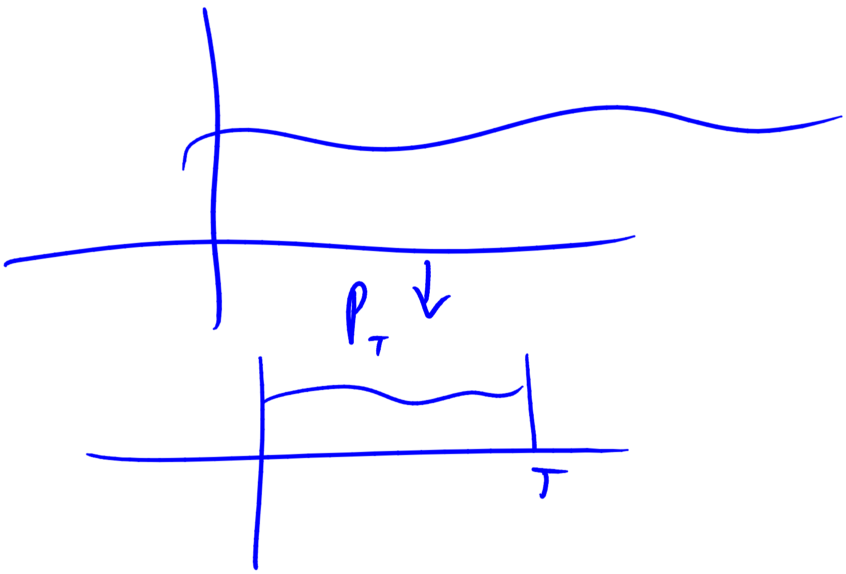
lecture17

Tuesday, March 29, 2011

8:24 AM

$$(P_T x)(t) = x(t) \quad \text{if } t < T$$
$$= 0 \quad \text{if } t > T$$

①

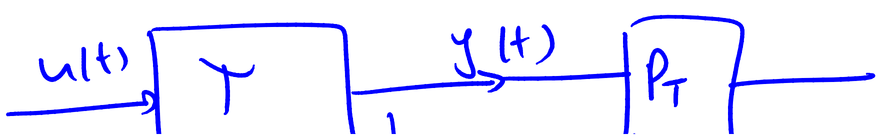


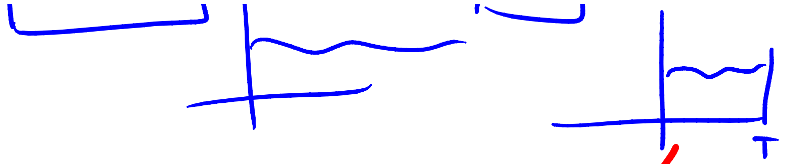
② $(S_\tau x)(t) = x(t-\tau)$ Shift operator

We say a system $T: L^n \to L^m$ is causal if

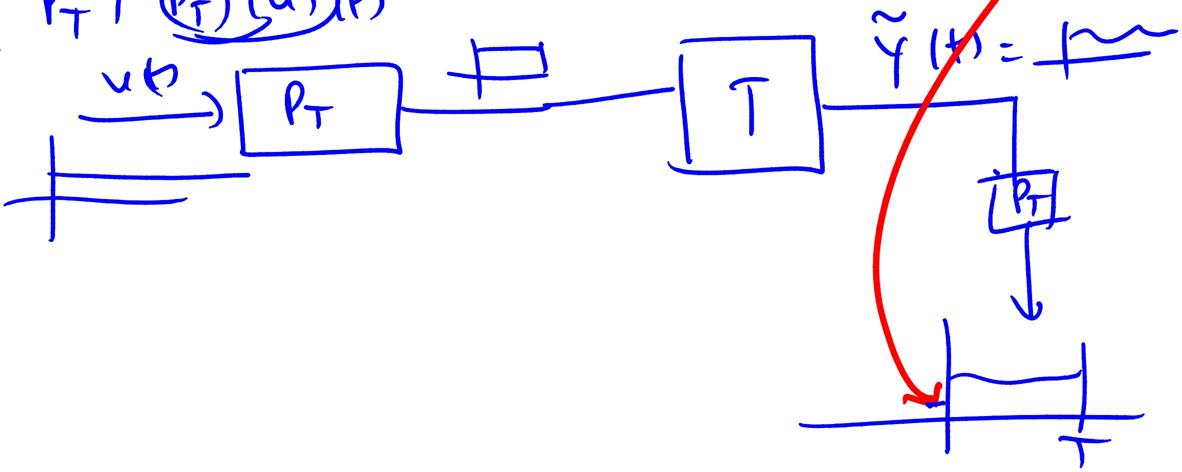
$$(P_t T) = \underline{P_t T P_t}$$

$$(\underline{P_T T u})(t) = (Tu)(t) \quad \text{if } t < T$$
$$= 0 \quad \text{otherwise}$$

