Dynamics and control of distributed systems

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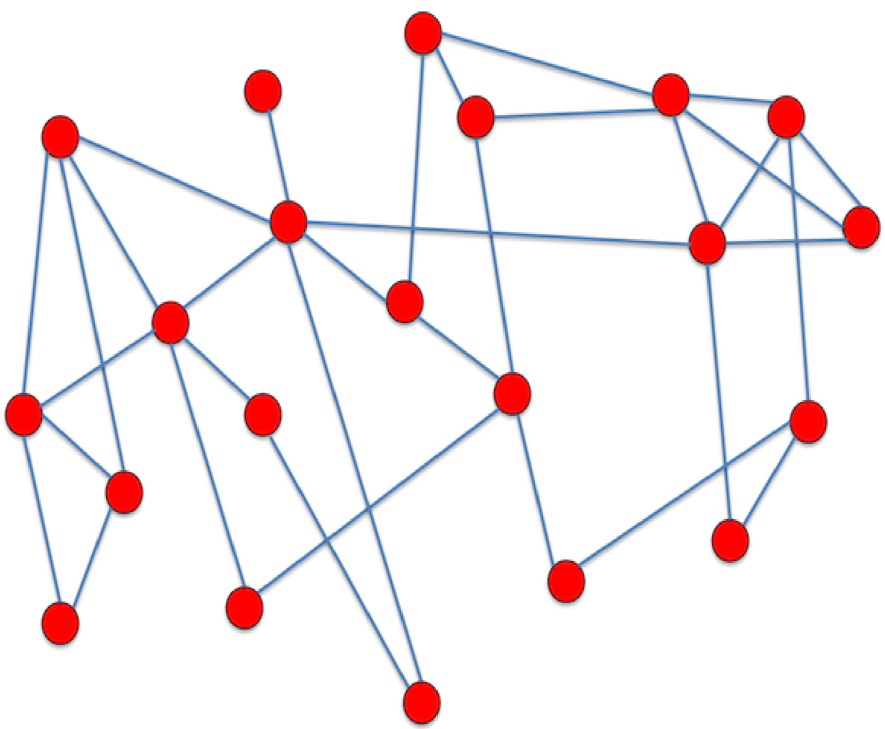
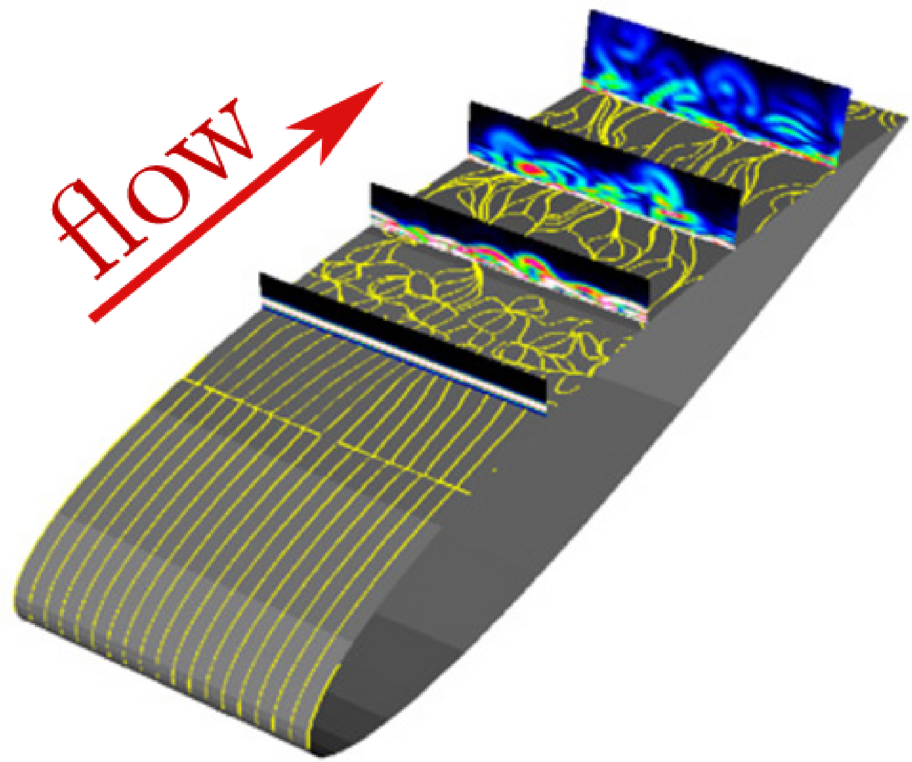




