Throughput and transmission capacity of ad hoc networks with channel state information

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***Abstract*— This paper develops a general framework for deriving the spatial throughput and transmission capacity of spatially Poisson distributed ad hoc networks for (i) random (e.g., fading) channels, and (ii) random transmission distances. In both of these scenarios the randomness of the channels and ranges invariably lowers both throughput and transmission capacity assuming that users randomly elect to transmit with some probability (i.e., Aloha). Assuming each node knows the channel state information (CSI, e.g., the channel gain and/or channel distance) to just its intended receiver, we propose a simple distributed threshold-based scheduling rule and derive the threshold that optimizes throughput. The gains are surprisingly large: a factor of three increase in throughput over Aloha is typical even with no further coordination between the nodes in the network.**

I. INTRODUCTION

This paper addresses two issues of contemporary interest in the field of ad hoc network capacity. First, we attempt to precisely characterize the effect that channel variations (e.g., shadowing and fading) have on the allowable density of simultaneous transmissions in the network. Second, we consider how channel state information available to the transmitter can be used to increase this density by utilizing a simple but effective distributed scheduling algorithm whose implementation and performance can be described quantitatively.

For both of these pursuits, our analytical tools center around the use of stochastic geometry and marked Poisson point processes, and their application to finding the maximum number of successful communication links that can be accommodated in a unit area, subject to an outage constraint. The metric used to quantify performance is termed the *transmission capacity*, and was introduced in [1] by the authors. Stated simply, transmission capacity measures the achievable rates, or equivalently the instantaneous mutual information, between an arbitrary pair of nearby (i.e., single-hop) transmitting and receiving nodes from an outage perspective. This metric succinctly quantifies the link-level performance of a large-scale ad hoc network or any unplanned network (e.g., open spectrum usage), upon which other networking functionalities, e.g., multi-hop, can be constructed.

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A. Relation to prior work

The transmission capacity is closely related to the ubiquitous transport capacity metric introduced by Gupta and Kumar [2] and extended to random channels in [3], with the distinctions that (i) a stochastic outage probability requirement is introduced, and (ii) the nodes are Poisson distributed. The stochastic outage probability requirement is practical as well as analytically pragmatic, and corresponds roughly to a packet error rate or other likely performance requirement in a real ad hoc network. The spatial Poisson assumption is taken for mathematical tractability, but has considerable empirical support; for example, the Poisson assumption for node positions has been used by numerous cellular service operators to successfully predict blocking probabilities [4].

The outage probability, throughput, and transmission capacity are each computable from the distribution of the normalized interference level seen by a typical receiver. This normalized interference has long been recognized as a spatial shot noise process [5], [6], [7]. Under the assumed power law channel model (capturing path loss attenuation) the normalized interference is known to be Lévy stable [6], [8], [7]. The impact of fading and other random non-distance dependent channel effects on the outage probability and transmission capacity has also been addressed [8], [9]. These characterizations are exploited in obtaining our performance bounds.

There has been some notable prior work on quantifying ad hoc network capacity in the presence of fading wireless channels. One of the earlier works [10] determined that fading actually increases the achievable rate regions (as opposed to the overall ad hoc network capacity) by providing statistical diversity, since the best set of transmit-receive pairs can be selected. This however, would require a global centralized search which is impractical. Using the transport capacity framework, some interesting recent results include a study on entirely random channels (no geometric dependence) that showed that shadowing or obstructions could increase the transport capacity [11] and a study on fading channels with geometric considerations valid for path-loss exponents greater than 3 that supported those authors' previous results in the lack of fading [3], [12]. Another notable paper [13] argued that although fading reduced a transport capacity lower bound by a logarithmic factor, fading actually increased the overall network capacity. Finally, a different line of investigation spurred by [14] has considered how mobility, throughput, and delay interact in the context of time-varying channels [15], [13], [16], [17], [18]. Essentially, in order to fully exploit fading, some delay must be introduced, which results in a delay-capacity tradeoff. We will not consider this tradeoff in this paper, however.

Generally speaking, prior work has concentrated primarily on the scaling of capacity upper and lower bounds with the number of nodes. In this work, we study an achievable set of rates, i.e., a capacity lower bound, from an outage perspective, and are able to accurately quantify scaling constants. Thus, we are able to precisely calculate the effect of channel fading and channel state information on a very pragmatic capacity lower bound.

B. Contributions

The results in this paper are distinct from the prior work on scheduling as we do not assume any optimal selection of nodes. Rather than optimally choosing the nodes with the best channels (which is infeasible in practice), our results are first based on an Aloha protocol where the users transmit with some probability independent of other users in the network. We derive an upper bound on the transmission capacity and show that when guaranteeing QoS, fading can only reduce the capacity. Second, we consider the realistic scenario where each user independently monitors the channel to its desired recipient (either through channel reciprocity or a very low rate feedback channel), and then transmits opportunistically only when the channel strength is above a threshold. We characterize the optimum such threshold, and show that this simple approach increases the capacity significantly (typically around 3X) over a channel-blind Aloha approach. Note that the proposed scheduling scheme is extremely simple, as it only requires transmitter channel state information; the majority of previous work on distributed scheduling for ad hoc networks (e.g., [19]) has assumed a higher level of interaction between nodes in the network.

II. AD HOC NETWORK MODEL

Consider a large ad hoc network, where the locations of *potential* transmitters at a typical point in time form a stationary Poisson point process $\Pi = \{X_i\}$ on the plane $\mathbb{R}^2$. Our attention will focus on a (typical) reference receiver, without loss of generality assumed to be located at the origin, o. The spatial density of the point process is denoted by λ, giving the average number of potential transmitters per unit area. We also assume that each potential transmitter, i, has an associated intended receiver (not in Π), and we let the index i refer to the pair. We consider two models.

