Performance of ALOHA and CSMA in Spatially Distributed Wireless Networks

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Abstract—In this paper the performance of unslotted ALOHA and CSMA are analyzed in spatially distributed wireless networks. Users/packets arrive randomly in space and time according to a Poisson process, and are thereby transmitted to their intended destinations using a fully-distributed MAC protocol (either ALOHA or CSMA). An SINR-based model is considered, and a packet transmission is successful if the received SINR is above a threshold value for the duration of the packet. Accurate bounds to the probability of outage, which is a function of the density of transmissions, are developed for both MAC protocols. These bounds are used to evaluate the performances of ALOHA and CSMA, and to gain insight into the design of general MAC protocols for ad hoc networks. Moreover, CSMA with receiver-sensing is proposed to improve the performance of CSMA.

I. INTRODUCTION

A major challenge in the field of wireless communications in spatially distributed networks is sharing the medium and the available resources in a distributed manner. Sharing the medium has the adherent problem of interference, which may lead to erroneous reception of packets. In this paper, we consider nodes that are randomly distributed in space and address the problem of interference through the MAC layer design, which is an essential source of efficient resource allocation. The MAC protocols ALOHA and CSMA are applied for the communication between transmitters and receivers, and we investigate how often the packets are received successfully.

We ask the following main questions: given a fixed transmission power and signal-to-interference-plus-noise ratio (SINR) threshold for each single-hop transmitter-receiver link in the network, (a) what is the probability of successful transmission if unslotted ALOHA (i.e., transmit immediately upon reception) is used by all nodes in the network, and (b) what is the probability of successful transmission if a simple and fully-distributed CSMA mechanism (i.e., check the measured SINR before transmitting) is used by all nodes?

We consider a network in which transmitter nodes are randomly located according to a Poisson point process (PPP) with a specified spatial density, and packets arrive randomly in time according to a 1-dimensional PPP. In order to derive precise results, we focus exclusively on single-hop communication and assume that each transmitter wishes to communicate with a receiver a fixed distance away from it. All multi-user interference is treated as noise, and our model uses the SINR to evaluate the performance (i.e., outage probability) of the communication system. The only source of randomness in the model is in the location of nodes and concurrent transmissions, which allows

us to focus on the relationships between transmission density, outage probability, and the choice of MAC protocol.

A. Related Work

There has been a notable amount of research done on the performance of ALOHA in ad hoc networks. A number of different researchers have analyzed slotted ALOHA using a Poisson model for transmitter locations, considering transmission capacity and success probability of the network [1] [3]. Ferrari and Tonguz [6] have analyzed the transport capacity of slotted ALOHA and CSMA, showing that for low transmission densities the performance of slotted ALOHA is almost twice that of CSMA. However, for increasing densities, while the capacity of ALOHA drops to zero, the capacity of CSMA increases, making CSMA more beneficial at higher densities.

Other recent works have also considered the performance of ALOHA, showing among others that the scaling of transport capacity depends on the amount of attenuation in the channel [7] [8]. However, most of the research done thus far, focuses on deterministic SINR models and employs deterministic channel access schemes, which thereby precludes the occurrences of outages. In order to best model the behavior of a distributed ad hoc network at the MAC layer, a stochastic SINR requirement must be used, as is the case in our model.

Within our Poisson model with random location of nodes, neither unslotted ALOHA nor CSMA appear to have been analyzed in detail, despite the fact that CSMA is one of the most widely used MAC protocols today. Perhaps the closest work is that of Hasan and Andrews [4] where success probability of ALOHA and CSMA is analyzed for a similar spatial model assuming that a scheduling mechanism creates an interference-free *guard zone* (i.e., circle) around the receiver and the optimal guard zone size is studied. The CSMA mechanism we consider is able to suppress some nearby interferers, but is not able to create a perfect guard zone; in the future we hope to utilize the results of [4] to study the optimal sensing zone for the flavor of CSMA considered here.

