The Simple Harmonic Oscillator

The deterministic Simple harmonic oscillator is governed by the equation $\hat{\chi}$ + $\gamma \hat{\chi}$ + $\omega \hat{\chi}$ = f(t) Or equivalently $dv = -\omega^2 x - rx^2 + f(t)$ $= -\omega^2 x - \gamma V + f(t)$ $dv = -\omega^2 x dt - \gamma v dt + f(t) dt$ **>** ord dx = v => dx= vat Assuming that the forcing is being caused by the Survounding nucleules and the damping is also due to the smoundry moleuter. We can model

Position, Velocity are Gaussian

the stochastic forcing due to surrounding molecules by $f(t) dt = \int \vec{p} dt N_{t}^{++} dt$ and thus, $dV = -\omega^2 X - YV + \sqrt{\beta^2} dt N_+(o_{11})$ and dx = Vat. Clearly as in earlier cases, assuming that X(0) and V(0) are deterministic we have that Y(dt) = - w² x(0) - 7 V(0) + Jp3dt N, (01) + V(0) E Span { No^{tadt}(011) } $X(H) = V(0)dt + X(0) \in$ $V(2dt) = -\omega^2 X(dt) - \gamma V(dt) + \sqrt{\beta^2} dt N_1(0_1)$ dt + V(dt)

Normality Monday, February 01, 2010 7:28 PM

Jointly Gaussanity of X and V

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 $< x(t); < x(t); < x(t); < x^{2}(t); < v(t);$ and Guis XH, VHI].

We will determine these now.

Mean of X and V

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Note that

$$V(t+dt) - V(t) = -\omega^{2} \times (t) - V(t)dt + \int \beta dt \eta \phi \eta$$

$$= \sqrt{(t+dt)} - (V(t)) = -\omega^{2} \times (t) - r \times V(t) dt$$
ond
$$= \sqrt{(t+dt)} - (\chi(t)) = -\omega^{2} \times (t) = \sqrt{(t+dt)} dt$$

$$\Rightarrow \lim_{d \to \infty} \frac{\langle v(t+d+1) \rangle - \langle v(t+) \rangle = [-\omega^2 - r] [\langle x(t+) \rangle]}{dt}$$

$$\Rightarrow \frac{d \langle v(t+) \rangle = [-\omega^2 - r] [\langle x(t+) \rangle]}{dt}$$

$$\frac{d \langle x(t+) \rangle}{dt} = [-\omega^2 - r] [\langle x(t+) \rangle]$$

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$$\Rightarrow \left[\frac{d \langle x(t+) \rangle}{dt} \right] = \left[\begin{array}{c} 0 & t \\ -\omega^2 & -r \end{array} \right] [\langle x(t+) \rangle]$$

$$\Rightarrow \left[\frac{d \langle x(t+) \rangle}{dt} \right] = \left[\begin{array}{c} 0 & 1 \\ -\omega^2 & -r \end{array} \right] [\langle x(t+) \rangle]$$

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Mean of X and V $% \left({{{\rm{A}}} \right) = {{\rm{A}}} \right)$

Monday, February 01, 2010 7:36 PM

Letting
$$A = \begin{bmatrix} 0 & 1 \\ -w^2 & 1 \end{bmatrix}$$

and letting $8 = \begin{bmatrix} < x(b7) \\ < x(b7) \end{bmatrix}$
we have
 $d8 = A8$ with $8(0) = \begin{bmatrix} < x(07) \\ < x(07) \end{bmatrix}$
and this have bolution
 $8(t) = e^{At} 8(0)$
where $e^{At} := (I + At + A^2 + 2^2 + \cdots)$
 $= \sum (At)^{n}$

Mean of X and V

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to obtain

$$(\chi(t)7 = e^{-\chi t/2} [\chi_0 C_0 S(\omega't) + (\chi_0 + \chi_0)]_{in(\omega)} + (\eta_0 + \eta_0) + (\eta_0 - \eta_0)$$

V(0)=70.

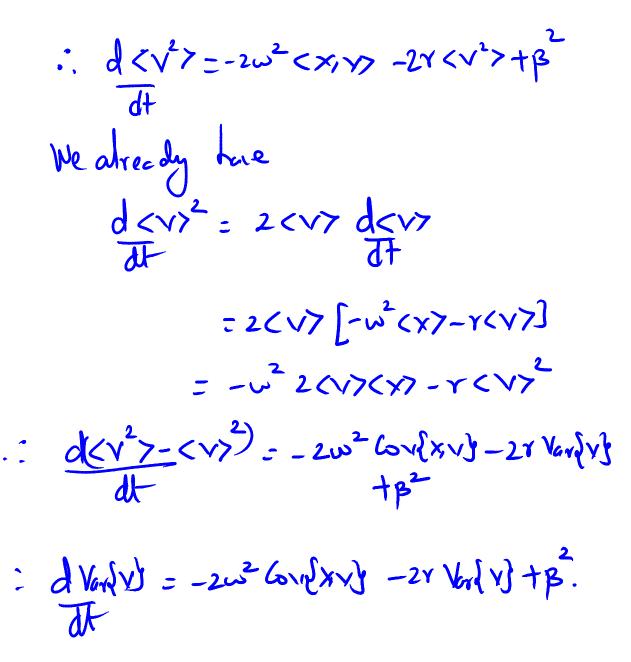
Variance of V Tuesday, February 02, 2010 12:46 AM