Model 1: fading channels. In the first model we consider a channel model of the form $h(d, m) = md^{-\alpha}$, where d is the distance separating the transmitter and receiver, m is the (random) non-distance dependent channel effect (incorporating aspects such as fading and shadowing), and $\alpha > 2$ is the path loss exponent. It is convenient to define $\delta = 2/\alpha < 1$. For simplicity, in this model we assume each transmitter has an assigned intended receiver (not in Π) at a fixed distance r. Of course in reality transmitter to receiver distances will vary across pairs, but this assumption permits us to focus on the impact of fading on network performance. The fading effects, denoted by M, are modeled as random variables with distribution $F_M(m)$. In particular, M_{ij} is the random non-distance dependent channel state from transmitter i to receiver j. The $\{M_{ij}\}$ are assumed to be iid. Note that the channel model $h(d, m)$ with random $\{M_{ij}\}$ incorporates a large number of realistic channel models, including, e.g., Rayleigh fading and lognormal shadowing. As we are primarily concerned with the reference receiver at the origin, we can represent the state of the system at the typical time as a stationary *marked* Poisson point process (MPPP). The required marks for the first model include both the channels connecting each potential transmitter and its receiver ($\{M_{ii}\}$), and the channels connecting each potential transmitter with the reference receiver ($\{M_{i0}\}$):

$$\Pi_{\rm chan} = \{(X_i, M_{ii}, M_{i0})\}. \quad (1)$$

We will use the phrase **chan** to denote quantities specific to this model.

Model 2: random distances separating pairs. In the second model we consider the case where the distance separating each transmitter and its intended receiver varies randomly across transmitter receiver pairs. For simplicity, in this model we assume a simplified channel model of the form $h(d) = d^{-\alpha}$ for $\alpha > 2$, i.e., we assume a pure path loss channel model. Of course this channel model ignores fading and other important random non-distance dependent effects, but this assumption permits us to focus on the impact of the random distances separating communicating pairs on network performance. The random distances, denoted by D, are modeled as random variables with distribution $F_D(d)$. In particular, D_i is the random distance between potential transmitter i and its intended receiver. The $\{D_i\}$ are assumed to be iid. The required marks for the second model are the random distances between each potential transmitter and its associated receiver ($\{D_{ii}\}$):

$$\Pi_{\rm dist} = \{(X_i, D_i)\}. \quad (2)$$

We will use the phrase **dist** to denote quantities specific to this model.

Note that both channel models suffer from a physically unrealistic singularity at $d = 0$. This singularity can be corrected for by using a channel model of the form $h(d, m) = m/(1+d)^{\alpha}$, for example, but any such correction significantly complicates the resulting expressions. We have shown in simulation that the impact of the singularity on network performance is in most cases negligible.

Outage probability. A reception is assumed successful provided the signal to interference ratio (SIR) seen at the receiver is acceptably high. Ambient noise is not included as it does not have a material impact on the character of the results but significantly complicates the expressions. All nodes are assumed to require an SIR exceeding a specified

$\beta > 0$, with an outage resulting if this condition is not satisfied. Let q denote the probability of outage:

$$q = \mathbb{P}^0(\text{SIR} < \beta), \quad (3)$$

where the notation $\mathbb{P}^0(\cdot)$ is the Palm probability giving the receiver-average outage probability computed by considering the reference receiver at the origin.

Transmitters are assumed to employ constant transmission power ρ, i.e., we do not consider the impact of power control. The SIR is independent of ρ since an increase in ρ achieves a linear increase in both the signal and interference levels. Recall that Π includes all *potential* transmitters, but the actual aggregate interference is computed by summing over all *actual* interferers. Let $\Phi \subset \Pi$ denote the set of actual interferers at the typical time under consideration; we will assume that the decisions to transmit or not are made independently by each node, and independent of their location. It follows that Φ is also a stationary MPPP, albeit with a smaller intensity, denoted as $\mu \leq \lambda$. We discuss transmission decision rules for obtaining Φ from Π in Sections III (for the random channel model) and IV (for the random distance model).

For the fading channel model, given $\Phi_{\text{chan}} \subset \Pi_{\text{chan}}$, the outage probability for the typical receiver at the origin is

$$q_{\text{chan}}(\mu) = \mathbb{P}^0\left(\frac{\rho M_{00} r^{-\alpha}}{\sum_{i \in \Phi_{\text{chan}}} \rho M_{i0} |X_i|^{-\alpha}} < \beta\right), \quad (4)$$

where $|X_i|$ is the distance of interfering transmitter i from the reference receiver located at the origin. For the random distance model, given $\Phi_{\text{dist}} \subset \Pi_{\text{dist}}$, the outage probability for the typical receiver at the origin is

$$q_{\text{dist}}(\mu) = \mathbb{P}^0\left(\frac{\rho (D_0)^{-\alpha}}{\sum_{i \in \Phi_{\text{dist}}} \rho |X_i|^{-\alpha}} < \beta\right). \quad (5)$$

It is apparent that $q(\mu)$ depends upon the sum of a random number of random variables, and as such a closed form expression for $q(\mu)$ is not available. As such we resort to bounds. In particular, we will establish lower bounds of the form

$$q_{\text{lb}}(\mu) = 1 - \exp\left\{-\kappa\mu\right\}, \quad (6)$$

where κ may depend on μ, depending on the transmission rule for obtaining Φ from Π.

Information theoretic interpretation. The SIR-based outage probability introduced above corresponds very simply to achievability in the information theoretic sense. If all nodes are assumed to transmit Gaussian symbols and the channel is narrowband, the mutual information between the transmitting (X_k) and receiving node (Y_k) is given by:

$$I(X_k; Y_k | \mathbf{H}, \Pi) = \log_2(1 + SIR), \quad (7)$$

where SIR is defined in either (4) or (5). Since only the term $I(X_k; Y_k | \mathbf{H}, \Pi)$ is considered, an implicit assumption is that multi-user interference is treated as noise. Although this is the only metric studied in this work, interference cancellation can indeed be incorporated into this framework [20].

