EE 5501 | Prof. N. Jindal
Digital Communication | Oct. 7, 2009

Homework 4

Due: Tuesday, Oct. 13, 12:30 PM

1. 3.15
   You do not need to solve part (c), but can use its result for the rest of the problem.

2. 3.13 (a) and (b)
   For part (b), find the intelligent union bound for an inner point and for an outer point, and combine these to get the intelligent union bound on the probability of error.

3. 3.17 (a) and (b)

4. Consider anitpodal binary signaling, where message 0 corresponds to the point $-A$ and message 1 corresponds to the point A. The message probabilities are $\pi(0)$ and $\pi(1)$.

   (a) The optimal decision rule is of the form

   $$\delta(y) = \begin{cases} 0 & y \leq T \\ 1 & y > T \end{cases}$$

   for the optimal threshold T. Derive an expression for T in terms of A, σ^2 and $\pi(0)$.

   (b) Is T increasing or decreasing in $\pi(0)$? Explain.

   (c) Assume $\pi(0) = 0.75$. What happens to the threshold T and the probability of error as the noise variance goes to infinity (i.e., $\sigma^2 \to \infty$)?

   (d) Show that the probability of error with $\pi(0) = 0.75$ is less than or equal to the probability of error when $\pi(0) = 0.5$.

5. Consider the following four waveforms:

$$s_0(t) = \begin{cases} 1 & 0 \leq t \leq T \\ 0 & \text{else} \end{cases} \qquad s_1(t) = \begin{cases} -1 & 0 \leq t \leq T \\ 0 & \text{else} \end{cases}$$

$$s_2(t) = \begin{cases} 1 & 0 \leq t \leq \beta \\ -1 & \beta \leq t \leq T \\ 0 & \text{else} \end{cases} \qquad s_3(t) = \begin{cases} -1 & 0 \leq t \leq \beta \\ 1 & \beta \leq t \leq T \\ 0 & \text{else} \end{cases}$$

where β is some constant between 0 and T. Assume equal message probabilities and white Gaussian noise with power spectral density $N_0/2 = \sigma^2$.

(a) Find an orthonormal basis for the four waveforms and provide the signal space representation in terms of this basis.

(b) Sketch the optimal decision regions in terms of the signal space representation.

(c) Compute E_s and E_b.

(d) Find the intelligent union bound (in terms of T, β, and σ^2) for the probability of error.

(e) Derive an expression for the exact probability of error (in terms of T, β, and σ^2).

6. In this problem we will determine what effect increasing the amplitude/energy of one or both signal vectors has on the probability of error. Consider binary transmission with equal message probabilities. For each of the following two statements, either prove the statement is true, or provide a counterexample.

(a) For any signal vectors $\vec{s}_1$ and $\vec{s}_2$ and any constant $\alpha > 1$, the error probability if vectors $\alpha\vec{s}_1$ and $\alpha\vec{s}_2$ are used is less than or equal to the error probability if vectors $\vec{s}_1$ and $\vec{s}_2$ are used?

(b) For any signal vectors $\vec{s}_1$ and $\vec{s}_2$ and any constant $\alpha > 1$, the error probability if vectors $\alpha\vec{s}_1$ and $\vec{s}_2$ are used is less than or equal to the error probability if vectors $\vec{s}_1$ and $\vec{s}_2$ are used?