EE 5505 | Prof. Jindal
Wireless Communication | May 4, 2010

Practice Final

1. **Signalling in Wideband Channels**

   Consider a wideband channel in which the channel response is given by:

   $$h(t) = \sum_{i=0}^{9} \alpha_i \delta(t - \tau_i),$$

   where the delays satisfy $0 = \tau_0 < \tau_1 < \cdots < \tau_9 = 10$ microseconds. Furthermore, assume each of the attenuation coefficients $\alpha_0, \ldots, \alpha_9$ is iid Rayleigh. The available bandwidth is 1 MHz, and thus the symbol period is $T_s = 1$ microsecond.

   (a) Approximately what length (in terms of number of taps) equalizer would be necessary for this channel?

   (b) Now assume OFDM is used over this channel.

      i. What length cyclic prefix is necessary?

      ii. How would frequency diversity be obtained in this system and what is the (approximate) maximum diversity order achievable over this channel?

2. **Diversity in Fixed Access Schemes**
   In this question we compare the available diversity when using two different fixed access schemes, FDMA and TDMA.

   (a) Consider a narrowband ($B << B_c$) downlink channel in which a single access point serves 10 users. The AP wishes to send a single codeword to each of the users, e.g., a different voice packet to each of the users. A single block lasts T_b seconds, and each voice packet must be served during this block (i.e., a codeword cannot be spread over multiple blocks). Assume the coherence time of the channel is half the block time: $T_c = \frac{1}{2}T_b$. The transmitter does not know the channel quality, and thus must use a fixed access scheme.

      i. Consider traditional FDMA, in which the bandwidth B is split into 10 equal parts: user 1 is allocated the first $\frac{B}{10}$ Hz, user 2 the next $\frac{B}{10}$ Hz, etc. How could each user obtain diversity in such a system? What is the maximum diversity order achievable by each user?

      ii. Consider traditional TDMA, in which the block T_b is split into 10 equal parts: user 1 gets the channel for the first $\frac{T_b}{10}$ seconds, user 2 for the next $\frac{T_b}{10}$ seconds, and so forth. How could each user obtain diversity in such a system? What is the maximum diversity order achievable by each user?

      iii. Is there a different TDMA strategy that could provide each user with more diversity than the strategy in (ii)?

(b) Now consider a wideband downlink channel with $B = 10B_c$, again with 10 users that each need to be sent a single codeword during a block. This time we assume that the coherence time is much larger than the block time T_b.

   i. Consider traditional FDMA, in which the bandwidth B is split into 10 equal parts: user 1 is allocated the first $\frac{B}{10}$ Hz, user 2 the next $\frac{B}{10}$ Hz, etc. How could each user obtain diversity in such a system? What is the maximum diversity order achievable by each user?

   ii. Consider traditional TDMA, in which the block T_b is split into 10 equal parts: user 1 gets the channel for the first $\frac{T_b}{10}$ seconds, user 2 for the next $\frac{T_b}{10}$ seconds, and so forth. Assume OFDM is used within each time slot. How could each user obtain diversity in such a system? What is the maximum diversity order achievable by each user?

   iii. Now assume the transmitter knows the channels of each of the users perfectly, and that a hybrid OFDMA/TDMA scheme is used, where users can be assigned a different set of subcarriers during each time slot. Briefly describe how the scheduler might allocate resources (3 sentences max).

3. **Orthogonal Cellular System**

Consider the downlink of a narrowband cellular system as shown in Figure 1. The mobile is 1 km away from its base station, and is 3 km away from the only interfering base station. We will ignore thermal noise in this problem.

(a) Assuming that signal strength (from both the intended BS as well as from the interfering BS) is only affected by path loss with an exponent of 2, what is the SIR at the mobile?

(b) Now assume that the link between the intended BS and the mobile suffers from Rayleigh fading in addition to path loss. What is the outage probability for a desired SIR of 0 dB? Note that there is no fading on the interfering link, i.e., the interference power still only depends on path loss.

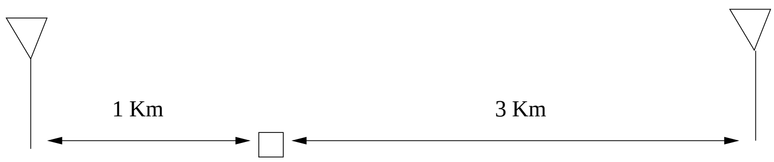

Figure 1: Downlink Model

(c) Consider the same scenario as in the part (b), but now assume the mobile has two receive antennas, each with iid Rayleigh fading. Compute the outage probability with respect to a 0 dB threshold for selection combining as well as for MRC. (Hint: the interference power on each receive antenna is the same as in the previous parts.)