Mutual information, or rate, is measured conditioned on channel conditions, node locations, and the instantaneous set of transmitters. Thus, the quantity in (7) measures the rate of reliable information flow from X_k to Y_k at a snapshot of the network. In the outage formulation, the instantaneous mutual information is treated as a random variable (a function of random interferer locations and channel conditions) and an outage occurs whenever this random variable falls below the desired rate of communication. Thus, for rate R the outage probability is given by $P_{out} = \mathbb{P}(I(X_k; Y_k | \mathbf{H}, \Pi) < R)$. Since there is a one-to-one mapping between mutual information and SIR in this expression, outage can equivalently be stated in terms of SIR, as in (4) and (5), with $\beta = 2^R - 1$.

Throughput. The achieved network throughput is

$$\tau(\mu) = \mu(1 - q(\mu)), \quad (8)$$

i.e., the product of the attempted transmission intensity (μ) times the average probability of success ($1 - q(\mu)$). Using the *lower* bound on the outage probability gives an *upper* bound on the throughput of the form

$$\tau_{\text{ub}}(\mu) = \mu e^{-\kappa\mu}. \quad (9)$$

For the case where κ is independent of μ the expression for the throughput matches the usual form of randomized MAC throughput $Ge^{-\kappa G}$, where G is the attempt rate and $e^{-\kappa G}$ is the success probability. In contrast to most "classical" randomized MAC throughput derivations, however, our throughput expression incorporates i) a spatial model, ii) a realistic channel model, and iii) a realistic reception model, all captured in κ.

Transmission capacity. Although the network throughput is an important system performance metric, it often obscures the fact that high throughput is sometimes obtained at the expense of unacceptably high outage. This is especially important in ad hoc networks as wasted transmissions both cause unnecessary interference for other nodes and they waste precious energy. As a simple example of high throughput achieved through high outage, note that classic slotted Aloha has a throughput of the form Ge^{-G}, which is maximized for an attempt rate of $G = 1$. The optimal throughput at $G = 1$ is $1/e \approx 0.32$, but the outage probability is $1 - 1/e \approx 0.68$. Thus 68% of all attempted transmissions must fail to achieve the optimal throughput. For many important network applications, e.g., streaming media, high levels of outage are unacceptable, and as such it is desirable that the network operate in a low outage regime. With this in mind, we define the *optimal contention density*, $\nu(\epsilon)$, as the maximum spatial density of *attempted* transmissions such that the corresponding outage probability is $\epsilon \in [0, 1]$. The parameter ϵ serves as a proxy for network quality of service. The optimal contention density is found by solving $q(\nu) = \epsilon$ for ν. Having found the optimal contention density, we define the *transmission capacity* as the corresponding spatial density of *successful* transmissions,

$$c(\epsilon) = \nu(\epsilon)(1-\epsilon). \tag{10}$$

The advantage of the transmission capacity framework is that it yields the maximum throughput that can be obtained subject to a maximum permissible outage probability, i.e., a QoS requirement.

III. First model: fading channels

Our purpose in this section is to address the following two questions. First: *what is the impact of fading on network performance when nodes make randomized decisions whether or not to transmit?* The motivation behind this question is the intuition that randomized transmission decisions may be harmful in the presence of fading because transmitting nodes may find the channel to their intended receiver in a deep fade, and thereby incur an unacceptably high outage probability. Second: *how can local channel state information be exploited to improve network performance?* The motivation behind this question is the intuition that transmitting only when the channel to one's intended receiver is strong may significantly improve performance above randomized transmission decisions. We emphasize there is no claim that the channel-aware transmission decisions are globally optimal: global optimality would require global channel state knowledge by each node, which is clearly unrealistic. Transmitter channel state information, however, is a realistic assumption, especially when channel coherence times extend across multiple transmission attempts.

Motivated by the above two questions, we introduce two transmission decision rules: $i)$ randomized transmissions, made independently across nodes, and independent of the channel states, and $ii)$ a threshold rule where each node only transmits if its channel is sufficiently strong.

A. Randomized transmission decisions

In this scenario each node makes a random decision to transmit, independent of its (unknown) channel state to its receiver, by transmitting with a specified probability p. In this case the intensity of attempted transmissions is $\mu = \mu(p) = \lambda p$ and, as shown below, κ in (6) is independent of μ. It follows from simple calculus that the optimal transmission probability to maximize the throughput bound is $p_{\rm opt} = \frac{1}{\lambda\kappa}$, and thus the optimal intensity of transmission attempts is $\mu_{\rm opt} = \frac{1}{\kappa} \wedge \lambda$. The corresponding bound optimal throughput is

$$\tau_{\rm opt} = \tau_{\rm ub}(\mu_{\rm opt}) = \frac{1}{e\kappa}, \tag{11}$$

assuming $\lambda > 1/\kappa$. We emphasize the optimality holds for the bound, not the throughput itself, however our numerical and simulation results have shown that the approximation is valid over most regimes of interest. Using the lower bound on the outage probability gives an upper bound on the transmission capacity of the form

$$c_{\rm ub}(\epsilon) = \frac{-(1-\epsilon)\ln(1-\epsilon)}{\kappa}. \tag{12}$$

Note that the outage probability that maximizes the transmission capacity bound is $\epsilon_{\rm opt} = 1 - 1/e$, with a corresponding bound optimal transmission capacity of

$$c_{\rm opt} = c_{\rm ub}(\epsilon_{\rm opt}) = \frac{1}{e\kappa} = \tau_{\rm opt}, \tag{13}$$

assuming $\lambda > 1/\kappa$. Thus the bound-optimal transmission capacity equals the bound-optimal throughput. If the throughput achieves $\tau_{\rm opt}$, then there is a corresponding outage probability associated with that throughput, and as such the transmission capacity at that outage probability must achieve the same throughput.

