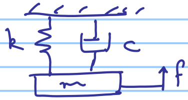
Homework 2:

Note Title 9/28/2009

Problem 1: [Second order Systems]

Consider the Spring-mass-damper System given by



where f is the force applied, k is the stiffness and c is the damping in the System.

The "natural frequency" ω_0 is defined by

$$\omega_0 := \sqrt{\frac{k}{m}}.$$

and the damping ratio is defined by

$$\xi_0 := \left(\frac{c}{2m}\right)\frac{1}{\omega_0}$$

The equation of motion of the mass is given by

$$m\ddot{z} + c\dot{z} + kz = f$$

$$\Rightarrow \quad \ddot{z} + \frac{c}{m}\dot{z} + \frac{k}{m}z = \frac{f}{m}$$

$$\Rightarrow \quad \boxed{\ddot{z} + 2\xi\omega_0\dot{z} + \omega_0^2 z = f/m.}$$

(a) Obtain the state-space description of the system.

(b) Let $k = 1\ N/M$; $m = 1\,kg$; $c = 0.45$;

Let $p_1 = \begin{bmatrix} 0.1591 + 0.6890J \\ -0.7071 \end{bmatrix}$

$p_2 = \begin{bmatrix} 0.1591 - 0.6890J \\ -0.7071 \end{bmatrix}$

where $J = \sqrt{-1}$

Show that

$$Ap_1 = (-0.2250 + 0.9744J)\,p_1$$

$$Ap_2 = (-0.2250 - 0.9744J)\,p_2$$

[If for a matrix A; if p is a vector such that $Ap = \lambda p$; with $p \neq 0$ then p is a eigenvector of A with eigenvalue λ]

(c) Let $\lambda_1 = -0.2250 + 0.9744J$

$\lambda_2 = -0.2250 - 0.9744J$

and let

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Show that

$$A = P \Lambda P^{-1}$$

with $P := [\, p_1 \;\; p_2 \,]$;

(d) Is the system asymptotically stable?

(e) Find the matrix $\exp(At)$.

(f) Find the initial condition response with initial condition $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(g) Find the state trajectory of the system when the initial condition is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $f(t)$ is a unit step $u(t) = 0$ if $t \leq 0$ and $u(t) = 1$ N when $t > 0$. Compare your result with MATLAB.

(h) Plot the step-response obtained in (g) and indicate the steady-state part of the forced response and the transient part of the forced response.