Information Theoretic View on Capacity of Hybrid Wireless Networks

Mikalai Kisialiou

Department of Electrical and Computer Engineering
University of Minnesota
200 Union Street SE
Minneapolis, MN 55455

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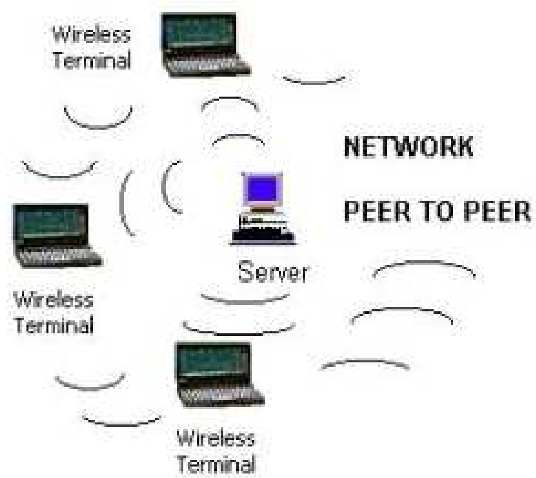
Summary

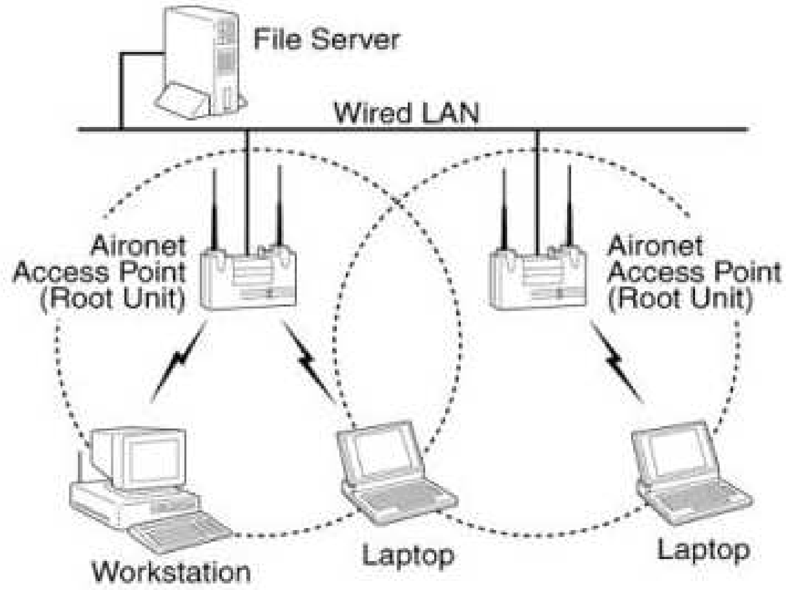
- For pure ad-hoc network the capacity per user goes to 0 as network size goes to infinity [Gupta-Kumar, Leveque-Talatar].
- We consider a hybrid network where both peer-to-peer and via-infrastructure connections are allowed.
- How many access points (base stations) is needed to ensure capacity increase?
- [Liu-Liu-Towsley] proposed a protocol for the hybrid network that suggests a capacity dominated by capacity of infrastructure if $m > \mathcal{O}(\sqrt{n})$ with both $m, n \to \infty$ (they assumed $C(n) = \mathcal{O}(\sqrt{n})$).
- We analyze the hybrid network from Information Theory point of view. We use Random Matrix Theory to calculate the asymptotic behavior of the capacity for the uplink channel and show that the growth

$$m > \mathcal{O}\left(n^{2/3}\right)$$

is enough to increase capacity, where $\alpha \geq 1$ is a path loss coefficient.