Theorem 1: Under the fading channels model with iid fades M (with distribution F_M), and with each node making a randomized transmission decision rule with parameter p, the intensity of attempted transmissions is $\mu = \lambda p$. Moreover, the constant κ in (6) is independent of p, and hence independent of μ, and is given by

$$\kappa^{\rm chan}_{\rm rand} = \pi\beta^\delta r^2 \mathbb{E}[M^\delta]\mathbb{E}[M^{-\delta}]. \tag{14}$$

The lower bound on the outage probability ($q^{\rm chan}_{\rm lb,rand}(\mu)$), the upper bound on the network throughput ($\tau^{\rm chan}_{\rm ub,rand}(\mu)$), the bound optimal throughput ($\tau^{\rm chan}_{\rm opt,rand}$), and the upper bound on the transmission capacity ($c^{\rm chan}_{\rm ub,rand}(\epsilon)$), are given by (6), (9), (11), and (12), respectively with $\kappa = \kappa^{\rm chan}_{\rm rand}$. The proof is found in the appendix.

The expressions for optimal throughput and transmission capacity are best understood in the context of spatial sphere packing. In particular, both optimal throughput and transmission capacity are proportional to $1/\kappa$, which in turn are proportional to $1/(\pi r^2)$. In this form it is clear that the impact of the path loss attenuation, α, the SIR threshold, β, and the channel statistics, F_M, determine the effective radius of the transmission-free disk that must surround each successful reception.

B. Channel aware transmission decisions

We now suppose that each potential transmitter is aware of the channel to its intended receiver. Given our stated assumption that all transmitter receiver pairs are at a fixed distance r, it follows that knowledge of the fading channel state M_{ii} suffices to characterize the channel. Each potential transmitter i only transmits if its channel is acceptably strong, i.e., $M_{ii} > t$, where t is the global channel state threshold. The attempted transmission intensity is

$$\mu = \mu(t) = \lambda \mathbb{P}(M > t) = \lambda \bar{F}_M(t). \tag{15}$$

As we will show below, the constant $\kappa = \kappa(t)$ in (6) in this case depends upon the threshold t, and thus on μ. It follows that the outage probability bound becomes

$$q_{\rm lb}(t) = 1 - e^{-\kappa(t)\mu(t)}, \tag{16}$$

and the corresponding throughput bound becomes

$$\tau_{\rm ub}(t) = \mu(t)e^{-\kappa(t)\mu(t)}. \tag{17}$$

Theorem 2: Under the fading channels model with fades M (with distribution F_M), and with each node making a *threshold based transmission decision with threshold t, the intensity of attempted transmissions is $\mu(t) = \lambda \bar{F}_M(t)$. Moreover, $\kappa(t)$ is given by*

$$\kappa^{\text{chan}}_{\text{thresh}}(t) = \pi \beta^\delta r^2 \frac{\mathbb{E}[M^\delta]\mathbb{E}[M^{-\delta}\mathbb{I}_{M>t}]}{\bar{F}_M(t)}, \quad (18)$$

where $\mathbb{I}_A$ is the indicator random variable of the event A. The lower bound on the outage probability ($q^{\text{chan}}_{\text{lb,thresh}}(t)$) and the upper bound on the network throughput ($\tau^{\text{chan}}_{\text{ub,thresh}}(t)$), are given by (16) and (17) respectively with $\kappa(t) = \kappa^{\text{chan}}_{\text{thresh}}(t)$. The throughput bound (17) is concave in t, and the optimal threshold t_{opt} that maximizes the throughput bound is the unique solution of the equation

$$\mu(t)\left(\kappa'(t)\frac{\mu(t)}{\mu'(t)} + \kappa(t)\right) = 1. \quad (19)$$

The transmission capacity with outage probability constraint ϵ, is given by

$$c^{\text{chan}}_{\text{ub,thresh}}(\epsilon) = \mu(t^*(\epsilon))(1-\epsilon), \quad (20)$$

where $t^*(\epsilon)$ is the unique solution to

$$\kappa(t)\mu(t) = -\ln(1-\epsilon). \quad (21)$$

The proof is found in the appendix.

C. Example: Rayleigh fading

Consider a Rayleigh fading channel where the random non-distance dependent channel state is given by an exponential random variable with parameter $u > 0$, i.e., $M \sim \text{Exp}(u)$. Then

$$\mathbb{E}[M^\delta] = u^{-\delta}\Gamma(1+\delta), \quad \mathbb{E}[M^{-\delta}] = u^\delta \Gamma(1-\delta), \quad (22)$$

and thus

$$\kappa^{\text{chan}}_{\text{rand}} = \pi\beta^\delta r^2 \frac{\pi\delta}{\sin(\pi\delta)}, \quad (23)$$

for $\Gamma(x)$ the Gamma function. Note the dependence on u vanishes. This calculation is all that is needed to compute all the quantities in Theorem 1. For Theorem 2 we compute

$$\mathbb{E}[M^{-\delta}\mathbb{I}_{M>t}] = u^\delta \Gamma(1-\delta, tu), \quad (24)$$

for $\Gamma(x,z)$ the incomplete Gamma function, and thus

$$\kappa^{\text{chan}}_{\text{thresh}}(t)\mu(t) = \pi\beta^\delta r^2 \Gamma(1+\delta)\Gamma(1-\delta, tu)\mu. \quad (25)$$

The quantity $t^*(\epsilon)$ is given by

$$t^*(\epsilon) = \frac{1}{u}\Gamma^{-1}\left(1-\delta, \frac{-\log(1-\epsilon)}{\Gamma(1+\delta)\pi\beta^\delta r^2 \lambda}\right), \quad (26)$$

where $z = \Gamma(x,s)^{-1}$ is the inverse incomplete Gamma function that solves $\Gamma(x,z) = s$. From here the transmission capacity is given by

$$c(\epsilon) = \lambda e^{-ut^*(\epsilon)}(1-\epsilon). \quad (27)$$

Figure 1 presents the outage probability, throughput, and transmission capacity for the fading channels model with Rayleigh fading, for both randomized and threshold based transmissions. For easy comparison the channel aware outage probability and throughput are shown versus the transmission probability p, which corresponds to a threshold $t = t(p)$ solving $\bar{F}_M(t) = p$. In the case of Rayleigh fading we have $t(p) = -\frac{1}{u}\ln(p)$. Numerical values for the parameters are shown in Table I. Besides the plots for the two transmission decision rules we also show the performance under randomized transmissions without any random channel effects. The plots illustrate both that $i)$ randomized transmissions perform worse under fading conditions, and $ii)$ the fades can be exploited through the threshold rule to achieve superior performance. For example, for 10% outage, randomized transmission provides a transmission capacity of only 0.0002, while the threshold rule corresponds to a capacity of approximately 0.0007.