Note that dV= -w2 x dt - vV dt + JR2 H Nt + dt (01)

and therefore $dy^{2} = y(t+dt) - y^{2}(t)$ = $(V+dv)^2 - v^2 = v^2 + 2vdv + (dv)^2 - v^2$ = $2vdv + dv)^2$ $d\langle v^2 \rangle = \langle dv^2 \rangle = 2 \langle v dv \rangle + \langle dv \rangle \rangle$ = 2 < -w² XVdt - vv²dt + (p²dt v N⁺(01)) + $(-\omega^2 x)dt^2 + (rvdt) + (Jpdt N_1^{+dt})$ + 2 $(-\omega^2 \times)(-\nu)/dt^2$ + 2 $(-\omega^2 \times \sqrt{p^2} dt N_+^{++} dt)$ + 2 < - rv 1 pat N+ (31) > dt = -2w2< xv7dt -2v< x2dt + 0 + $<-\omega^2 x > dt^2 + <r^2 v > dt^2 + .p^2 dt$ - $2\omega^2 r < xv > dt^2 + 0 + 0$

Ode for var{V}

Tuesday, February 02, 2010 1:00 AM



Now.

Ode for Var{X} Tuesday, February 02, 2010 1:05 AM

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$$dx^{2} = x(t+dt)^{2} - x(t)$$

$$= (x+dx)^{2} - x^{2}$$

$$= x^{2} + (dx)^{2} + 2x dx - x^{2}$$

$$= (dx)^{2} + 2x dx$$

$$= (ydt)^{2} + 2x dx$$

$$= (ydt)^{2} + 2x dx$$

$$= x^{2} dt^{2} + 2x dx = 0 \sqrt{2} dt^{2} + 2x \sqrt{2} dt$$

$$dx^{2} = (x^{2} dt^{2})^{2} + 2 \sqrt{x} \sqrt{2} dt$$

$$dx^{2} = (x^{2} dt^{2})^{2} + 2 \sqrt{x} \sqrt{2} dt$$

$$dx^{2} = 2 \sqrt{x} \sqrt{y} dt$$

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$$dt$$

$$dx^{2} = 2 \sqrt{y} \sqrt{y}$$

$$dt$$

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$$dt$$

$$dx^{2} = 2 \sqrt{y} \sqrt{y}$$

Ode for Cov{X,V} Tuesday, February 02, 2010 1:08 AM

6 3

$$d(xv) = x(t+dt) v(t+dt) - x(t+)v(t+)$$

$$= (x+dx)(v+dv) - xv$$

$$= xv + xdv + vdx + dxdv - xu$$

$$= xdv + vdx + dxdv$$

$$= xdv + v^{2}dt + vdv dt$$

$$d(xv) = (dxv) = (xdv) + (v^{2}dt)^{2}$$

$$= (x(-w^{2}xdt - rvdt + N_{1}^{t+d}(o))) + (v^{2}ydt + (v^{2}ydt + (v^{2}xdt - rvdt + N_{1}^{t+d}(o)))) + (v^{2}ydt + (v^{2}ydt + (v^{2}xdt - rvdt + N_{1}^{t+d}(o)))) + (v^{2}ydt + (v^{2}ydt + (v^{2}xdt - rvdt + N_{1}^{t+d}(o)))) + (v^{2}ydt + (v^{2}ydt + (v^{2}xdt - rvdt + N_{1}^{t+d}(o)))) + (v^{2}ydt + (v^{2}ydt$$

Solution for second order statistics

Tuesday, February 02, 2010 1:13 AM

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$$= C \times 7 \left(-\omega^{2} C \times 7 - \gamma C \vee 7 \right)$$

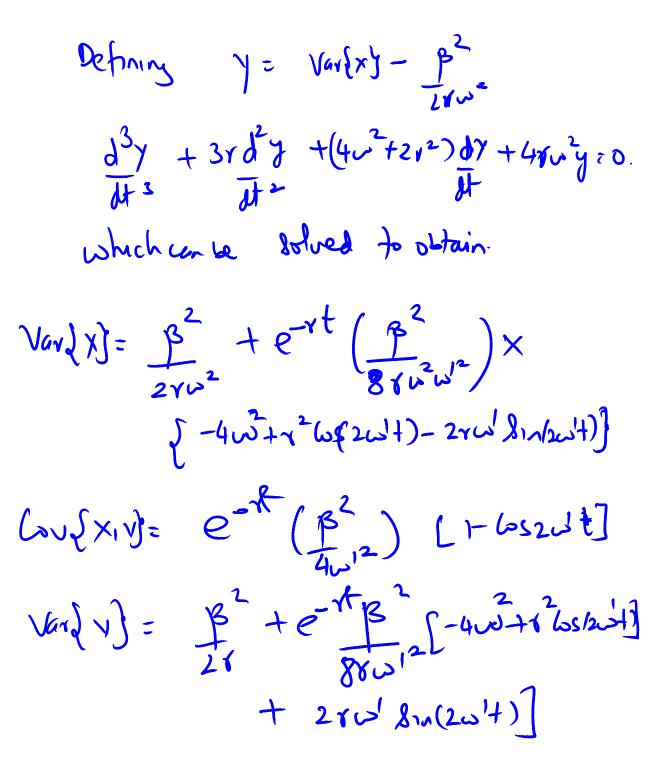
$$+ C \vee 7 C \vee 7$$

$$= -\omega^{2} C \times^{3} \gamma^{2} - \gamma C \times 7 C \vee 7 + C \vee 7^{2}$$

$$\Rightarrow d = \omega \sqrt{2} \times 1 \sqrt{3} = -\omega^{2} \vee C + \sqrt{2} + \frac{1}{2} \sqrt{2} + \frac{$$

Solution for second order statistics

Tuesday, February 02, 2010 1:22 AM



Thermal Equilibrium

Tuesday, February 02, 2010 1:25 AM

Brt- 00 Var {v}-P/21) ~ M Var(V) ard k 1 8 シ ks 2