II. PRELIMINARIES

A. System Model

We wish to analyze the performance of CSMA in a network with randomly located users and random transmission times. One possible model that could be used is as follows: transmitters are located on an infinite 2-D plane according to a homogeneous PPP with spatial density λ^s, and each transmitter
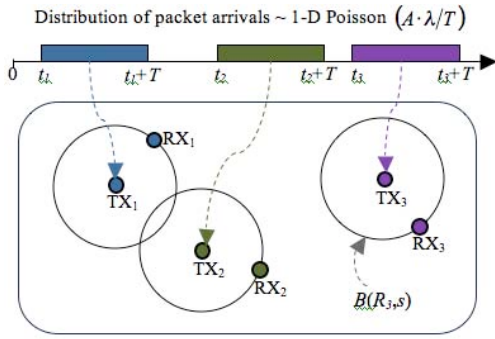
Fig. 1. Each new packet arrival is assigned to a transmitter-receiver pair, which is positioned randomly on the plane.

receives packets (in time) according to an independent 1-D Poisson process with parameter λ^t. These packets are then transmitted to the dedicated receiver, which is located a fixed distance R away (with random orientation). The transmission power ρ is constant for all transmitters. Assumed that each packet has a fixed duration, T, at each time instant the density of transmitters that have received a packet in the last T seconds is: $\lambda = T \; \lambda^s \; \lambda^t$. In order to analyze this network, it would be necessary to average over the spatial (to fix locations) and temporal (packet arrivals) statistics, which is rather difficult.

An alternative model is to assume that packets/users arrive at a random point in space and time and then disappear after their packet is served (successful or not). In the above model user locations are first fixed and then traffic is generated, while in this model user/packet locations are also random. As a result, there is a single process that describes both the spatial and temporal variations which greatly simplifies analysis. We consider a finite area A, and model packet arrivals according to a 1-D PPP with arrival rate $(A/T)\lambda$. Upon arrival each packet is assigned to a random transmitter location (uniformly distributed in area A) and a receiver is randomly located a distance R away, as shown in Fig. 1.

Note that the number of packet arrivals during a time interval of T seconds is Poisson($A\lambda$). When A is made large this translates to a spatial density of λ , which is the same as in the fixed model that was initially discussed. Therefore, results generated with our model can be fairly compared to the first network model with density λ. Furthermore, note that if slotted ALOHA is used (and the area A is taken to infinity) the two models are the same, because the set of transmitters during each time slot is a homogenous 2-D process with density λ. The parameter for our temporal Poisson distribution is $\lambda^{tempral} = A \; \lambda^s \; \lambda^t = A \; \lambda/T$.

For the channel model, only path loss attenuation effects (with exponent $\alpha > 2$) are considered, i.e. additional channel effects such as shadowing and fast fading are ignored, and the channel is considered to be constant for the duration of a transmission. Note that it is feasible to extend the work to include fading using the techniques developed in [5]. Each receiver sees interference from all the transmitters, and these interference powers are added to the channel noise η to result in a certain SINR at each receiver. If this SINR falls below the required SINR threshold β at any time during the packet

transmission, the packet is received in *outage*. With an outage probability constraint of ϵ, this is given as:

$$Pr\left(\frac{\rho R^{-\alpha}}{\eta + \sum_i \rho |r_i|^{-\alpha}} \le \beta\right) \le \epsilon \tag{1}$$

where r_i is the distance between the node under observation and the i-th interfering transmitter.

In the case of ALOHA, each transmission starts as soon as the nodes are placed, regardless of the channel condition. *Slotted* ALOHA improves performance by removing partial outages, but this system requires synchronization. In the CSMA protocol the incoming transmitter listens to the channel in the beginning of the packet, and if the measured SINR is below β, it drops its packet. No retransmissions are applied.

