Information Theoretic View on Hybrid Wireless Networks

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Abstract

The capacity of large hybrid wireless networks is considered in this project. The hybrid network is based on an ad-hoc network with an embedded wired infrastructure. Both types of wireless transmissions are allowed in hybrid networks: peer-to-peer and through infrastructure. The report focuses on the case when the number of base stations in the hybrid network grows sub-linearly with the network size. Previous results suggest that there exists a protocol that provides an improvement in capacity as compared to a pure ad-hoc network if the infrastructure growth is at least as fast as square root of the size of the network. In this work, we consider this problem from Information Theory point of view and derive the similar result for a general hybrid network without specifying any protocol.

Key Words: *Hybrid Wireless Networks, Multiuser Information Theory, Random Matrix Theory.*

1 Introduction

The success in the development of wireless communications in late 90s has resulted in the installation of commercial cellular networks. The ease of use and importance of mobility has led to the exploded use of cell phones. In many cases such systems are limited in

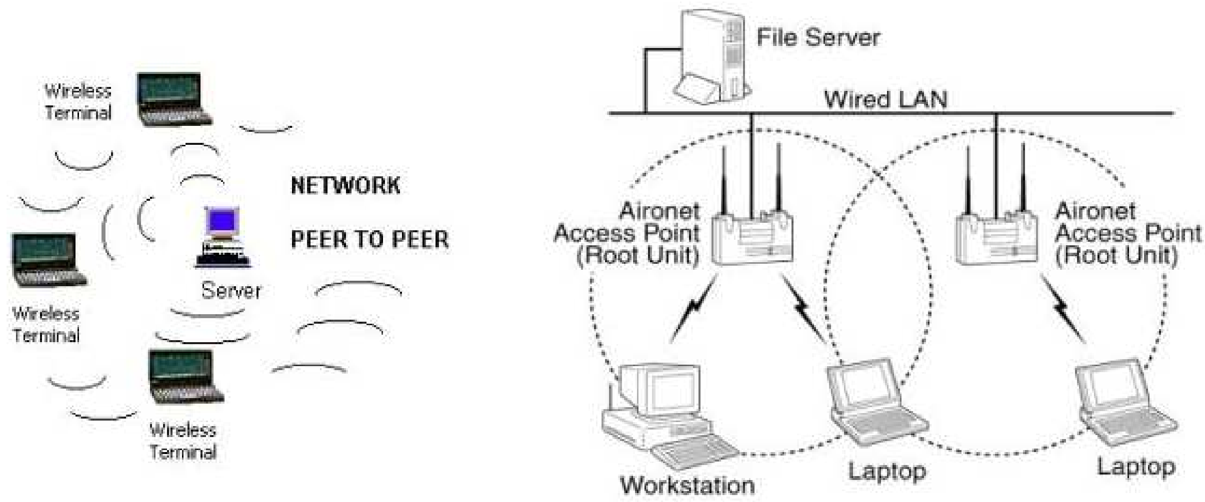

Figure 1: Hybrid network allows both type of communications: peer-to-peer and through-infrastructure.

the number of mobile users they can simultaneously handle. Market competition drives the costs lower which results in the increased number of users willing to pay for the wireless service. Unfortunately, bandwidth constraints and stringent FCC regulations on the irradiated power limit the capacity of such systems. Being limited in wireless resources, the only way to survive in the market is to offer the highest possible efficiency of resource utilization which directly translates into the efficient use of system capacity.

