

Homework 3

Due: Tuesday, Feb. 26, 5:00 PM

1. In class we linearized the function $C(x) = E[\log(1 + x|h|^2)]$ about the point $x = P$ to come up with the following approximation to the optimal number of training symbols in a block-Rayleigh fading channel:

$$\tau^* \approx \sqrt{Tab + ab^2} - b \quad (1)$$

where

$$a = \frac{P\dot{C}(P)}{C(P)} = \frac{E_H \left[\frac{P|H|^2}{1+P|H|^2} \right]}{C(P)} \quad (2)$$

$$b = 1 + \frac{1}{P} \quad (3)$$

- (a) Using the above approximation and a suitable high-SNR approximation of $C(P)$, show that $\tau = 1$ becomes optimal at approximately the following SNR:

$$P_{dB} \approx (2.5 \log_{10}(e))T + 2.5.$$

- (b) Show that the approximation to τ^* in (1) converges to $\frac{T}{2}$ as $P \rightarrow 0$.
(Hint: use an appropriate Taylor series approximation for (1))

2. Let us now consider the case where the receiver has 2 antennas, and there is independent fading to each of the antennas (within the block fading model). If we let h_1 and h_2 denote the fading coefficients to antennas 1 and 2 (the two random variables are iid complex normal), respectively, show that the lower bound to the mutual information achieved with τ training symbols and iid Gaussian data symbols is:

$$\left(1 - \frac{\tau}{T}\right) E_H \left[\log \left(1 + \left(\frac{\tau}{\tau + 1 + \frac{1}{P}} \right) P(|h_1|^2 + |h_2|^2) \right) \right]. \quad (4)$$

Do you expect a significant change in the optimal number of training symbols relative to a single receive antenna system?