III. Outage Probability for ALOHA

A. Slotted ALOHA

Weber et al. consider the ALOHA protocol in a slotted version of our network model [1], i.e. transmitters can only start their packet transmissions at the beginning of the next time slot after the packet has been formed. Thus there is no partial overlap of transmitted packets, something that is intuitively expected to decrease the probability of outage. Define s to be the distance between the receiver under observation and its closest interfering transmitter that causes the SINR to fall just below the threshold β. This distance is found to be:

$$s = \left(\frac{R^{-\alpha}}{\beta} - \frac{\eta}{\rho}\right)^{-\frac{1}{\alpha}} \tag{2}$$

Consider the area $B(R_1, s)$, which is a circle of radius s around the receiver under observation, RX_1. One situation that would result receiver RX_1 to go into outage is if at least one active transmitter, except TX_1, falls within $B(R_1, s)$. Based on this, the lower bound (LB) for the probability of outage is [1]:

$$
\begin{aligned}
P_{out}^{LB} &(\text{Slotted ALOHA}) \\
&= P(\ge 1 \text{ TX inside } B(R_i, s) \text{ during } [0, T]) \quad (3) \\
&= 1 - e^{-\lambda \pi s^2}
\end{aligned}
$$

Using the Taylor expansion formula for small values of the density, equation (3) may be approximated by:

$$P_{out}^{LB}(\text{Slotted ALOHA}) \approx \lambda \pi s^2 \tag{4}$$

This shows that the outage probability is approximately a linear function of the spatial density. This will be referred to later for the sake of comparison and intuition. In the following we will use (3) to develop expressions for continuous-time transmissions for both the ALOHA and CSMA protocols.

B. Unslotted ALOHA

If no synchronization is possible in the system, the nodes will have to use unslotted ALOHA for communication.

Theorem 1: The lower bound for the probability of outage for continuous-time ALOHA is:

$$P_{out}^{LB}(\text{Unslotted ALOHA}) = 1 - e^{-2\lambda \pi s^2} \tag{5}$$

Proof of Theorem 1: Consider the equation for the lower bound of the outage probability of slotted ALOHA, and note
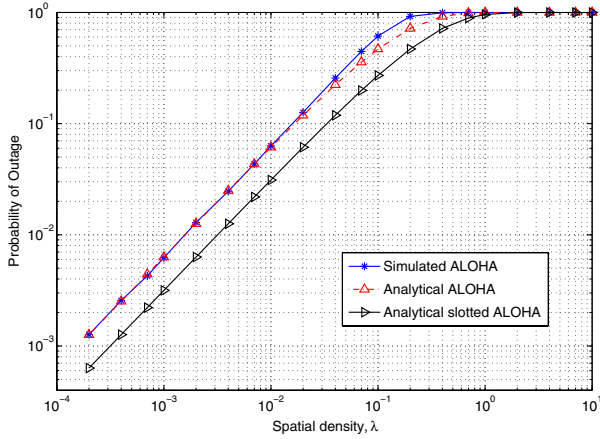
Fig. 2. Probability of outage for slotted and unslotted ALOHA along with the simulation results.

that this indicates that there are no interfering transmissions inside $B(R_1, s)$ during the time period $[0, T]$. Now, within a continuous-time system, we know that any transmission that started less than time T before the transmission between TX_1 and RX_1 starts, is still an ongoing transmission, and will thus contribute to the outage probability of RX_1. Hence, we now require that except for TX_1 there are no other active transmitters inside $B(R_1, s)$ during the period $[-T, T]$, i.e.:

$$P_{out}^{LB}(\text{Unslotted ALOHA})$$
$$= P(\text{outage in } [-T, 0] \cup \text{outage in } [0, T])$$
$$= 2 \cdot \left(1 - e^{-\lambda \pi s^2}\right) - \left(1 - e^{-\lambda \pi s^2}\right) \cdot \left(1 - e^{-\lambda \pi s^2}\right)$$
$$= 1 - e^{-2\lambda \pi s^2}$$