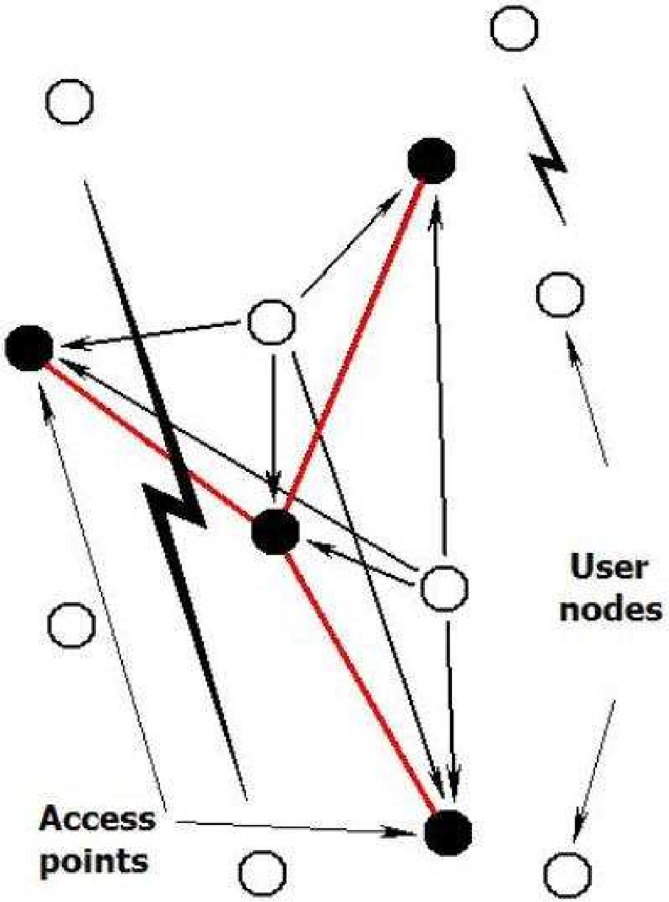
Network Structure





- Ad-hoc networks: peer-to-peer links between nodes.
- Networks with infrastructure: nodes communicate with each other via a backbone.
- Hybrid networks allow both type of connections.

Hybrid Network



- n is the number of user nodes.
- m is the number of access points (base stations).
- Access points do not generate or absorb traffic.
- Nodes are independently and uniformly distributed over area A.
- Each user has power P at their disposal.
- We consider network with constant density, that is area A grows linearly with n.
- Access points are independently and uniformly placed over the area A.
- No cellular structure is imposed.

Known Results

- In few scenarios we know the capacity exactly, we focus instead on the order of growth.
- For an ad hoc network the capacity scales roughly as $\mathcal{O}(\sqrt{n})$.
- From information theory point the upper and lower bounds have been obtained:

$$\mathcal{O}\left(\sqrt{n}\,(\log n)^{-1/2-\alpha}\right) \leq C(n) \leq \mathcal{O}\left(\sqrt{n}\; n^{1/(2\alpha)}\; \log n\right).$$

- Per user capacity of an ad hoc network goes to 0 if $\alpha > 1$ [Leveque-Talatar]. To overcome it we embed a wired infrastructure.
- Cellular system deploys $m = \mathcal{O}(n)$ access points. Can we allow a sub-linear growth of m? Interested in the question: how many access points do we need to improve the asymptotic of hybrid network capacity?
- Assuming $C(n) = \mathcal{O}(\sqrt{n})$ it has been shown [Liu-Liu-Towsley] that there is a protocol that provides the capacity increase if $m = \mathcal{O}(\sqrt{n})$.
- Can we prove it from Information Theory point of view without any protocol assumptions?

Main Results

- The channel fading is modelled as $h(r) = r^{-\alpha},\ r > r_0, \alpha > 1$.

- The capacity of the uplink of the hybrid network is dominated by the infrastructure capacity if
$$m > \mathcal{O}\left(n^{2/3}\right).$$

- When $\alpha = 1$ the minimal access point growth can be shown to be $m > \mathcal{O}(\sqrt{n})$ which agrees with previously derived results.

- For path loss coefficient $\alpha \to \infty$ the required number of access points in the infrastructure tends to linear $m \sim n$. (Average distance to an access point grows while an average distance to a peer is a constant.)

Hybrid network capacity

- Let γ fraction of all users use ad hoc communications and $(1 - \gamma)$ fraction use infrastructure.
- Split a unit of time into two parts - first for ad-hoc links and second for MAC links:
  - This approach sacrifices at most half of the capacity.
  - Time sharing does not affect the asymptotic behavior.
  - Ad hoc and infrastructure communications do not interfere.
- The total capacity can be represented as follows:

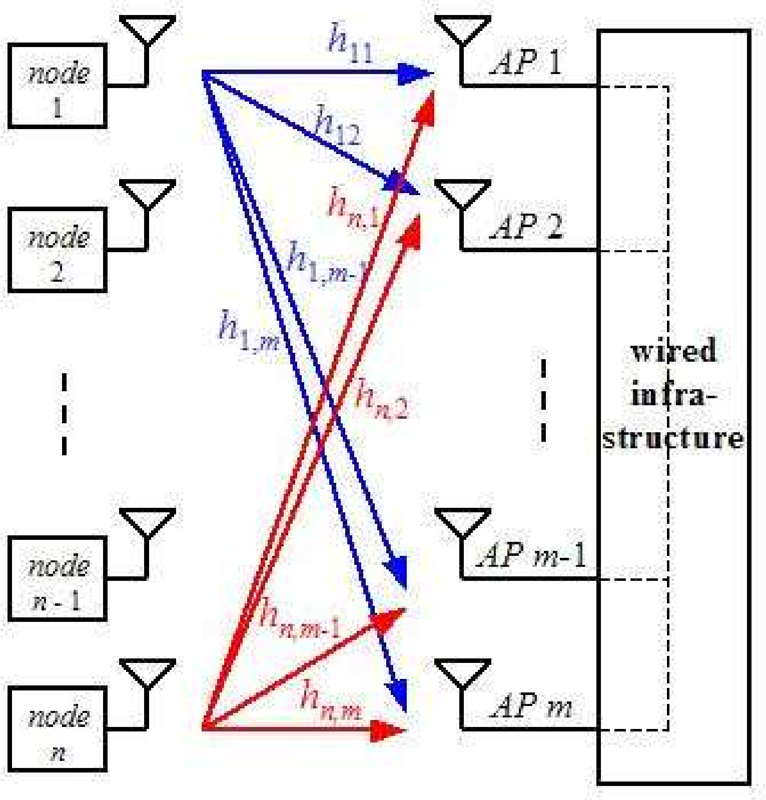
$$C_{hybrid} = \gamma C_{ad-hoc} + (1 - \gamma) C_{mac}.$$

- Taking the results for ad hoc network from [Leveque-Talatar] we have:

$$C_{ad-hoc}(n) \leq \mathcal{O}\left(\sqrt{n}\, n^{1/(2\alpha)} \, \log n\right).$$

- More precise expression for lower and upper bounds derived from Information Theoretic approach can be used to easily generalize the results.

Vector MAC capacity



Vector MAC channel capacity:

$$\sum_{i=1}^{n} R_i \leq \frac{1}{2} \log \det \left(\mathbf{I} + \sum_{k=1}^{n} P \mathbf{h}_k \mathbf{h}_k^T \right)$$

where $\mathbf{h}_k$ is vector of channel coefficients for user k.

Hence,

$$C_{mac} = \frac{1}{2} \log \det \left(\mathbf{I} + P \mathbf{H} \mathbf{H}^T \right)$$

where

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_n] .$$

Vector MAC capacity

- The area of the network is growing $\sim n$.
- The area per access point $\sim n/m$.
- The average distance to the access point $\sim \sqrt{n/m}$.
- Normalize the variance value of $H_{i,k}$ to be $1/n$ by introducing $G_{i,k} = \sqrt{n/m}\ H_{i,k}$ to be constant:

$$C_{mac} = \frac{1}{2} \log\det \left(\mathbf{I} + P \left(\frac{m}{n} \right)^{\alpha} n \left(\frac{1}{n} \mathbf{G}\mathbf{G}^T \right) \right)$$

- We will use Random Matrix Theory [Tulino-Verdu] to calculate the asymptotic behavior:

$$\lim_{n,m\to\infty} \log\det \left(a\mathbf{I} + \frac{1}{n}\mathbf{G}\mathbf{G}^T \right) = \int_0^\infty \log(a+x) f_\beta(x)dx,$$

where $\beta = \lim_{n\to\infty} m/n$
and $f_\beta(x)$ is the limit of the empirical distribution of $\frac{1}{n}\mathbf{G}\mathbf{G}^T$.

Asymptotic behavior of MAC capacity

- We are interested in the case $m = o(n)$, and $\beta = 0$ the limit of empirical distribution is $\delta(x-1)$.
- If $3 > \alpha \geq 1$ pick $\mathcal{O}\left(n^{\frac{\alpha-1}{\alpha}}\right) < m < \mathcal{O}\left(n^{\frac{\alpha-1}{\alpha+1}+\frac{1}{2\alpha}}\right)$ and $C_{mac} = \mathcal{O}\left(\frac{m^{\alpha+1}}{n^{\alpha-1}}\right)$.
- If $\alpha \geq 3$ pick $m > \mathcal{O}\left(n^{\frac{\alpha-1}{\alpha}}\right)$ and $C_{mac} = \mathcal{O}(m \log(m^{\alpha}\, n^{1-\alpha}))$.
- For ad-hoc network capacity we have an upper bound $C_{ad-hoc}(n) \leq \mathcal{O}\left(\sqrt{n}\, n^{1/(2\alpha)}\, \log n\right)$.
- Hence, for $\alpha \geq 1$ the ratio tends to infinity $\frac{C_{mac}}{C_{ad-hoc}} \longrightarrow \infty$.
- Thus, for $\alpha \geq 1$:

$$m > \mathcal{O}(n^{2/3}) > \mathcal{O}\left(n^{\max(\frac{\alpha-1}{\alpha}, \frac{\alpha-1}{\alpha+1}+\frac{1}{2\alpha})}\right)$$

is enough for the infrastructure to dominate capacity.

Thank you

References

[1] P. Gupta, and P.R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

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