TABLE I
NUMERICAL VALUES FOR PARAMETERS

Symbol	Description	Value
α	path loss exponent	4
β	SIR requirement	10
r	tx-rx distance	5 (meters)
λ	density of pot. trans.	0.01 (1/meters^2)
u	Rayleigh fading param.	1.0

IV. SECOND MODEL: RANDOM TRANSMISSION DISTANCES

Our purpose in this section is similar to that of the previous section in that our focus is to understand how knowledge of the channel can be exploited to improve network performance. Recall in this model there are no non-distance dependent random channel effects, i.e., the attenuation is solely due to path loss. However, the distances separating transmitters and their intended receivers vary across pairs, which is physically realistic. The intuition is that blind randomized transmission decisions, irrespective of the pair separation distance, could have a significant negative impact on network performance. On the other hand, knowledge of the separation distance can be exploited to significantly improve network performance, say as compared with the fixed pair distance case.

A. Randomized transmission decisions

In this scenario each node makes a random decision to transmit, independent of its (unknown) distance to its receiver, by transmitting with a specified probability p. Again we have $\mu = \mu(p) = \lambda p$ and κ in (6) is independent of μ.

Theorem 3: Under the second model with random pair distances D with distribution F_D, and with each node making a randomized transmission decision rule with parameter p, the intensity of attempted transmissions is $\mu = \lambda p$. Moreover, the constant κ in (6) is independent of p, and hence independent of μ, and is given by

$$\kappa^{\text{dist}}_{\text{rand}} = \pi\beta^\delta \mathbb{E}[D^2]. \quad (28)$$

The lower bound on the outage probability ($q^{\text{dist}}_{\text{lb,rand}}(\mu)$), the upper bound on the network throughput ($\tau^{\text{dist}}_{\text{ub,rand}}(\mu)$), the bound optimal throughput ($\tau^{\text{dist}}_{\text{opt,rand}}$), and the upper bound *on the transmission capacity ($c^{\rm dist}_{\rm ub,rand}(\epsilon)$), are given by (6), (9), (11), and (12), respectively with $\kappa = \kappa^{\rm dist}_{\rm rand}$. The proof is found in the appendix.*

B. Distance aware transmission decisions

We now suppose that each potential transmitter is aware of the distance to its intended receiver. Each potential transmitter i only transmits if its receiver is acceptably close, i.e., $D_i < t$, where t is the global distance threshold. The attempted transmission intensity is

$$\mu = \mu(t) = \lambda \mathbb{P}(D < t) = \lambda F_D(t). \qquad (29)$$

As before, the constant $\kappa = \kappa(t)$ in (6) in this case depends upon the threshold t, and thus on μ.

Theorem 4: Under the second model with random pair distances D with distribution F_D, and with each node making a threshold based transmission decision with threshold t, the intensity of attempted transmissions is $\mu(t) = \lambda F_D(t)$. Moreover, $\kappa(t)$ is given by

$$\kappa^{\rm dist}_{\rm thresh}(t) = \pi \beta^{\delta} \frac{\mathbb{E}[D^2 \mathbb{I}_{D<t}]}{F_D(t)}, \qquad (30)$$

The lower bound on the outage probability ($q^{\rm dist}_{\rm lb,thresh}(t)$) and the upper bound on the network throughput ($\tau^{\rm dist}_{\rm ub,thresh}(t)$), are given by (16) and (17) respectively with $\kappa(t) = \kappa^{\rm dist}_{\rm thresh}(t)$. The throughput bound (17) is concave in t, and the optimal threshold $t_{\rm opt}$ that maximizes the throughput bound is the unique solution of (19). The transmission capacity with outage probability constraint ϵ, is given by (20), where $t^*(\epsilon)$ is the unique solution to (21). The proof is found in the appendix.

C. Example: nearest neighbor transmissions

Consider the case when each node elects to transmit to its nearest neighbor. For given potential transmitter $i \in \Pi$, the distance, D_i, to its nearest neighbor has distribution

$$\bar{F}_D(d) = \mathbb{P}(D > d) = \mathbb{P}(\Pi \cap b(o,d) = \emptyset) = e^{-\pi\lambda d^2}. \quad (31)$$

Then $\mathbb{E}[D] = \frac{1}{2\sqrt{\lambda}}$, and $\mathbb{E}[D^2] = \frac{1}{\pi\lambda}$; this is all that is needed to compute all the quantities in Theorem 3. For Theorem 4, we can compute

$$\mathbb{E}[D\mathbb{I}_{D<t}] = \frac{1 - 2Q(\sqrt{2\pi\lambda}t)}{2\sqrt{\lambda}} - te^{-\pi\lambda t^2}, \qquad (32)$$

and

$$\mathbb{E}[D^2\mathbb{I}_{D<t}] = \frac{1 - (1 + \pi\lambda t^2)e^{-\pi\lambda t^2}}{\pi\lambda}, \qquad (33)$$

where Q is the CCDF for a standard $\mathcal{N}(0,1)$ random variable. Defining $a = \pi\lambda t^2$, we can parameterize $q_{\rm lb}$ in terms of a:

$$q_{\rm lb}(a) = 1 - \exp\left\{-z^{-\delta}(1+a)e^{-a}\right\}. \qquad (34)$$

Solving $q_{\rm lb}(a) = \epsilon$ for a yields

$$a(\epsilon) = -1 - W_{-1}\left(\frac{-1 - z^{\delta}\log(1-\epsilon)}{e}\right), \qquad (35)$$

where W_{-1} is the $k = -1$ branch of Lambert's $W(z)$ function, which solves $z = W(z)e^{W(z)}$. Note that we can write $\mathbb{P}(D < t) = 1 - e^{-a}$. The transmission capacity is then

$$c_{\rm ub}(\epsilon) = \lambda\left(1 - e^{-a(\epsilon)}\right)(1-\epsilon). \qquad (36)$$