Note that this derivation is valid because the probability of outage in $[-T, 0]$ is independent of the probability of outage in $[0, T]$, since all packets are of equal length T. That is, the set of active transmissions at time 0 is independent of those at time T. For small values of the density λ the probability of outage for unslotted ALOHA may be approximated with:

$$P_{out}^{LB}(\text{Unslotted ALOHA}) \approx 2\lambda \pi s^2 \qquad (6)$$

Comparing this with equation (4) we see that slotted ALOHA performs better than unslotted ALOHA by a factor of 2 in terms of probability of outage. This is expected and consistent with the results obtained from the conventional model for the slotted and unslotted ALOHA protocols.

For the Matlab simulations we apply a constant transmission power ρ of 1, R equal to 1, path loss exponent α of 3, and a SINR threshold β of 0 dB. The latter is chosen for the outage probability to have little dependence on the path loss exponent α. Fig. 2 shows the outage probability versus density for both slotted and unslotted ALOHA. The analytical bounds given in (3) and (5) are also plotted, with the latter following the simulations tightly. As expected, slotted ALOHA outperforms unslotted ALOHA by approximately a factor of 2.

IV. OUTAGE PROBABILITY FOR CSMA

In the CSMA protocol, a transmitter backs off or drops its packet if the accumulation of the interference from all other active transmitters results in a measured SINR that is below β at the *beginning* of the packet. The probability of this is called *backoff probability*, P_b. If the transmitter decides to transmit, but the measured SINR at the receiver is below β any time along the packet duration, the packet is received in outage. In the following sections we derive expressions for the probability of backoff and the total probability of outage for CSMA, both when the transmitter senses the channel SINR, and when the receiver senses the channel SINR upon arrival.

A. Probability of Backoff

Due to the complexity of the analysis, as in the ALOHA case, we consider the lower bound for the outage probability, which may be obtained by only considering the effect of the nearest interferer (TX_2 in Fig. 3) on the receiver under observation (RX_1). Denoting this distance by d, we have that d^2 follows a Poisson distribution. Note that because of the backoff property of CSMA, the number of transmitters on the plane no longer follows a PPP. Nevertheless, as an approximation, we assume that the transmitters are still Poisson distributed, and the simulation results show that this assumption is reasonable.

Theorem 2: The approximate probability that a transmitter using CSMA backs off is given by:

$$P_b = 1 - e^{-\lambda(1-P_b)\pi s^2} \qquad (7)$$

The solution to this can be given in closed form in terms of the Lambert function, $W_o(\cdot)$, as:

$$P_b = 1 - \frac{W_o\left(\lambda \pi s^2\right)}{\lambda \pi s^2} = 1 - \frac{1}{\lambda \pi s^2} \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} \left(\lambda \pi s^2\right)^n \qquad (8)$$

Proof of Theorem 2: Consider a new packet arrival that is assigned to a transmitter-receiver pair. In order for this new transmitter to start its transmission, we require that the closest transmission that is already active on the plane is at least a distance s away. Only the time period $[-T, 0]$ is of interest for the probability of backoff, because the decision on whether to back off or not is made at the beginning of each packet. Also, due to the backoff property of CSMA the density of nodes on the plane is now: $\lambda(1 - P_b)$. Hence, using equation (3), the probability of backoff for a new packet arrival is derived to be (8). For small values of the density, the probability of backoff increases as a linear function of λ. The analytical backoff probability for CSMA is plotted versus density in Fig. 4, and shown to follow the simulation results tightly. As expected, for higher densities, there is a greater probability that a node backs off its transmission due to the higher level of interference.

B. Outage Probability for CSMA – Transmitter-Sensing

Next we consider the probability that a packet goes into outage *during* its transmission (i.e., we assume that TX_1 has already sensed its own channel at the start of its transmission, and has decided to transmit).

