A high bandwidth cantilever probe sensor

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Consider a simple model of a cantilevered bridge given by

$$\ddot{p} + \frac{\omega_0}{Q}\dot{p} + \omega_0^2 p = g(t) + d(t) \tag{1}$$

where ω_0 is the resonant frequency of the bridge and Q is the Quality factor that characterizes the dissipative properties of the bridge. $g(t)$ is a forcing that is an input given by the user and $d(t)$ is the disturbance (that is not manipulable). The position of the end of the bridge is given by p and there is a sensor that measures this value.

- Determine the state space model of the dynamics. Note that there are two inputs $g(t)$ and $d(t)$.

  Hint: The state space model is described by

  $$\begin{array}{rcl} \dot{x} &=& Ax + B_1 g + B_2 d \\ y &=& Cx + Du \end{array} \tag{2}$$

  Determine the matrices A, B_1, B_2, C, and D.

- Obtain the realization circuit of the above state space in terms of integrators, adders and multipliers. Clearly indicate the inputs d and g and the output y.

- Determine the state space model of an observer for the state space described in (2) with the observer gain L.

  Hint: The state space model of the observer is given by described by

  $$\begin{array}{rcl} \dot{\hat{x}} &=& A\hat{x} + B_1 g + L(y - \hat{y}) \\ \hat{y} &=& C\hat{x} \end{array} \tag{3}$$

- Derive the dynamics of the error $e := x - \hat{x}$.

- Realize the observer in terms of integrators, adders and multipliers. Clearly indicate the inputs and the output $\hat{y}$. Hint: note that d is not an input to the observer

- Implement the observer in simulink with the model of the bridge also implemented. Let $\omega_0 = 2\pi\ rad/s$ and quality factor $Q = 100$. Determine the observer gain $\bar{L}$ by using commands $P = [-5\ -7]'; K = place(A', C', P); L = K'$. This places the eigenvalues of $A - LC$ at -5 and -7.

- Investigate the error response due to initial condition mismatch by having the initial condition of the cantilever set to $x(0) = [4\ 0]'$ and that of the observer set to $\hat{x}(0) = [-4\ 0]'$. Do this step for $L = 0$ and with $L = \bar{L}$ as determined above.

- Set the initial conditions of all integrators to zero. Let $g(t) = \sin \omega_0 t$. Introduce a pulse $d(t) = 1$ for $(10)\frac{2\pi}{\omega_0}$ seconds, *after the observer and the cantilever are in "sync" (the error due to transient response has died out).* Simulate the response for two cases $L = 0$ and $L = \bar{L}$ chosen in the step above.

  Hint: The transient response should die out within $10Q$ cycles of the forcing (i.e in a time $10Q\omega_0/(2\pi)$. Verify this by simulations first

- Comment on the tracking behavior of both $L = 0$ and $L = \bar{L}$.