Figure 2 presents the outage probability, throughput, and transmission capacity for the random pair distances model with nearest neighbor transmissions, for both randomized and threshold based transmission rules. For easy comparison the distance aware outage probability and throughput are shown versus the transmission probability p, which corresponds to a threshold $t = t(p)$ solving $\bar{F}_D(t) = p$. In the case of random nearest neighbor distances we have $t(p) = \sqrt{\frac{-\ln p}{\pi}}$. All three plots demonstrate the significant improvement in performance attainable by exploiting local channel state information. At 10% outage, for example, we see a tremendous difference in transmission capacity between the randomized and threshold rules. Table I gives the values of the constants (except for r and u which don't apply in this case). As in Figure 1, we also show the case when there are no variations in pair distance, i.e., the fixed distance between transmitters and their intended receivers is $r = \mathbb{E}[D] = \frac{1}{2\sqrt{\lambda}}$.

V. Conclusion

The two primary observations from this work are $i)$ randomized transmissions perform poorly in the presence of either fading or variable channel distances, and $ii)$ these channel variations can easily be exploited through the use of simple threshold schemes. We emphasize the practical nature of the threshold schemes, which require no sharing of information among nodes other than with the intended receiver. There are numerous extensions to this framework, most notably including power control and an OFDM-like case where each transmitter may select from one of K known channels. Our current work is focused on evaluating the performance of our threshold scheme with the throughput optimal scheduling schemes found in the literature.

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APPENDIX

PROOF OF THEOREM 1

Defining the constant $y = \frac{1}{\beta r^\alpha}$, and the normalized interference

$$Y(\mu) = \sum_{i \in \Phi} W_i |X_i|^{-\alpha}, \quad W_i = \frac{M_{i0}}{M_{00}}, \tag{37}$$

we can express the outage probability as the complementary cumulative distribution function (CCDF) of Y:

$$q(\mu) = \mathbb{P}^0(Y(\mu) > y). \tag{38}$$

It is known that the random variable $Y = Y(\mu)$ is Lévy stable with stability parameter $\delta = 2/\alpha < 1$ [5], [8]. Bounds of this form are shown to be asymptotically tight as $y \to \infty$ in [9]. In the case of randomized transmissions we have both $M_{i0} \sim F_M$ and $M_{00} \sim F_M$, which means the distribution of the $\{W_i\}$ is

$$W_i \sim F_W(w) = \int_0^\infty F_M(mw) dF_M(m). \tag{39}$$

In [9] we show that the lower bound on outage under the single channel randomized transmission model is

$$q_{\rm lb}(\mu) = 1 - \exp\{-\pi y^{-\frac{2}{\alpha}} \mathbb{E}[W^{\frac{2}{\alpha}}] \mu\}. \tag{40}$$

To get the expression in Theorem 1, we simply exchange the order of integration and use a change of variable $x = mw$:

$$\begin{aligned}
\mathbb{E}[W^{\frac{2}{\alpha}}] &= \int_0^\infty w^{\frac{2}{\alpha}} f_W(w) dw \\
&= \int_0^\infty w^{\frac{2}{\alpha}} \left(\int_0^\infty m f_M(mw) f_M(m) dm \right) dw \\
&= \int_0^\infty m f_M(m) \left(\int_0^\infty w^{\frac{2}{\alpha}} f_M(mw) dw \right) dm \\
&= \int_0^\infty m f_M(m) \left(\int_0^\infty \left(\frac{x}{m}\right)^{\frac{2}{\alpha}} f_M(x) \frac{1}{m} dx \right) dm \\
&= \mathbb{E}[M^{\frac{2}{\alpha}}] \int_0^\infty m^{-\frac{2}{\alpha}} f_M(m) dm \\
&= \mathbb{E}[M^{\frac{2}{\alpha}}] \mathbb{E}[M^{-\frac{2}{\alpha}}].
\end{aligned}$$

PROOF OF THEOREM 2

By our assumption that channel states are independent across receivers, even from a common transmitter, it follows that the distribution of the M_{i0} is independent of the threshold t, for all $i \neq 0$. Considering (37), however, it is clear that the distribution of $M_{ii} = M_{ii}(t)$ *is* affected by the transmission rule. In particular, $M_{00}(t)$ has a distribution given by:

$$F_{M|t}(m) = \mathbb{P}(M \leq m \mid M > t) = \frac{F_M(m) - F_M(t)}{\bar{F}_M(t)}, \tag{41}$$

for $m \geq t$. It follows that the mark ratios $W_i(t) = M_{i0}/M_{00}(t)$ in (37) have distribution

$$\begin{aligned}
F_{W|t}(w) &= \int_t^\infty F_M(mw) dF_{M|t}(m) && (42) \\
&= \frac{1}{\bar{F}_M(t)} \int_t^\infty F_M(mw) dF_M(m). && (43)
\end{aligned}$$