While cellular wireless systems are easily scalable they still have a number of unsolved problems. For example, one of them is related to the mobility management that assigns a mobile user to the certain "home" location. If both users assigned to home place A happen to be at place B their calls to each other will be routed through location A mobile switching center [1]. This incurs a substantial delay sometimes reaching the order of minutes which is clearly an unacceptable Quality-of-Service(QoS). It seems reasonable to allow those users to have a direct connection call if they are located close enough to each other. Thus, we naturally arrive at the idea of allowing an ad-hoc type of connections in a backbone based network. In fact, with the development of Personal Wireless Area Networks and cognitive radio technology, the importance of a hybrid network structure is going to increase significantly. In addition to that, there is an ongoing FCC discussions to abolish the stringent frequency band regulations by introducing a more flexible system based on user access priorities. This change will allow customers and service providers to compete for the wireless resources and negotiate the terms of frequency use *locally.* Such a shift in FCC mentality seems to indicate that the era of hybrid networks is inevitable. On the other hand, the other extreme of not using an infrastructure results in a substantial loss of system capacity. Recently, it has been shown [2] that the capacity per user of wireless ad-hoc network goes to 0 as the size of the network goes to infinity. In practice, network scalability is an essential requirement for a business model of any wireless service provider. Therefore, practical feasibility of network deployment demands a constant rate $\mathcal{O}(1)$ allocated for each user. A couple of different approaches were suggested to overcome the deterioration of per-user capacity of an ad-hoc network. Among the brightest thoughts we can point out the use of mobility [3] to provide asymptotically $\mathcal{O}(1)$ rate for each user. Unfortunately, this approach leads to higher transmission delays. In fact, the capacity and delay for such a network reveal an interesting tradeoff [4].

The hybrid network enjoys advantages of both types of networks: it offers local flexibility of ad-hoc networks with efficient long-distance routing strategies of wired infrastructure, see Fig. 1. In this paper we focus on capacity scaling laws of hybrid networks and the relative size of infrastructure.

Instead of specifying a certain connection protocol, we want to look at the network capacity from Information Theory point of view. Such a general benchmark allows engineers to estimate how much of the capacity they sacrifice by making specific design decisions. The matter is complicated by the fact that the current development of Information Theory can not provide capacity regions of some single user channels, for example, non-degraded broadcast channels (BC), relay channels (RC) etc. Network capacity is intrinsically more complicated and contains RC, BC, multiple access channel (MAC) capacity regions as special cases. Therefore, we do not hope to obtain the full characterization of the capacity region for such a general model as a hybrid network. Instead, we are going to focus on the asymptotic behavior as the network size grows to infinity.

The organization of this paper is as follows: in Section 2 we formally define the hybrid network, channel model and discuss our assumptions. Section 3 will state the main result of this work. Section 4 will be devoted to proving the result we claim in Section 3. Section 5 will summarize the key ideas and References will conclude the paper.

In this paper we will often use terms *(network) capacity* and *per-user (network) capacity.* The term *capacity* will refer to the total network capacity, maximum sum-rate of all users, unless stated explicitly that we are talking about *per-user capacity.*

Throughout the paper we will use the following asymptotic notations:

- For any two functions $f(x)$ and $g(x)$ we write $f(x) = o(g(x))$ to imply that

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0.$$

- We write $f(x) = \mathcal{O}(g(x))$ to imply that

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} < \infty \text{ and } \lim_{x\to\infty} \frac{f(x)}{g(x)} < \infty.$$

2 Hybrid Networks

Consider a network of n mobile users who are randomly and uniformly distributed with a constant density over area A as the network size increases. Thus, total network area A is proportional to the number of users n. The randomness serves as an important tool in our analysis, since the capacity of a random network is much easier to analyze than a predefined network setup without any symmetry in it. Suppose, that we randomly and uniformly select m locations in such a network for base stations. The base stations are connected by a wired backbone with bandwidth significantly greater than that of wireless links. For theoretical tractability of further analysis we will assume that base stations are connected by wired infrastructure with infinite bandwidth, thus, allowing base stations to cooperate in signal reception. Each user in the network has a handset with limited power P and one antenna. Each base station is also modelled to have one antenna, although our analysis can be extended to the case of any number of antennas at a base station in a straightforward manner. The purpose of base stations is to relay the transmitted messages of network users, so base stations do not generate or absorb any traffic. For the purpose of generality we do not impose any cellular structure on the network architecture. We consider the large scale path loss model for the wireless channel (without shadowing and multi-path fading for simplicity):

$$h(r) = \frac{1}{r^\alpha}, \quad r > r_0, \tag{1}$$

where α is the path loss coefficient which in practice is typically between 1.6(line-of-sight resonance, indoor) and 6(no line-of-sight, blocked by walls, indoor), and r_0 is the minimum distance between users. Since we maintain the constant user density in the network, the probability of two users to be within distance r_0 of each other vanishes when the size of the network grows to infinity. Thus, we assume that we can neglect these events. In practice, there is a distance (lower bounded by half of the wavelength, typically several wavelengths) that is necessary to avoid induced currents in transmitter/receiver circuitry. For theoretic analysis we will also assume that $\alpha > 2$ which is a very common scenario for outdoor wireless channels.