Theorem 3: Considering an active transmitter-receiver pair, the probability that the packet is received in outage is:

$$P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \qquad (9)$$
$$= \int_{(s-R)^2}^{s^2} \left[1 - \frac{1}{\pi} cos^{-1}\left(\frac{d^2 + R^2 - s^2}{2Rd}\right)\right] \pi \lambda e^{-\pi \lambda d^2} d(d^2)$$
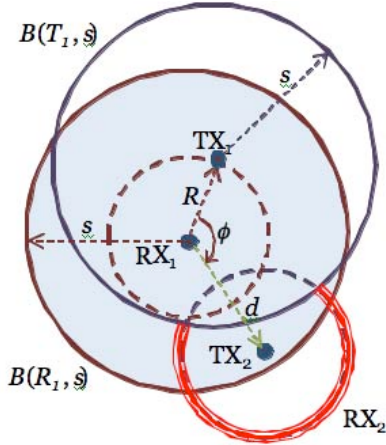
Fig. 3. The setup used to analyze CSMA. TX_1 and RX_1, are assumed to be active when the new arrival of TX_2 and RX_2 occurs.

Fig. 4. CSMA's total probability of outage, backoff probability, and the probability that packet goes into outage after it decides to transmit.

Proof of Theorem 3: Consider an ongoing packet transmission between say TX_1 and RX_1 in Fig. 3. Then a new packet arrives and is assigned to TX_2. This new transmission will cause RX_1 to go in outage if TX_2 falls inside $B(R_1, s)$. Moreover, due to the properties of CSMA, in order for TX_2 to not back off and cause interference, it has to be placed at least a distance s away from the ongoing transmission of TX_1, i.e., it has to be outside of $B(T_1, s)$. That is, TX_2 has to fall inside $B(R_1, s) \cap \overline{B(T_1, s)}$, in order for the arrival of a new packet to result in outage for an ongoing packet transmission. Note that this outage probability only covers the transmitter-receiver pairs that are already active on the plane. We know that the pdf of d^2 is approximately $(\pi\lambda e^{-\lambda\pi d^2})$, and we denote the angle rotating around RX_1 by ϕ, as shown in Fig. 3. Using that the pdf of ϕ is $1/(2\pi)$, we double-integrate over ϕ and d^2 to find the area in which the existence of TX_2 may cause outage for an ongoing transmission. That is:

$$P_{out}^{LB}(\text{CSMA}|\text{no backoff}) = \int_{(s-R)^2}^{s^2} \int_{\nu(d)}^{\omega(d)} \frac{\lambda}{2} \, e^{-\pi\lambda d^2} \, d\phi \, d(d^2)$$

where the integration limits for the angle ϕ are:

$$\nu(d) = cos^{-1}\left(\frac{d^2 + R^2 - s^2}{2Rd}\right) \quad ; \quad \omega(d) = 2\pi - \nu(d)$$

Solving this integral as far as it is analytically possible, we obtain (9) as the probability that an already active packet goes into outage anytime during its transmission.

To find the total outage probability of CSMA, we note that a packet is received in outage if either the transmitter backs off its transmission, or if the receiver is in outage upon arrival, or if the packet goes into outage any time during its transmission. After some manipulation of the expressions, we obtain:

$$P_{out}^{LB}(\text{CSMA}) = P_b + (1 - P_b)P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \quad (10)$$
$$+P_b\left[1-P_{out}^{LB}(\text{CSMA}|\text{no backoff})\right]\left[1-P_{out}^{LB}(\text{RX beg.}|\text{backoff})\right]$$

