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Practice Problems for Midterm 2

1. Consider the $R = 1/2$ binary convolutional code with memory 2 that outputs the following two coded bits for each information bit:

$$u[k] + u[k-1]$$
$$u[k] + u[k-2]$$

   (a) Draw the trellis diagram (with outputs labeled for each branch) for this code.

   (b) Compute d_{free} for this code. Find all error events at d_{free}, and for each write down the corresponding input sequence, output sequence, and input weight.

   (c) Write down the expression for the nearest neighbor approximation.

   (d) On the following page you are given the Matlab code for the Viterbi decoder for the $[7, 5]$ (nonrecursive & nonsystematic) convolutional code that we examined in class. Modify the Matlab code so that it implements the Viterbi decoder for the convolutional code in this problem. The code is terminated in state 00 by setting two additional information bits to zero.
   Hint: You should only have to modify a small number of lines in the Matlab code.

2. For the systematic recursive $[7, 5]$ binary convolutional code we studied in class (and which we used for our parallel concatenated turbo code), compute the BCJR algorithm (by hand, and in the linear domain) for the following length 10 input sequence:

$$+1, +2, -3, 4, -2, 0, -1, 2, 0, 0 \tag{1}$$

   Assume there are three information bits followed by two terminating bits (such that the code terminates in state 00). Draw the extended trellis and indicate the α, β, and γ values. Also compute the LLR's for the 3 information bits. In your computation, assume $\sqrt{E_s} = 1$ and $\sigma^2 = 1$.

3. Consider a system where the impulse response of the TX filter and of the channel are given by:

$$g_{\text{TX}}(t) = \begin{cases} 1 & 0 \le t \le 3 \\ 0 & \text{else} \end{cases} \qquad g_C(t) = \delta(t) + 1.5\delta(t-2)$$

   (a) Assuming a sample period of $T = 3$, compute $p(t)$ and $h[n]$.

   (b) Draw the trellis diagram for the Viterbi algorithm that implements MLSE, and write down the formulas for the branch metrics.

   (c) Assume the TX filter remains the same, but now consider the following channel:

$$g_C(t) = \delta(t) + 1.5\delta(t-10)$$

   Assuming binary modulation, how many states would the Viterbi implementation of MLSE have?