3 Main Result

We are interested in the influence of the wired backbone on the overall capacity of the hybrid network. It is clear that when number of base stations m grows proportional to the network size n then the capacity will be dominated by the capacity of the uplink connection to the infrastructure. We consider the case when the number of base stations m grows sub-linearly with n. From a practical perspective we ask if we can increase the capacity of a hybrid network with less than linear growth in the number of base stations?

Claim 1. The infrastructure dominates the capacity of a hybrid network if

$$m = \mathcal{O}\left(n^\beta\right),$$

where

$$\beta > \max\left\{\frac{1}{2} + \frac{1}{2\alpha}, \frac{\alpha-1}{\alpha+1} + \frac{1}{2\alpha}\right\}, \text{with } \alpha > 2.$$

The behavior of the power exponent is shown in Fig. 2. This is the sufficient condition but not necessary. We notice that $\beta \to 1$ as $\alpha \to \infty$. This result can be intuitively understood because the average distance between nodes is a constant (proportional to density) whereas the average distance to a base station increases if $m = o(n)$. Therefore, asymptotically with $\alpha \to \infty$ the signals can not reach wired infrastructure while the ad-hoc transmissions are not affected as much as the uplink connections.

4 Proof of main result.

Suppose that at the specific time moment γ fraction of users wants to communicate directly in the ad-hoc multi-hop fashion and $(1-\gamma)$ fraction of users prefers to use infrastructure. We split the transmission interval into 2 halves - the first half for all ad-hoc transmissions and the second half for uplink connections to the wired infrastructure,

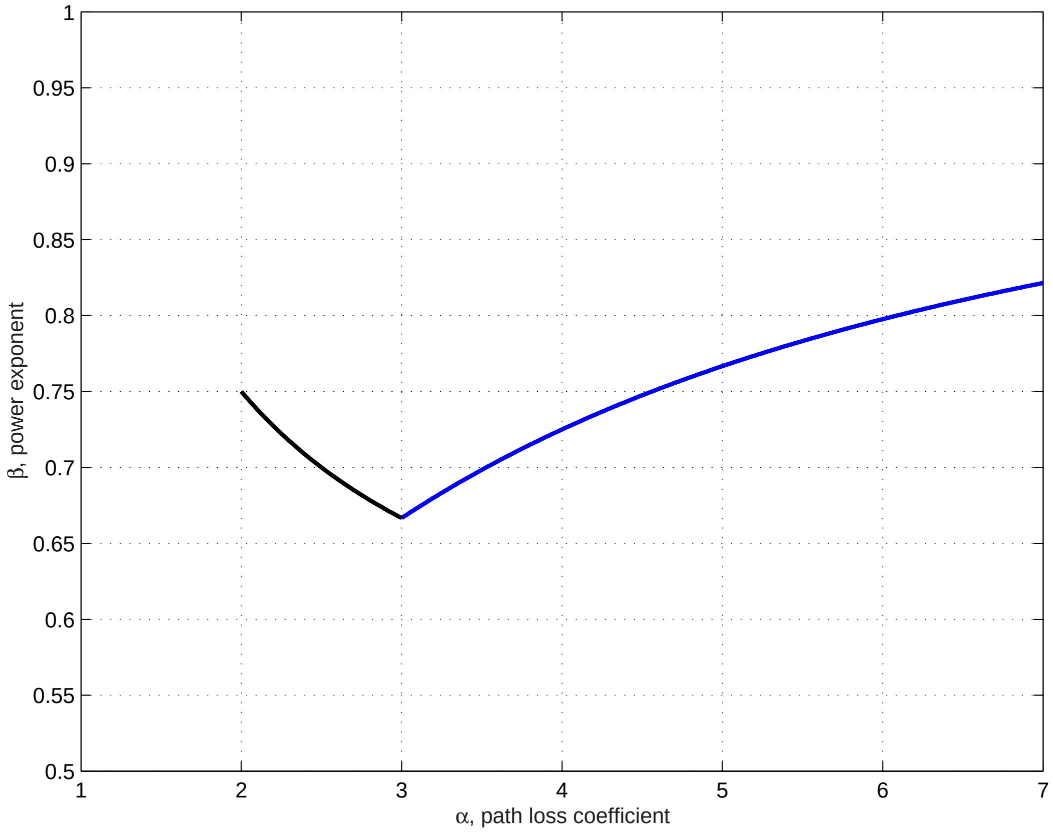

Figure 2: Base station growth exponent for the curves $m = n^{\beta}$.

