}\boldsymbol{A}\boldsymbol{E}_{22} - \boldsymbol{E}_{22}\boldsymbol{A}\boldsymbol{E}_{11} + \boldsymbol{E}_{21}\boldsymbol{A}\boldsymbol{E}_{12} \\ &= \boldsymbol{E}_{12}\boldsymbol{A}^T\boldsymbol{E}_{21} - \boldsymbol{E}_{12}\boldsymbol{A}^T\boldsymbol{E}_{12} - \boldsymbol{E}_{21}\boldsymbol{A}^T\boldsymbol{E}_{21} + \boldsymbol{E}_{21}\boldsymbol{A}^T\boldsymbol{E}_{12}. \end{aligned}$$

Let $\boldsymbol{F}_{i+1} = \text{adj}(\boldsymbol{H}_i)$ where $\boldsymbol{H}_i = \boldsymbol{E}_{11}\boldsymbol{F}_i^*\boldsymbol{R}_1 + \boldsymbol{E}_{22}\boldsymbol{F}_i^*\boldsymbol{R}_2$. Since $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are Hermitian matrices, $\boldsymbol{H}_i^T = \boldsymbol{R}_1^T\boldsymbol{F}_i^*\boldsymbol{E}_{11} + \boldsymbol{R}_2^T\boldsymbol{M}_i^*\boldsymbol{E}_{22}$. Then, we can express $\boldsymbol{F}_{i+1}$ as

$$\begin{aligned} \boldsymbol{F}_{i+1} = \boldsymbol{E}_{12}\boldsymbol{H}_i^T\boldsymbol{E}_{21} - \boldsymbol{E}_{12}\boldsymbol{H}_i^T\boldsymbol{E}_{12} \\ -\boldsymbol{E}_{21}\boldsymbol{H}_i^T\boldsymbol{E}_{21} + \boldsymbol{E}_{21}\boldsymbol{H}_i^T\boldsymbol{E}_{12}. \end{aligned}$$

After simplifications, we have

$$\begin{aligned} \boldsymbol{F}_{i+1}^* = \boldsymbol{E}_{12}\boldsymbol{F}_i^T\boldsymbol{R}_2^*\boldsymbol{E}_{21} - \boldsymbol{E}_{21}\boldsymbol{F}_i^T\boldsymbol{R}_1^*\boldsymbol{E}_{21} \\ -\boldsymbol{E}_{12}\boldsymbol{F}_i^T\boldsymbol{R}_2^*\boldsymbol{E}_{12} + \boldsymbol{E}_{21}\boldsymbol{F}_i^T\boldsymbol{R}_1^*\boldsymbol{E}_{12}. \end{aligned} \quad (15)$$

Substituting (15) in $\boldsymbol{H}_{i+1} = \boldsymbol{E}_{11}\boldsymbol{F}^*_{i+1}\boldsymbol{R}_1 + \boldsymbol{E}_{22}\boldsymbol{F}^*_{i+1}\boldsymbol{R}_2$ using the property that $\boldsymbol{R}_1 = \boldsymbol{R}_1^*$ and $\boldsymbol{R}_2 = \boldsymbol{R}_2^*$, we get

$$\begin{aligned}\boldsymbol{H}^T_{i+1} &= \boldsymbol{R}_1^T\boldsymbol{E}_{12}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{21} - \boldsymbol{R}_1^T\boldsymbol{E}_{21}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{21}\\ &-\boldsymbol{R}_2^T\boldsymbol{E}_{12}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{12} + \boldsymbol{R}_2^T\boldsymbol{E}_{21}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{12}. \qquad (16)\end{aligned}$$

Substituting (16) in $\boldsymbol{F}_{i+2} = \boldsymbol{E}_{12}\boldsymbol{H}^T_{i+1}\boldsymbol{E}_{21} - \boldsymbol{E}_{12}\boldsymbol{H}^T_{i+1}\boldsymbol{E}_{12} - \boldsymbol{E}_{21}\boldsymbol{H}^T_{i+1}\boldsymbol{E}_{21} + \boldsymbol{E}_{21}\boldsymbol{H}^T_{i+1}\boldsymbol{E}_{12}$, we have the equation on the top of the following page.