ICME Colloquium, Stanford University; April 8, 2013

Part 1: Large dynamic networks

Fundamental performance limitations

- **Network coherence**
  - ⋆ **can local feedback provide robustness to external disturbances?**

- **Roles of topology and spatial dimension**
  - ⋆ **1D vs 2D vs 3D**

Optimal design

- **Sparsity-promoting optimal control**
  - ⋆ **performance vs sparsity**

Part 2: Fluids

DYNAMICS AND CONTROL OF SHEAR FLOWS

- **The early stages of transition**
  - ⋆ **initiated by high flow sensitivity**
- **Controlling the onset of turbulence**
  - ⋆ **simulation-free design for reducing sensitivity**

Key issue:

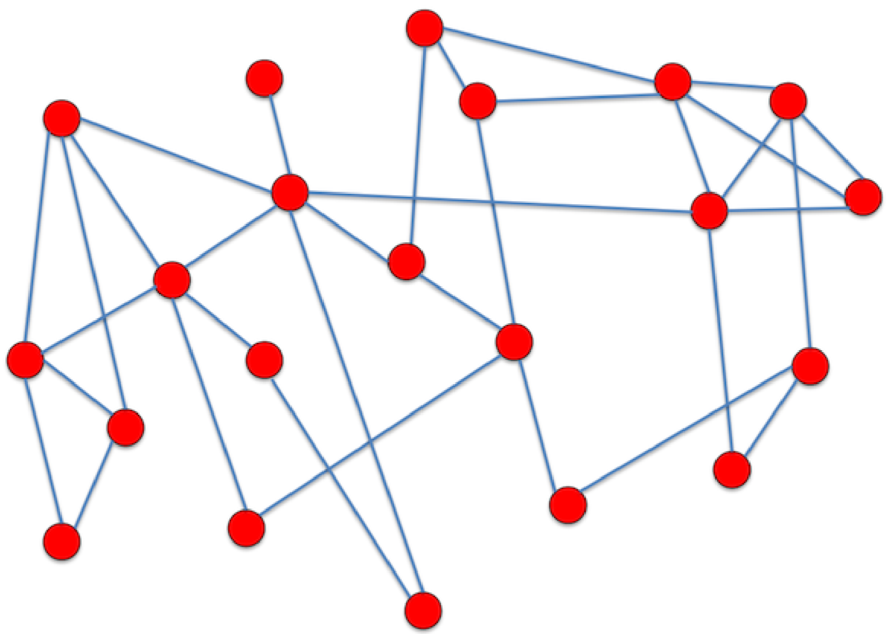
high flow sensitivity

SUMMARY AND OUTLOOK

Consensus by distributed averaging

- CHALLENGE

⋆ **how to quantify performance of large dynamic networks**



RELATIVE INFORMATION EXCHANGE WITH NEIGHBORS

⋆ **simplest distributed averaging algorithm**

$$\dot{x}_i(t) \;=\; -\sum_{j\,\in\,\mathcal{N}_i} \Big(x_i(t) \;-\; x_j(t) \Big)$$

Convergence and convergence rate

- NETWORK DYNAMICS

  ⋆ **diffusion on a graph with Laplacian** $L = L^T$

$$\begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_M(t) \end{bmatrix} = \begin{bmatrix} & & \\ & -L & \\ & & \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_M(t) \end{bmatrix}$$

  ⋆ **e-values of** L**:** $0 = \lambda_1 \le \lambda_2 \le \cdots \le \lambda_M$

connected network $\lambda_2(L) > 0$	$\Rightarrow$	**convergence to the average** $x_i(t) \xrightarrow{t \to \infty} \bar{x}(t) := \frac{1}{M} \sum x_i(t)$