so the total network capacity can be represented as:

$$C_{net} = \gamma C_{ad-hoc} + (1-\gamma) C_{uplink}.$$

By doing that we lose at most half of capacity for both type of transmission. Since, eventually we are concerned with the asymptotic behavior of the network capacity the loss of one half is not going to affect the final result. On the other hand, such a transmission scheme allows us to avoid interference between the ad-hoc and uplink transmissions. Note that we do not get rid of interference among users within each half, it is only the mixed-connection interference that we want to avoid. Thus, asymptotically as $n, m \to \infty$ we can write

$$C_{net} = \mathcal{O}(C_{ad-hoc} + C_{uplink}). \tag{2}$$

Based on the adopted channel model let us look at the two modes of communications available to users in a hybrid wireless network.

Ad-hoc mode.

Ad-hoc network capacity was first considered in [2] where the authors argued that the capacity of such a network grows sub-linear with rate $\mathcal{O}(\sqrt{n})$. Later, O. Leveque and E. Telatar has proven this pessimistic result from Information Theory point of view by providing an upper bound on the ad-hoc network capacity [5]:

$$C(n) \leq \mathcal{O}(\sqrt{n}\ n^{1/(2\alpha)}\ \log n).$$

A cellular TDMA scheme provides an achievable sum-rate for an ad-hoc network:

$$C(n) \geq \mathcal{O}(\sqrt{n}\ (\log n)^{-1/2-\alpha}).$$

Infrastructure mode.

Due to infinite bandwidth of the wired backbone (in practice, several orders bigger than bandwidth of wireless links) we can assume perfect cooperation among base stations. From that perspective, the whole wired infrastructure can be viewed as a multi-antenna receiver. Therefore, the uplink connection of n users to the infrastructure has a natural description of a vector Multiple Access Channel with n independent users and one receiver with m antennas. In the same light, the downlink connection can be represented by a Broadcast Channel with m transmit antennas and n target users. In this work we will focus on the uplink capacity, the downlink can be analyzed in the similar way. Let $x_i, i = 1, \ldots, n$ denote the transmitted signal of the i-th user and $y_k, k = 1, \ldots, m$ denote the signal received by k-th base station. Then the uplink channel can be modelled as

$$\mathbf{y} = \sum_{i=1}^{n} \mathbf{h}_i x_i + \mathbf{v},$$

where $\mathbf{h}_i$ is a vector of channel coefficients from i-th user to all base stations, and v_k is AWGN, zero mean, unit variance noise at the k-th base station. The transmitted signals must satisfy the power constraint $E\{x_i^2\} \leq P$ for every user $i = 1, \ldots, n$. The capacity region of the vector MAC is given by:

$$C_{mac} = \bigcup_{0 \leq P_i \leq P} \left\{ (R_1, \ldots, R_n) : \sum_{i \in \mathbf{S}} R_i \leq \frac{1}{2} \log\det \left(\mathbf{I} + \sum_{i \in \mathbf{S}} P_i \mathbf{h}_i \mathbf{h}_i^T \right),\ \forall \mathbf{S} \subseteq \{1, \ldots, n\} \right\}.$$

Since we are interested in the total network capacity we are going to focus on sum-rate capacity only:

$$C_{mac,sum-rate} = \max_{0 \leq P_i \leq P} \frac{1}{2} \log\det \left(\mathbf{I} + \sum_{i=1}^{n} P_i \mathbf{h}_i \mathbf{h}_i^T \right).$$

Stack vectors $\mathbf{h}_i$ as columns into matrix $\mathbf{H}$:

$$\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2\ \ldots\ \mathbf{h}_n] \in \mathcal{R}^{m \times n}.$$