Finally $\boldsymbol{F}_{i+2} = \text{adj}(\boldsymbol{R}_2)\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{11} + \text{adj}(\boldsymbol{R}_1)\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{22}$ is the first column of $\text{adj}(\boldsymbol{R}_2)\boldsymbol{R}_1\boldsymbol{F}_i$. By recurrence, we obtain

$$\boldsymbol{F}_{2i} = (\text{adj}(\boldsymbol{R}_2)\boldsymbol{R}_1)^i\boldsymbol{F}_0\boldsymbol{E}_{11} + (\text{adj}(\boldsymbol{R}_1)\boldsymbol{R}_2)^i\boldsymbol{F}_0\boldsymbol{E}_{22}. \qquad (17)$$

The first column of $(\text{adj}(\boldsymbol{R}_2)\boldsymbol{R}_1)^i\boldsymbol{F}_0\boldsymbol{E}_{11}$ in (17) is the beamforming vector for user 1, while the second column is equal to 0. The term $\boldsymbol{F}_0\boldsymbol{E}_{11}$ is equivalent to applying a random initial vector $\boldsymbol{w}_0 \neq 0$ to the matrix $\text{adj}(\boldsymbol{R}_2)\boldsymbol{R}_1$, and performing power iterations. As $i \to \infty$, $(\text{adj}(\boldsymbol{R}_2)\boldsymbol{R}_1)^i\boldsymbol{w}$ converges to the eigenvector of $\boldsymbol{R}_2^{-1}\boldsymbol{R}_1$ for any $\boldsymbol{w} \neq 0$. Similarly, $(\text{adj}(\boldsymbol{R}_1)\boldsymbol{R}_2)^i\boldsymbol{F}_0\boldsymbol{E}_{22}$ is the beamforming vector for user 2. As $i \to \infty$, $(\text{adj}(\boldsymbol{R}_1)\boldsymbol{R}_2)^i\boldsymbol{w}_0$ converges to the eigenvector of $\boldsymbol{R}_1^{-1}\boldsymbol{R}_2$.

With Hermitian matrices $\boldsymbol{R}_1 = \boldsymbol{H}_1^*\boldsymbol{H}_1$ and $\boldsymbol{R}_2 = \boldsymbol{H}_2^*\boldsymbol{H}_2$, we define $\boldsymbol{G} = \boldsymbol{R}_1^{-1}\boldsymbol{R}_2\text{det}(\boldsymbol{R}_1)$. Note that $\boldsymbol{G}$ is not Hermitian. Then the eigenvalues of $\boldsymbol{G}$ with $d_1 < d_2$ and $\boldsymbol{GU} = \boldsymbol{UD}$ are

$$\begin{aligned}d_1 &= \frac{1}{2}\Big(g_{11} + g_{22} - \sqrt{g_{11}^2 - 2g_{11}g_{22} + g_{22}^2 + 4g_{21}g_{12}}\Big)\\ d_2 &= \frac{1}{2}\Big(g_{11} + g_{22} + \sqrt{g_{11}^2 - 2g_{11}g_{22} + g_{22}^2 + 4g_{21}g_{12}}\Big)\end{aligned}$$

where $\boldsymbol{D}$ is a diagonal matrix that contains the eigenvalues of $\boldsymbol{G}$, and $\boldsymbol{U}$ is a unitary matrix whose columns are the eigenvectors of $\boldsymbol{G}$. Then we can express the transmit beamforming matrix $\boldsymbol{F} = [\boldsymbol{f}_1\ \boldsymbol{f}_2]$ before normalizing the transmit beamformers as

$$\boldsymbol{F} = \begin{pmatrix} (g_{11} - g_{22}) - \sqrt{(g_{11} - g_{22})^2 + 4g_{21}g_{12}} & 2g_{21} \\ (g_{11} - g_{22}) + \sqrt{(g_{11} - g_{22})^2 + 4g_{21}g_{12}} & 2g_{21}\end{pmatrix}^T. \qquad (18)$$