where expressions for P_b and $P_{out}^{LB}(\text{CSMA}|\text{no backoff})$ have been derived earlier, while $P_{out}^{LB}(\text{RX beg.}|\text{backoff})$ is derived by finding for the probability that the closest interferer, which
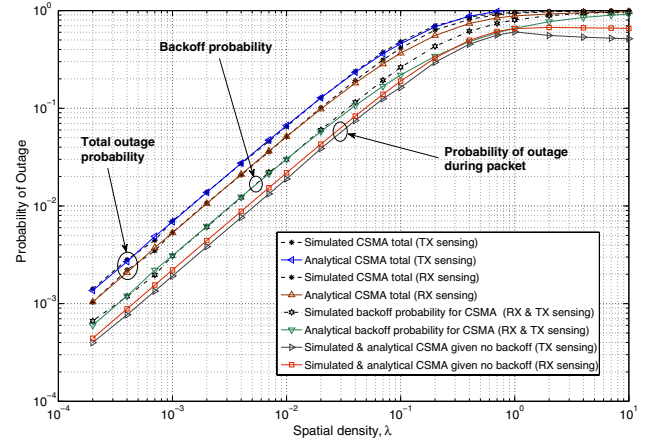
is given to be inside $B(T_1, s)$, is also inside $B(R_1, s)$. That is:

$$P_{out}^{LB}(\text{RX beg.}|\text{backoff}) = \frac{2}{\pi}cos^{-1}\left(\frac{R}{2s}\right) - \frac{R}{\pi s}\sqrt{1 - \left(\frac{R}{2s}\right)^2}$$

Fig. 4 shows the total probability of outage for CSMA, as well as the backoff probability and the probability that an active transmission goes into outage during its packet length. The simulated results follow the analytical results tightly, hence validating our method and formulas. For lower densities, about 30% of total outage probability is due to backoffs, while 45% is due to outages occurring during packet transmission.

C. Outage Probability for CSMA – Receiver-Sensing

For the sake of comparison and with the goal of improving the performance of CSMA, we consider a modified version of CSMA in which the *receiver* senses the channel and informs its transmitter over a control channel whether to start its transmission or not. This adds an extra factor to our expressions for the probability of outage, namely the relative position of the receiver RX_2 with respect to TX_1 and TX_2.

Theorem 4: Considering an active transmitter-receiver pair and receiver-sensing, the probability that an ongoing packet transmission is received in outage is:

$$P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \quad (11)$$
$$= \int_0^{s^2} \int_{\alpha(d)}^{\gamma(d)} \frac{1}{2\pi} P(\text{active}|d,\phi) \, \pi\lambda e^{-\pi\lambda d^2} \, d\phi \, d(d^2)$$

where $P(\text{active}|d,\phi)$, $\alpha(d)$, and $\gamma(d)$ are given as:

$$P(\text{active}|d,\phi) = 1 - \frac{1}{\pi}cos\left(\frac{d^2 + 2R^2 - s^2 - 2Rd cos\phi}{2R\sqrt{d^2 + R^2 - 2Rd cos\phi}}\right) \quad (12)$$

$$\alpha(d) = cos^{-1}\left(\frac{d^2 + 2Rs - s^2}{2Rd}\right) \quad ; \quad \gamma(d) = 2\pi - \alpha(d) \quad (13)$$

Proof of Theorem 4: As earlier, outage is caused if TX_2 falls within $B(R_1, s)$. However, in order for TX_2 to not back off, we now require that RX_2 falls outside of $B(T_1, s)$. Since the distance R between the transmitter and receiver is constant, the new receiver must be positioned on the part of the circle centered on TX_2 that is at least a distance s away from TX_1.

Given the location of the new transmitter TX_2 through d and ϕ, whose distributions are known, the probability that TX_2 starts its transmission is derived to be (12). The probability of outage given that the transmitter decides to transmit is then found by double-integrating $P(\text{active}|d,\phi)$ with respect to ϕ and d^2, obtaining (11), which can be solved numerically.

In order to find the *total* outage probability when the receiver is sensing the channel, we no longer need to consider the situation when TX_1 is in outage upon arrival. Then the total probability of outage for CSMA is given as a summation of the probability that outage occurs at the beginning of the packet at the receiver, P_b, and the probability that the transmitted packet goes into outage during its duration T, i.e.:

$$P_{out}^{LB}(\text{CSMA}) = P_b + (1 - P_b)P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \quad (14)$$

where expressions for P_b and $P_{out}^{LB}(\text{CSMA}|\text{no backoff})$ are given by respectively by equations (8) and (11).