Then, sum-rate capacity of the network uplink can be represented in the form:

$$C_{mac,sum-rate} = \max_{0 \leq P_i \leq P} \frac{1}{2} \log\det \left(\mathbf{I} + \mathbf{H}\mathbf{P}\mathbf{H}^T \right),$$

where $\mathbf{P}$ is the diagonal matrix with P_i on the diagonal. We assume that users do not have information about the channel state of the other users, therefore everyone uses as much power as possible to maximize their own transmission rate. With this assumption we arrive at the final form of sum-rate capacity:

$$C_{mac,sum-rate} = \frac{1}{2} \log\det \left(\mathbf{I} + P\mathbf{H}\mathbf{H}^T \right). \tag{3}$$

Note, that each entry H_{ki} of matrix $\mathbf{H}$ is function (1) of the distance from user i to base station k. Hence, the sum-rate capacity $C_{mac,sum-rate}$ is a complicated function of user and base station locations. Instead of evaluating this function for all possible locations we consider a different approach. We will calculate the sum-rate capacity *for a random* *network asymptotically as both $n, m \to \infty$.* That is, we treat user locations and base station locations as random variables. In practice, it is justified because the locations of mobile users are random and what matters for our model is the distance between users and base stations. As the size of network grows the typical random distances to base stations will be viewed by a user as the same random variables as if base stations were randomly placed over the area of interest. The magic here comes from Random Matrix Theory that suggests that asymptotically the distribution of eigenvalues of properly normalized matrix $\mathbf{H}\mathbf{H}^T$ converges to a deterministic distribution. This deterministic limit does not depend on the distribution of entries as long as a couple of conditions are satisfied.

Random Matrix Theory results.

To deal with the expression in (3) we will need several results from Random Matrix Theory:

1. If the entries of matrix $\mathbf{G} \in \mathcal{R}^{a \times b}$ are i.i.d., zero mean, with variance $1/a$ and fourth moments of order $\mathcal{O}(1/a^2)$ the empirical distribution of eigenvalues of $\mathbf{G}^T\mathbf{G}$ converges almost surely, as $a, b \to \infty$ with $b/a \to \beta$, so the so-called Marcenko-Pastur law whose density function is given by [6]-(page 9):
$$f_\beta(x) = \left(1 - \frac{1}{\beta}\right)^+ \delta(x) + \frac{\sqrt{(x-a)^+(b-x)^+}}{2\beta\pi x}, \tag{4}$$
where $(z)^+ = \max(0, z)$ and $a = (1 - \sqrt{\beta})^2, b = (1 + \sqrt{\beta})^2$.

2. Zero mean condition can be relaxed to having identical mean [6]-(page 54).

Now, we are going to apply the results from Random Matrix Theory to the sum-rate capacity obtained in (3). We are going to focus on the case when $m = o(n)$.

Sub-linear growth of base stations.

Since the network coverage area growth is proportional to $\mathcal{O}(n)$, the average area per base station grows as $\mathcal{O}(n/m)$. Hence, the average distance to the base station from a randomly placed user grows as $\mathcal{O}(\sqrt{n/m})$. First, we have to eliminate the dependence of the expected value of $H_{ik} \sim r^{-\alpha}$ on n and m. We introduce
$$\tilde{\mathbf{H}} = \left(\frac{n}{m}\right)^{\alpha/2} \mathbf{H}.$$
To ensure that the variance decreases as $\mathcal{O}(1/n)$ (required to apply the results of Random Matrix Theory) we introduce the scaling factor $1/\sqrt{n}$:
$$\mathbf{F} = \frac{1}{\sqrt{n}} \tilde{\mathbf{H}} = \frac{1}{\sqrt{n}} \left(\frac{n}{m}\right)^{\alpha/2} \mathbf{H}. \tag{5}$$
It is also worth pointing out that as long as $\alpha > 2$ all moments of random variable (1) are finite because we have a minimum distance $r > r_0$ constraint. This is the place where we need to use this minimum distance constraint. Substituting the properly normalized channel matrix (5) into the expression for sum-rate capacity (3) we obtain:
$$\begin{aligned} C_{mac,sum-rate} &= \frac{1}{2} \log\det\left(\mathbf{I} + P\, n\, \left(\frac{m}{n}\right)^\alpha \mathbf{F}\mathbf{F}^T\right) \\ &= \frac{1}{2} \sum_{i=1}^{m} \log\left(1 + P\, n\, \left(\frac{m}{n}\right)^\alpha \lambda_i(\mathbf{F}\mathbf{F}^T)\right), \end{aligned}$$

