EE 5505 Prof. Jindal
Wireless Communication February 19, 2010

Homework 4

Due: Monday, March 1, 4:00 PM

Please turn in your MATLAB scripts in addition to your solutions and plots.

1. In this problem we will complete the proof showing that maximum ratio combining (MRC) is optimal, in the sense that no information is lost when MRC is performed. We will do this for the case of two antennas ($M = 2$), although it is easy to see that the proof holds for arbitrary M.

   Recall that our model for the received signals at the different antennas, after performing co-phasing, is:

   $$\begin{aligned} y_1[i] &= |h_1[i]|x[i] + z_1[i] \\ y_2[i] &= |h_2[i]|x[i] + z_2[i], \end{aligned}$$

   and that the maximum likelihood (ML) decoder performs the following minimization:

   $$\text{argmin}_{(x[1],\ldots,x[n])\in\mathcal{C}} \sum_{i=1}^{n} \left|\left| \mathbf{y}[i] - \begin{bmatrix} |h_1[i]| \\ |h_2[i]| \end{bmatrix} x[i] \right|\right|^2$$

   where $\mathbf{y}[i] = [y_1[i] \;\; y_2[i]]^T$.

   By decomposing $\mathbf{y}[i]$ into two vectors, one that is parallel to $[|h_1[i]| \;\; |h_2[i]|]^T$ and one that is orthogonal to $[|h_1[i]| \;\; |h_2[i]|]^T$, show that the above minimization is equivalent to performing ML decoding on the MRC output:

   $$\text{argmin}_{(x[1],\ldots,x[n])\in\mathcal{C}} \sum_{i=1}^{n} \left(|h_1[i]|y_1[i] + |h_2[i]|y_2[i] - \left(|h_1[i]|^2 + |h_2[i]|^2\right) x[i]\right)^2$$

2. Using the same procedure as in problem 3(b) from Homework 3, show that the probability of bit error for QPSK with M receive antennas and MRC is given by:

   $$\frac{1}{2}\mathbb{P}\left[X \leq \frac{1}{M\text{SNR}}\right]$$

   where X is F-distributed$(2M, 1)$.

3. Show that the output SNR for equal gain combining (signals are phased, but are then equally weighted) is

   $$\frac{\left(\sum_{i=1}^{M} |h_i|\right)^2}{M} \text{ SNR}$$

   and that the expected output SNR is

   $$(1 + (M-1)(\pi/4))\,\text{SNR}.$$

4. Write a Matlab script that produces a plot of probability of bit error (for QPSK) vs. SNR for 3 receive antennas, and three different combining methods: MRC, equal gain combining, and selection combining. You should generate the plot using Monte Carlo simulation. The SNR in this plot should be the single-antenna average signal-to-noise ratio, and should range from -5 to 15 dB.