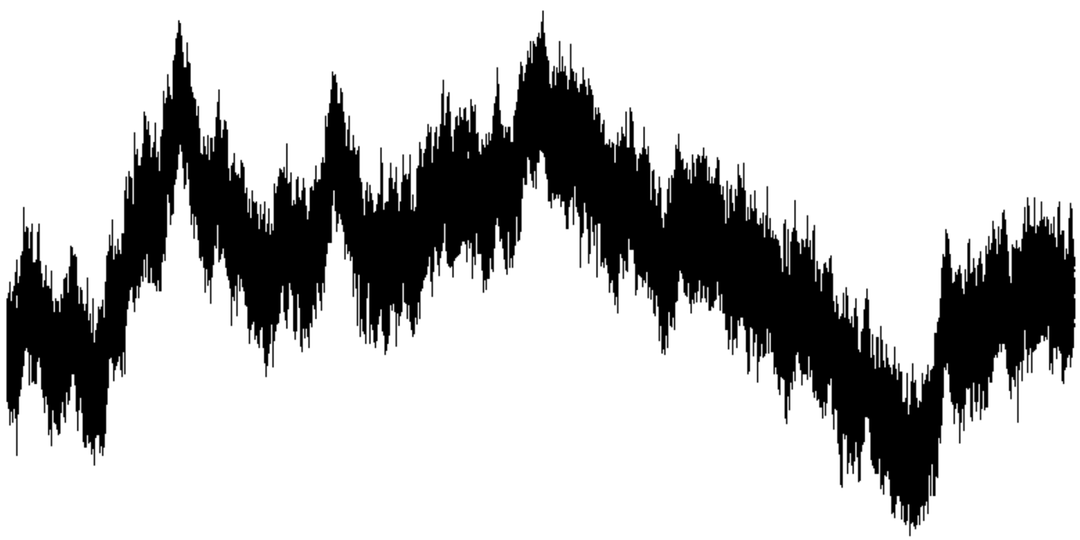
convergence time
(network time constant) $\sim \dfrac{1}{\lambda_2(L)}$

Consensus with stochastic disturbances

$$\dot{x}_i(t) \;=\; -\sum_{j\,\in\,\mathcal{N}_i} \Big(x_i(t) \;-\; x_j(t) \Big) \;+\; d_i(t)$$

$d_i(t) \downarrow$ **white noise**

- NETWORK AVERAGE
  - ⋆ **undergoes random walk**



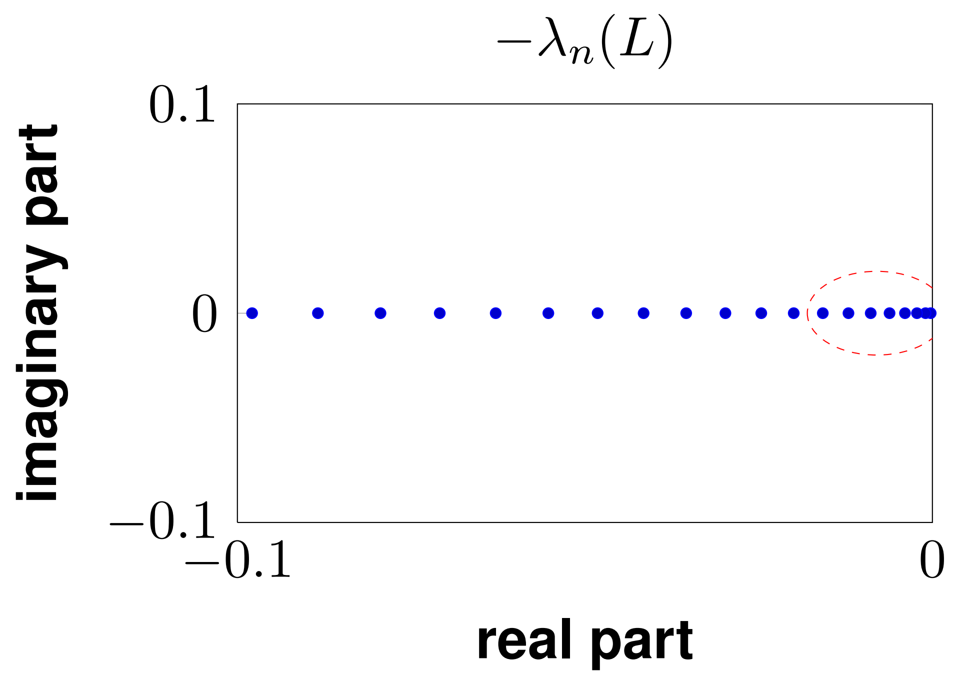
$$\lambda_2(L) \;>\; 0 \;\;\Rightarrow\;\; \begin{cases} \textbf{each } x_i(t) \textbf{ fluctuates around } \bar{x}(t) \\ \textbf{deviation from average: } \; \tilde{x}_i(t) \;:=\; x_i(t) \;-\; \bar{x}(t) \end{cases}$$

Variance of the deviation from average

$$\lim_{t \to \infty} \sum \mathcal{E}\left(\tilde{x}_i^2(t)\right) \;=\; \sum_{n \neq 1} \frac{1}{2\,\lambda_n(L)}$$

- AS NETWORK SIZE GROWS
  - ⋆ **spectrum clusters towards stability boundary**

  ASYMPTOTICS OF VARIANCES
  - ⋆ **determined by accumulation of e-values around zero**