As in the proof of Theorem 1, we employ the result that a lower bound on the outage probability when the channel ratios are iid $\{W_i\}$ is given by (40). To get the expression in Theorem 2, we again exchange the order of integration and use a change of variable $x = mw$:

$$\begin{aligned}
\mathbb{E}[W(t)^{\frac{2}{\alpha}}] &= \int_0^\infty w^{\frac{2}{\alpha}} f_W(w) dw \\
&= \frac{1}{\bar{F}_M(t)} \int_0^\infty w^{\frac{2}{\alpha}} \left(\int_t^\infty m f_M(mw) f_M(m) dm \right) dw \\
&= \frac{1}{\bar{F}_M(t)} \int_t^\infty m f_M(m) \left(\int_0^\infty w^{\frac{2}{\alpha}} f_M(mw) dw \right) dm \\
&= \frac{1}{\bar{F}_M(t)} \int_t^\infty m f_M(m) \left(\int_0^\infty \left(\frac{x}{m}\right)^{\frac{2}{\alpha}} f_M(x) \frac{dx}{m} \right) dm \\
&= \frac{1}{\bar{F}_M(t)} \mathbb{E}[M^{\frac{2}{\alpha}}] \int_t^\infty m^{-\frac{2}{\alpha}} f_M(m) dm \\
&= \frac{1}{\bar{F}_M(t)} \mathbb{E}[M^{\frac{2}{\alpha}}] \mathbb{E}[M^{-\frac{2}{\alpha}} \mathbb{I}_{M>t}].
\end{aligned}$$

Proof of Theorem 3

Defining the constant $z = 1/\beta$ and the normalized interference

$$Z(\mu) = \sum_{i\in\Phi} \left(\frac{|X_i|}{D_0}\right)^{-\alpha}, \tag{44}$$

we can express the outage probability as the complementary cumulative distribution function (CCDF) of Z:

$$q(\mu) = \mathbb{P}^0(Z(\mu) > z). \tag{45}$$

Select from Φ the set of dominant interferers, with dominance level z:

$$\Phi_z = \left\{(X_i, D_i) \in \Phi \ : \ \left(\frac{|X_i|}{D_0}\right)^{-\alpha} > z\right\}. \tag{46}$$

The process Φ_z is non-stationary with local intensity at location $x \in \mathbb{R}^2$ of

$$\mu_z(x) = \mu\mathbb{P}\left(\left(\frac{|x|}{D_0}\right)^{-\alpha} > z\right) = \mu\mathbb{P}(D_0 > |x|z^{\frac{1}{\alpha}}). \tag{47}$$

Define the normalized interference from dominant interferers:

$$Z_z = \sum_{i\in\Phi_\beta} \left(\frac{|X_i|}{D_0}\right)^{-\alpha}. \tag{48}$$

Then a lower bound on the outage probability is

$$q(\mu) = \mathbb{P}(Z > z) > \mathbb{P}(Z_z > z) = 1 - \mathbb{P}(\Phi_z = \emptyset). \tag{49}$$

The RHS is a void probability for the nonstationary process Φ_z, which may be evaluated as

$$\begin{aligned} q_{\rm lb}(\mu) &= 1 - \exp\left\{-\int_{\mathbb{R}^2} \mu_z(x)dx\right\} \\ &= 1 - \exp\left\{-2\pi\mu \int_0^\infty \bar{F}_D\left(rz^{\frac{1}{\alpha}}\right) rdr\right\} \\ &= 1 - \exp\left\{-\pi z^{-\frac{2}{\alpha}}\mathbb{E}[D^2]\mu\right\}, \end{aligned}$$

where the last step is obtained by writing $\bar{F}_D(d) = \int_d^\infty dF_D(d)$, and exchanging the order of integration.

Proof of Theorem 4

The distribution of distances from transmitters to their intended receivers under a distance threshold of t is

$$F_{D|t}(d) = \frac{F_D(d)}{F_D(t)}, \quad 0 \le d \le t. \tag{50}$$

Form $\Phi_z \subset \Phi$, the dominant interferers with dominance level z, as in (46). Conditioned on the reference transmitter electing to transmit, the process Φ_z is non-stationary with local intensity

$$\begin{aligned} \mu_z(x) &= \lambda F_D(t)\bar{F}_{D|t}\left(|x|z^{\frac{1}{\alpha}}\right) \\ &= \lambda\left[F_D(t) - F_D\left(|x|z^{\frac{1}{\alpha}}\right)\right], \quad |x| < z^{-\frac{1}{\alpha}}t, \end{aligned}$$

and intensity of 0 outside $b(o, z^{-\frac{1}{\alpha}}t)$. Using the lower bound (49), we obtain

$$\begin{aligned} q_{\rm lb}(t) &= 1 - \exp\left\{-\int_{\mathbb{R}^2} \mu_z(x)dx\right\} \\ &= 1 - \exp\left\{-2\pi\lambda \int_0^{z^{-\frac{1}{\alpha}}t} \left[F_D(t) - F_D\left(rz^{\frac{1}{\alpha}}\right)\right] rdr\right\} \\ &= 1 - \exp\left\{-\pi\lambda z^{-\frac{2}{\alpha}}\mathbb{E}[D^2\mathbb{I}_{D<t}]\right\}, \end{aligned}$$

where the last step is obtained by writing the CDF as a PDF and exchanging the order of integration.