The convergence of the iteration in (17) is a classical problem [35]. If we compute a classical iteration problem, $\boldsymbol{x}(i+1) = \boldsymbol{A}\boldsymbol{x}(i)$, then $\boldsymbol{x}$ will converge to a left principal eigenvector of $\boldsymbol{A}$, given the following assumptions are true: i) the largest eigenvalue of $\boldsymbol{A}$ is unique and ii) $\boldsymbol{A}(0)$ is not orthogonal to a left principal eigenvector. If the assumption i) is false, the closed-form expression is not valid. This event, however, occurs with probability zero. For the assumption ii), it is true in practice, so for the closed-form expression we assume we can choose such a vector. This is the reason why sometimes the iterative coordinated beamforming takes a very long time to converge. The convergence time is dictated by the ratio of the second largest by the largest eigenvalue. If this is close to one, then the power iterations will take a long time to converge. This problem is avoided by using the closed-form expression derived in the paper.

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$$
\begin{aligned}
\boldsymbol{F}_{i+2} =& -\boldsymbol{E}_{21}(\boldsymbol{R}_1^T\boldsymbol{E}_{12}\boldsymbol{Q}_2\boldsymbol{F}_i\boldsymbol{E}_{21} - \boldsymbol{R}_1^T\boldsymbol{E}_{21}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{21} - \boldsymbol{Q}_2^T\boldsymbol{E}_{12}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{12} + \boldsymbol{R}_2^T\boldsymbol{E}_{21}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{12})\boldsymbol{E}_{21} \\
&+ \boldsymbol{E}_{21}(\boldsymbol{R}_1^T\boldsymbol{E}_{12}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{21} - \boldsymbol{R}_1^T\boldsymbol{E}_{21}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{21} - \boldsymbol{R}_2^T\boldsymbol{E}_{12}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{12} + \boldsymbol{R}_2^T\boldsymbol{E}_{21}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{12})\boldsymbol{E}_{12} \\
=& -\boldsymbol{E}_{12}\boldsymbol{R}_2^T\boldsymbol{E}_{12}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{11} + \boldsymbol{E}_{12}\boldsymbol{R}_2^T\boldsymbol{E}_{21}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{11} - \boldsymbol{E}_{12}\boldsymbol{R}_1^T\boldsymbol{E}_{12}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{22} + \boldsymbol{E}_{12}\boldsymbol{R}_1^T\boldsymbol{E}_{21}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{22} \\
&- \boldsymbol{E}_{21}\boldsymbol{R}_2^T\boldsymbol{E}_{12}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{11} - \boldsymbol{E}_{21}\boldsymbol{R}_2^T\boldsymbol{E}_{21}\boldsymbol{R}_1\boldsymbol{F}_i\boldsymbol{E}_{11} + \boldsymbol{E}_{21}\boldsymbol{R}_1^T\boldsymbol{E}_{12}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{22} - \boldsymbol{E}_{21}\boldsymbol{R}_1^T\boldsymbol{E}_{21}\boldsymbol{R}_2\boldsymbol{F}_i\boldsymbol{E}_{22} \\
=&(-\boldsymbol{E}_{12}\boldsymbol{H}_2^T\boldsymbol{E}_{12} + \boldsymbol{E}_{12}\boldsymbol{H}_2^T\boldsymbol{E}_{21} + \boldsymbol{E}_{21}\boldsymbol{H}_2^T\boldsymbol{E}_{12} - \boldsymbol{E}_{21}\boldsymbol{H}_2^T\boldsymbol{E}_{21})\boldsymbol{H}_1\boldsymbol{F}_i\boldsymbol{E}_{11} \\
&+ (-\boldsymbol{E}_{12}\boldsymbol{H}_1^T\boldsymbol{E}_{12} + \boldsymbol{E}_{12}\boldsymbol{H}_1^T\boldsymbol{E}_{21} + \boldsymbol{E}_{21}\boldsymbol{H}_1^T\boldsymbol{E}_{12} - \boldsymbol{E}_{21}\boldsymbol{H}_1^T\boldsymbol{E}_{21})\boldsymbol{H}_2\boldsymbol{F}_i\boldsymbol{E}_{22}.
\end{aligned}
$$

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