Robust Control: HW 6

Problem 1: Consider the true plant to be

$$G_p(s) = \frac{3e^{-0.1s}}{(2s+1)(0.1s+1)^2}.$$

1. Derive the additional uncertainty weight with the nominal weight to be
$$G(s) = 3/(2s+1).$$
2. Derive the corresponding robust stability condition
3. Apply this test to the controller $K(s) = k/s$ and find the values of k that yield stability. Is this condition tight?
4. Obtain a multiplicative uncertainty weight with the nominal plant as given above.
5. With the multiplicative uncertainty description, first choose a performance objective of tracking with tracking required till a frequency ω_p (choose ω_p sensibly) with tracking error to be within m_p (choose a small m_p). Thus form a reasonable weight W_p. For the chosen W_p and the multiplicative uncertainty weight chosen, design a controller that achieves robust performance

Problem 3:[Designing a controller to satisfy asymptotic properties]

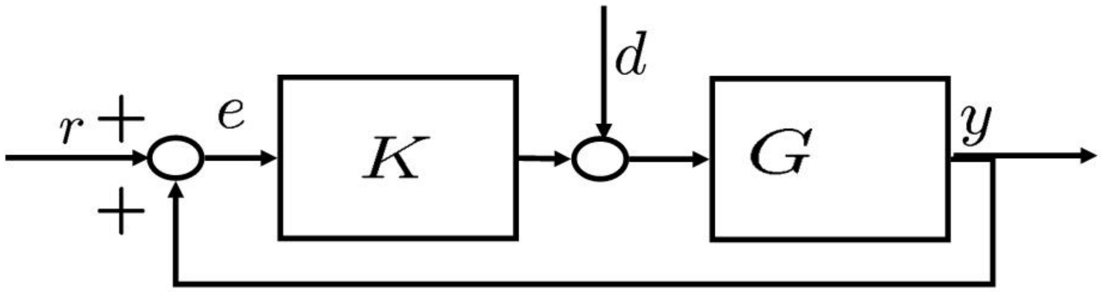

Figure 1: Positive feedback configuration

For the positive feedback configuration shown in Figure 1 suppose $G = N/M$ is a coprime factorization with $K = Y_1/X_1$ a stabilizing controller with

$$MX_1 - NY_1 = 1$$

where $M,\ N,\ X_1,\ Y_1$ are stable proper transfer functions.

1. Is it possible that M and N are such that they have a common zero at ∞ (that is, $\lim_{s\to\infty} M(s) = \lim_{s\to\infty} N(s) = 0$)

2. Consider the plant $G = \frac{1}{(s-1)(s-2)}$.

   (a) State a reason why we cannot choose $M = \frac{(s-1)(s-2)}{(s+1)^3}$ and $N = \frac{1}{(s+1)^3}$. Note that M and N are proper stable rational transfer functions with $G = NM^{-1}$.

   (b) Let $N = \frac{1}{(s+1)^2}$ and $M = \frac{(s-1)(s-2)}{(s+1)^2}$. Let $X_1 = \frac{s+6}{s+1}$ and $Y_1 = -\frac{19s-11}{s+1}$.

      i. Show that $G = NM^{-1}$ and that $MX_1 - NY_1 = 1$. Thus all stabilizing controllers K in terms of the Youla parameter Q are given by

$$K = \frac{Y_1 - MQ}{X_1 - NQ}. \tag{1}$$

ii. Choose a stable proper Q such that the controller has the all the following properties (1) The steady state step response y is 1 with $d = 0$ (that is when r is a unit step y settles to a constant 1.) (2) The final value of y is zero when d is a sinusoid of 10 rad/s with $r = 0$. [Hint: use the final value theorem.]