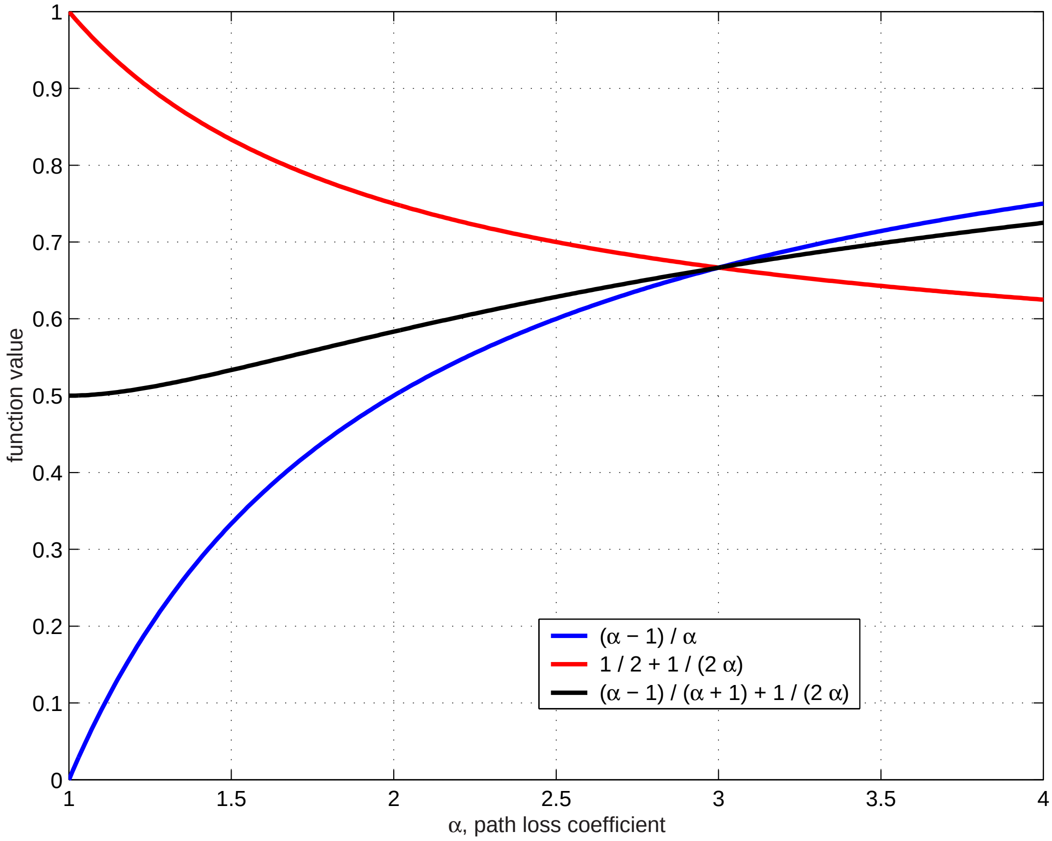

Figure 3: Power exponents for the curves $m = n^{\beta}$.

where $\lambda_i(\mathbf{FF}^T)$ is the i-th eigenvalue of matrix $\mathbf{FF}^T \in \mathcal{R}^{m\times m}$. Now, we can calculate the asymptotic of $C_{mac,sum-rate}$. Taking into account that $m = o(n)$ (for similar derivations see [6]):

$$\begin{aligned} C_{mac,sum-rate} &= \frac{m}{2}\,\frac{1}{m}\sum_{i=1}^{m}\log\left(1+P\;n\;\left(\frac{m}{n}\right)^{\alpha}\lambda_i(\mathbf{FF}^T)\right) \\ &\to \frac{m}{2}\;E_{f_\beta(\lambda)}\left\{\log\left(1+P\;n\;\left(\frac{m}{n}\right)^{\alpha}\;\lambda(\mathbf{FF}^T)\right)\right\} \\ &\to \frac{m}{2}\int_0^{\infty}\log\left(1+P\;n\;\left(\frac{m}{n}\right)^{\alpha}\;\lambda\right)f_\beta(\lambda)d\lambda, \end{aligned}$$

where $f_\beta(\lambda)$ is the limit of the empirical distribution of eigenvalues of matrix $\mathbf{FF}^T$. Since $m/n \to 0$ we have $\beta = 0$, therefore, the distribution (4) goes to $\delta(\lambda-1)$ as $\beta \to 0$. (Note that support $[a, b]$ of distribution (4) shrinks to one point $\lambda = 1$.) Thus, we have

$$\begin{aligned} C_{mac,sum-rate} &\to \frac{m}{2}\int_0^{\infty}\log\left(1+P\;n\;\left(\frac{m}{n}\right)^{\alpha}\;\lambda\right)\delta(\lambda-1)d\lambda \\ &\to \frac{m}{2}\;\log\left(1+P\;n\;\left(\frac{m}{n}\right)^{\alpha}\right) \end{aligned}$$