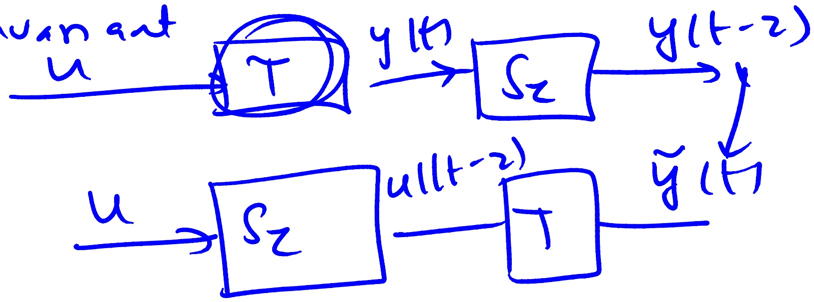




$(P_T T (P_T)(u))(t)$



(a) A time-invariant



$$\sup_{x\neq 0} \frac{\|Tx\|_p}{\|x\|_p} = \sup_{\|x\|=1} \|Tx\|_p$$

if T is a linear map

$$\left\|T \frac{x}{\|x\|_p}\right\| = \frac{\|Tx\|}{\|x\|_p}$$

$T: L^n \to L^m$

$$T(\alpha x+\beta y) = \alpha T(x) + \beta T(y)$$

$\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$

$$\left(\frac{\|T x\|}{\|x\|} < \infty\right) \quad \forall x \in L_p'$$

but $\displaystyle \sup_{x \neq 0} \frac{\|Tx\|}{\|x\|} = \infty$

Aryabhatta, Bezout, Diophantine:

→. m and n are co-prime with respect to each other (i.e. m and n have only 1 as the common factor)
iff
$mp + nq = 1$ for some integers p and q.

$$\frac{15}{4}$$

$$15 \cdot p + 4q = 1$$

$\Leftarrow$ $mp + nq = 1$
TP and suppose $m = m_r r$.
$n = n_r r$
with $(m_r, r, n_r, r) \in \mathbb{Z}$

$(p m_r r + n_r r q) = 1$
$\Rightarrow r(p m_r + n_r q) = 1$
$r = 1$ and $p m_r + n_r q = 1$

Euclids theorem. m and n are coprime

. then you can construct

p and q s.t. $mp + nq = 1$.

$\deg(m) > \deg(n)$

$m = n q_0 + r_0$; $\deg(r) < \deg(q)$

$s+1$) $s^2 + ...$ (q

$q_0 = n q_1 + r_1$

all stable rational transfer functions with no ~~com~~ common zeros.

M and N

$MP + NQ = 1$.

$G = \frac{N}{M}$; Y/X

$D = NY + MX$ has all zeros ~~poles~~ in the LHP.

$N \frac{Y}{D} + M \frac{X}{D} = 1$

D^{-1} has all poles in the LHP

$N \underbrace{Y D^{-1}}_{\tilde{P}} + M \underbrace{X D^{-1}}_{\tilde{Q}} = 1$

$N P + M Q = 1$.

$$\frac{1}{(s+1)^2} \qquad \begin{bmatrix} s-1 & 2(s-1) \\ 0 & 0 \end{bmatrix}$$

$$\underline{\underline{x^* B = 0}} \quad . \quad \underline{x^* A = \lambda x^*}$$

$$x^* [B \;\; AB \;\; \cdots \;\; A^{n-1}B]$$

$$= [x^* B \;\; \underline{\underline{x^* A}}B \;\; x^* A^2 B \;\; \cdots \;\; x^* A^{n-1} B]$$

$$= [0 \;\; x^* \lambda B \;\; \lambda^2 x^* B \;\; \cdots \;\; \lambda^{n-1} x^* B]$$

$$= [0 \;\; 0 \;\; \cdots \;\; 0]$$