(d) Would the outage probabilities calculated in parts (b) - (d) increase or decrease if the path loss exponent was larger than 2?

(e) Again consider the single antenna scenario from part (b), with path loss exponent of 2. In part (b), we assumed the interference power was constant because it was determined only by path loss. Now let us consider the case where there is shadowing on the interfering link such that the interference power (in dB) can be written as:

$$P_I = \overline{P_I} + U$$

where $\overline{P_I}$ is the average interference power due only to path loss (i.e., the interference power you used in the earlier calculations) and U (in dB units) is uniformly distributed between -3 dB and 3 dB. Would the outage probability in this scenario be much different than your answer to part (b)? More specifically, would the outage probability differ from your answer to part (b) by more than a factor of three (i.e., larger or smaller by a factor of three or more)?

4. **Indoor vs. Outdoor Channels**
In this problem we compare and contrast an indoor wireless system and an outdoor wireless system. For the sake of fairness, we assume the same carrier frequency for both systems.

(a) How would you expect indoor and outdoor wireless channels to differ in terms of coherence bandwidth, i.e., which would have a larger coherence bandwidth? Would you expect the coherence time to be larger in an outdoor wireless system or an indoor wireless system?

(b) In which environment, indoor or outdoor, is it easier to realize time diversity? How about frequency diversity?

(c) If you design an OFDM indoor system and an OFDM outdoor system, which of the two systems would you expect to be more efficient, assuming the the same number of OFDM tones (e.g., 32) are used in both systems?

(d) Now assume you maximize the efficiency of the indoor OFDM system by using the maximum number of OFDM tones (denoted N_1) such that orthogonality of the subcarriers is preserved, and similarly you maximize the efficiency of the outdoor OFDM system by using the maximum number of tones (denoted N_2). Which system will be more efficient?

5. **OFDM: Frequency Diversity vs. Channel Estimation Resources** (35 pts)

Consider an OFDM system with $N = 1000$ subcarriers in a 10-user uplink channel. The fading environment is such that the coherence bandwidth of each user is the width of 100 subcarriers; this means that for each user, the first 100 subcarriers (the first coherence band) have the same channel gain, the next 100 subcarriers (the second coherence band) have the same channel gain, and so forth. We use $\gamma_{i,j}$ to denote the SNR of user j on the i-th coherence band. Assume the standard Rayleigh fading model, which means the $\gamma_{i,j}$'s are iid exponential random variables.

During each OFDM symbol, each user is allocated 100 subcarriers and transmits one codeword over these subcarriers; although this is a short code, for the purposes of this problem assume that it is possible to transmit at rate equal to mutual information. Assume the coherence time is longer than the duration of an OFDM symbol and that no retransmissions are allowed; this means that outage capacity is the relevant metric. As usual, each user is allocated a different set of subcarriers.

In order for the receiver to obtain an accurate estimate of the SNR, a user must transmit pilot symbols on 10 subcarriers on *every* coherence band that he uses. For example, if user 1 is allocated the first 100 subcarriers, then 10 of these are used for training while the remaining 90 are be used for data. If user 2 is allocated 50 subcarriers in band 2 and 50 subcarriers in band 3, then user 2 must send pilots on 10 subcarriers in band 2 and 10 subcarriers in band 3 which leaves 40 subcarriers for data in band 2 and 40 in band 3.

(a) The simplest allocation is to assign the first 100 subcarriers (the first coherence band) to user 1, the next 100 subcarriers to user 2, and so forth. If each user's codeword contains 100 information bits, write the expression for each user's outage probability (this expression should reflect the fact that some subcarriers are used for training).

(b) Another possibility is to allocate each user 50 subcarriers in 2 different coherence bands; for example, user 1 could be allocated 50 subcarriers in band 1 and 50 subcarriers in band 2, and so forth. If we still require a codeword to contain 100 information bits, write the expression for each user's outage probability.

(c) We could further increase the diversity order by allocating each user subcarriers on even more bands, but at the cost of sending more training symbols.

What would you expect the optimal allocation to generally be? What if a very small outage probability (e.g 0.01) is required?

(d) Is there a similar tradeoff between frequency diversity and channel estimation for the downlink channel?

6. **MIMO Systems**

(a) Consider a 2 by 2 MIMO system with channel matrix:

$$H = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Why would V-BLAST, in which a different symbol is sent on each of the two transmit antennas (i.e., symbol x_1 is sent on antenna 1, symbol x_2 is sent on antenna 2) not work over this channel? Explain why it would be nearly impossible to detect x_1 and x_2 if each of the symbols are independently chosen BPSK symbols.