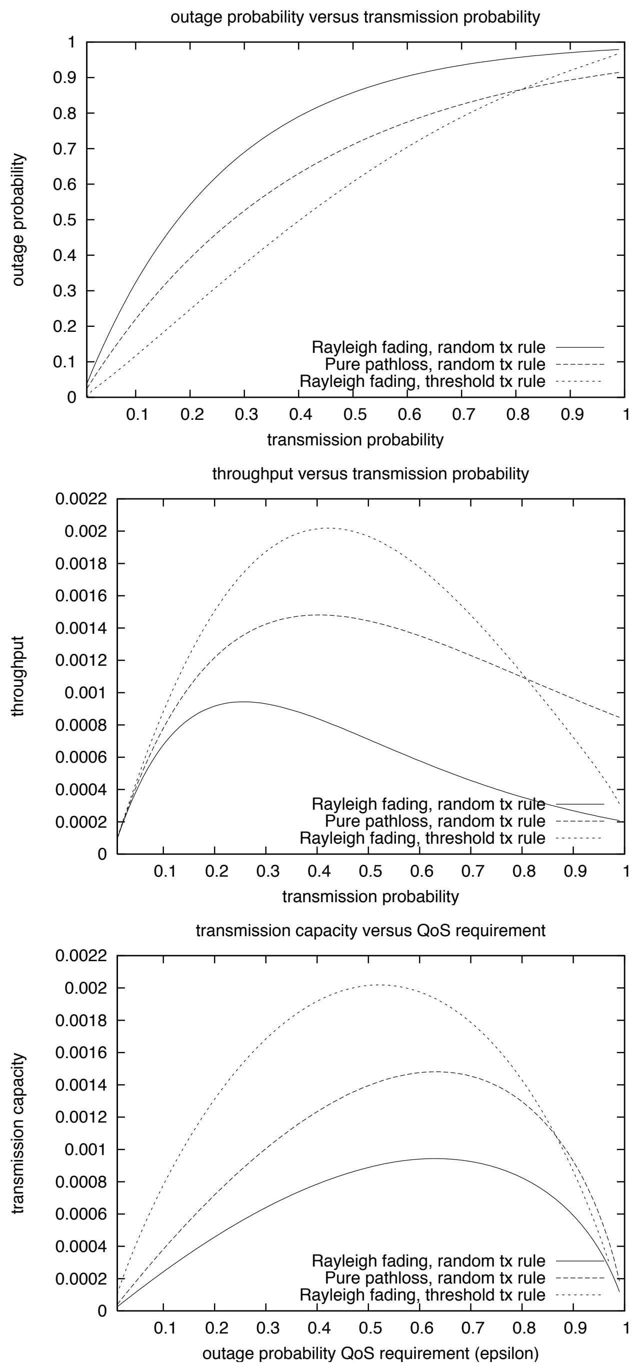

Fig. 1. Numerical results for the Rayleigh fading channel model with a fixed distance separating each transmitter and its intended receiver. The three curves on each plot are i) Rayleigh fading ($M \sim \mathrm{Exp}(1)$) with a randomized transmission rule (with parameter p), ii) pure path loss attenuation with a randomized transmission rule, and iii) Rayleigh fading with a threshold transmission rule (with $t(p)$ satisfying $p = F_M(t)$). The three plots are of outage probability, throughput, and transmission capacity. All three performance metrics show dramatic improvements when channel information is exploited.

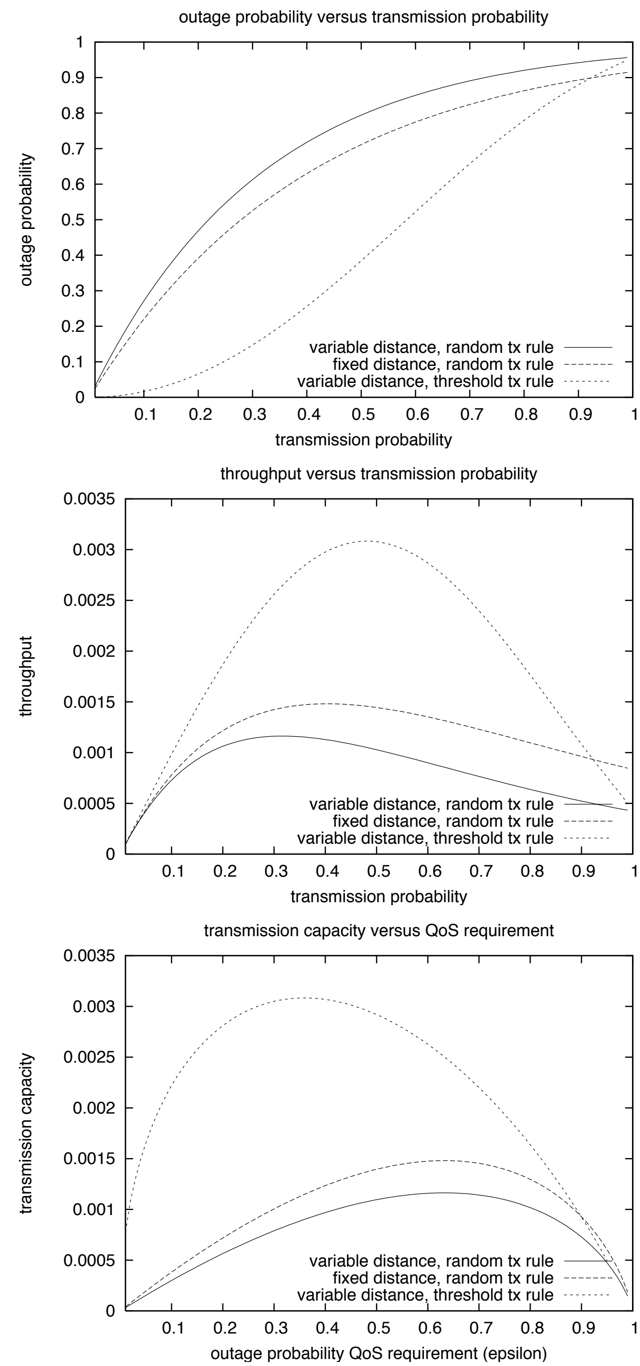

Fig. 2. Numerical results for the pure path loss channel model with random nearest neighbor distances between transmitters and receivers. The three curves on each plot are i) variable transmitter receiver distances with a randomized transmission rule (with parameter p), ii) fixed transmitter receiver distances (with $r = \mathbb{E}[D]$) with a randomized transmission rule, and iii) variable transmitter receiver distances with a threshold transmission rule (with $t(p)$ satisfying $p = F_D(t)$). The three plots are of outage probability, throughput, and transmission capacity. All three performance metrics show dramatic improvements when channel information is exploited.