Hence, we can point out three cases here:

- If $m^{\alpha}/n^{\alpha-1} \to const$ then $C_{mac,sum-rate} = \mathcal{O}(m)$;
- If $m^{\alpha}/n^{\alpha-1} \to 0$ then $C_{mac,sum-rate} = \mathcal{O}\left(m^{\alpha+1}/n^{\alpha-1}\right)$;

- If $m^\alpha/n^{\alpha-1} \to \infty$ then $C_{mac,sum-rate} = \mathcal{O}\left(m \log\left(m^\alpha/n^{\alpha-1}\right)\right) = \mathcal{O}(m \log n)$.

To complete the proof of Claim 1 we consider two cases:

- Suppose that $2 < \alpha \leq 3$ then we pick

$$m = \mathcal{O}\left(n^{1/2+1/(2\alpha)+\delta}\right), \text{ for arbitrary small } \delta > 0. \tag{6}$$

  For $2 < \alpha \leq 3$ we notice that $1/2 + 1/(2\alpha) \geq (\alpha-1)/\alpha$ (see Fig. 3), therefore, we also have $m > \mathcal{O}\left(n^{(1-\alpha)/\alpha}\right)$. For this asymptotic behavior of m we have derived that the capacity of wired infrastructure is $\mathcal{O}(m \log n)$. Hence, we use the upper bound on the capacity of an ad-hoc network to claim the following bound:

$$\frac{C_{mac,sum-rate}}{C_{ad-hoc}} \geq \mathcal{O}\left(\frac{m \log n}{n^{1/2+1/(2\alpha)} \log n}\right) = \mathcal{O}\left(\frac{m}{n^{1/2+1/(2\alpha)}}\right) \to \infty,$$

  which implies that the capacity of infrastructure dominates the capacity of ad-hoc communication.

- Now, suppose that $\alpha > 3$ then we pick

$$m = \mathcal{O}\left(n^\beta\right),$$

  where

$$\beta = \frac{\alpha-1}{\alpha+1} + \frac{1}{2\alpha} + \delta, \text{ for arbitrary small } \delta > 0.$$

  For any value $\alpha > 3$ arbitrary close to 3 there exists small $\delta > 0$ (see Fig. 3) such that

$$m = \mathcal{O}\left(n^{\frac{\alpha-1}{\alpha+1}+\frac{1}{2\alpha}+\delta}\right) < \mathcal{O}\left(n^{\frac{\alpha-1}{\alpha}}\right).$$

  Thus, as derived before the capacity of wired infrastructure has the following asymptotic for this case:

$$C_{mac,sum-rate} = \mathcal{O}\left(\frac{m^{\alpha+1}}{n^{\alpha-1}}\right).$$

  Hence,

$$\frac{C_{mac,sum-rate}}{C_{ad-hoc}} \geq \mathcal{O}\left(\frac{m^{\alpha+1}}{n^{\alpha-1}} \frac{1}{n^{1/2+1/(2\alpha)} \log n}\right) = \mathcal{O}\left(\frac{n^\delta}{\log n}\right) \to \infty,$$

  which means the wired infrastructure capacity dominates.

From Fig. 3 we see that we can rewrite both cases in the form stated in Claim 1.

5 Future directions

- Prove (if possible) that the capacity per user of a hybrid network goes to 0 if $m = o(n)$. For $m = \mathcal{O}(n)$ the capacity scales as $\mathcal{O}(n \log n)$ and, therefore, per user capacity goes to infinity. Are there other scenarios when per-user capacity does not go to 0?

- Consider the feedback from wired infrastructure to implement power control. Will it affect the capacity?

- Add the capacity of downlink to complete the analysis based on vector broadcast channel.

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