(b) Could we use the Alamouti code over this channel? How many information symbols would be sent per symbol period? What would the resulting SNR per symbol be (assuming transmit power E_s and noise power N_0)?

(c) The ergodic capacity of a fading MIMO channel with CSI only at the RX is given by:

$$C = E\left[\log_2\left(\det\left(\mathbf{I} + \frac{P}{M_t}\mathbf{H}\mathbf{H}^H\right)\right)\right],$$

while the outage capacity for outage probability P_{out} is defined to be the rate R satisfying:

$$\Pr\left[\log_2\left(\det\left(\mathbf{I} + \frac{P}{M_t}\mathbf{H}\mathbf{H}^H\right)\right) < R\right] = P_{out}.$$

i. Is there a fixed ordering between ergodic and outage capacity, i.e., can you say one is always greater than or equal to the other?

ii. For what outage probability is the outage capacity nearly equal to the ergodic capacity?

iii. In a MIMO-OFDM system (recall that in MIMO-OFDM, each subcarrier is a narrowband MIMO system) in which the system bandwidth is orders of magnitude larger than the coherence bandwidth, which of the two capacity metrics is more applicable? Assume that there is no time diversity, i.e., that the coherence time is extremely long.

7. **MIMO** (30 pts)

Consider a 2×2 MIMO system in a Rayleigh fading narrowband environment with perfect CSI at the receiver but no CSI at the transmitter. The channel matrix $\mathbf{H}$ is written as:

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix},$$

where the channel model implies that the entries of the matrix are iid complex Gaussian (with unit variance).

(a) One possible way of transmitting over this channel is to use the normal Alamouti code to send 2 modulation symbols in 2 channel symbols. Describe a simple receiver that would provide a per-symbol SNR of:

$$\frac{P}{2}(|h_{1,1}|^2 + |h_{2,1}|^2 + |h_{1,2}|^2 + |h_{2,2}|^2).$$

(b) An alternative to the Alamouti code is to transmit two modulation symbols per channel symbol using spatial multiplexing. The corresponding outage probability is given by:

$$P_{out}(R) = P\left[\log_2\left|\mathbf{I} + \frac{P}{2}\mathbf{H}\mathbf{H}^H\right| < R\right],$$

while the outage probability for the Alamouti strategy is:

$$P_{out}(R) = P\left[\log_2\left(\frac{P}{2}(|h_{1,1}|^2 + |h_{2,1}|^2 + |h_{1,2}|^2 + |h_{2,2}|^2)\right) < R\right].$$

In Figure 1 rate is plotted versus average SNR P for a fixed outage level $P_{out} = 0.1$. In other words, the rate R is adjusted to give an outage of 10% for each value of P.

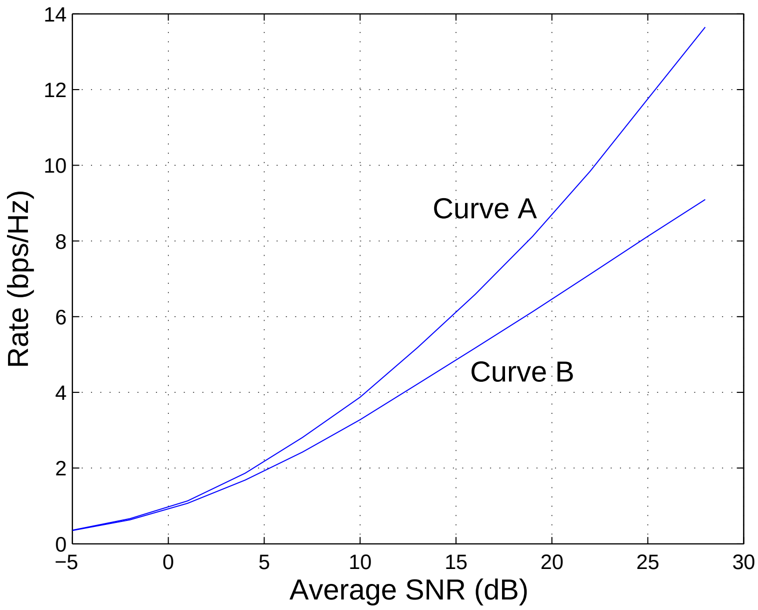

Figure 2: Rate vs. Average SNR ($P_{out} = 0.1$)

i. Which of the two curves corresponds to spatial multiplexing and which corresponds to Alamouti?

ii. Explain why the curves are nearly identical for low SNR while one of the curves has a larger slope (corresponding to a much higher rate) at high SNR.

(c) For which system, spatial multiplexing or Alamouti, is it more challenging to design a system that operates very near the outage capacity bound? How would this affect your choice between Alamouti and spatial multiplexing?