Along with the transmitter-sensing results in Fig. 4, we plot the total outage probability and the probability that an active transmission goes into outage during its packet length for the case of receiver-sensing. The simulated results follow the analytical results tightly, validating our obtained formulas. Furthermore, we see that approximately 40% of the total outage probability is due to backoffs, and the remaining 60% are outages occurring during the packet transmissions.

V. Comparing the MAC Protocols

The total outage probabilities of ALOHA (slotted and unslotted) and CSMA (transmitter-sensing and receiver-sensing) are all plotted in Fig. 5. Interestingly, we see that for lower densities, CSMA with transmitter-sensing actually performs worse than ALOHA, having about 10% more outage probability. In fact, the performance of slotted ALOHA is almost two orders of magnitude higher than that of CSMA (as was also concluded in [6]). As the density increases, however, the use of CSMA becomes more advantageous. This is because the backoff probability of CSMA increases resulting in fewer interferers, and also, the chance of backing off in cases where the packet would have been correctly received, decreases.

By allowing the *receiver* to sense the channel and decide whether to back off or not, the performance of CSMA may be improved by approximately 23%. Moreover, for a fixed probability of outage, the density of nodes may be increased by approximately 20% by using CSMA (RX-sensing) over unslotted ALOHA. Note that the backoff probability of CSMA with transmitter-sensing is approximately the same as that with receiver-sensing, because whether we choose to look at the transmitter or the receiver of a new packet arrival, they are both randomly placed on the plane, and their distance to the closest active *transmitter* is what determines the backoff probability.

VI. Conclusion and Future Work

The contribution of this paper is analyzing the performance of the MAC protocols ALOHA and CSMA in terms of probability of outage in a new framework. In this framework nodes are randomly placed in space, and transmissions are
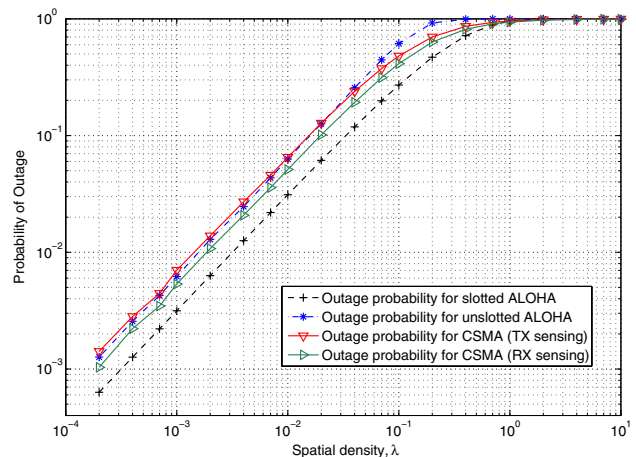

Fig. 5. Simulated probability of outage of ALOHA (slotted and unslotted) and CSMA (transmitter-sensing and receiver-sensing) as a function of density.

continuous in time, hence giving the closest representation to an actual ad hoc network. In this SINR-based model, we derive expressions for the probability of outage for unslotted ALOHA and CSMA (both with transmitter and with receiver sensing), and the probability of backoff for CSMA. The simulated results are consistent with our derived analytical expressions. It is shown that CSMA is most effective for higher densities, as there is a greater probability of backoff. CSMA performs worse than ALOHA when the transmitter senses the channel, but the performance of CSMA is significantly improved if the receiver senses the channel and informs the transmitter over a control channel whether to transmit or not.

For future work we will add retransmissions to the model and analyze the delay introduced, and thereby derive the transmission rate of packets. Furthermore, we will investigate the effect of applying adaptive rate and power control to our system, with the goal of improving the performance of CSMA in spatial ad hoc networks.

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