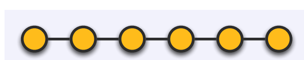
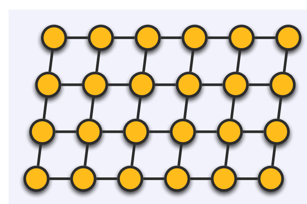
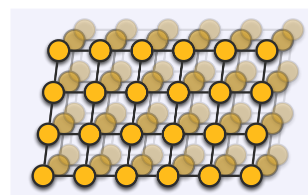
rate at which:

$$\sum_{n \neq 1} \frac{1}{\lambda_n(L)} \to \infty$$

unpredictable only from network time constant

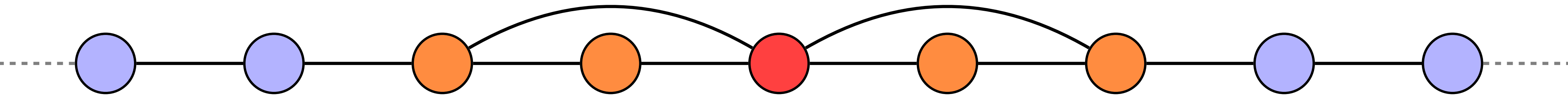
- SCALING DEPENDS ON NETWORK'S TOPOLOGY

asymptotic scaling for regular lattices:

Spatial dimension				d-dimensional lattice ($d \geq 4$)
Time constant	M^2	M	$M^{2/3}$	$M^{2/d}$
Variance (per node)	M	$\log(M)$	bounded	bounded

Questions

- CAN WE DO BETTER BY
  - ⋆ **going deeper into the lattice?**



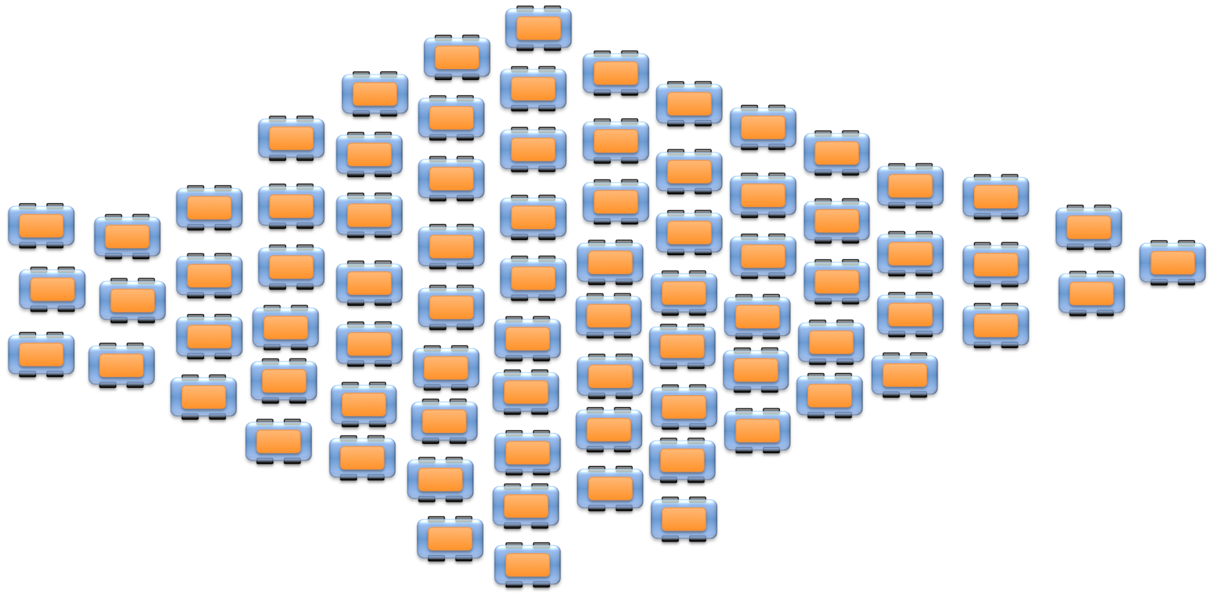
  - ⋆ **optimizing edge weights?**

$$\dot{x}_i(t) \;=\; -\sum_{j\,\in\,\mathcal{N}_i} K_{ij}\left(x_i(t) \,-\, x_j(t)\right) \,+\, d_i(t)$$

- ROLES OF
  - ⋆ **node dynamics**
  - ⋆ **spatial dimension**

Cooperative control of formations

- **Coherence: similarity between large formation and solid object**



1D:

snow geese formation

2D:

herd migration

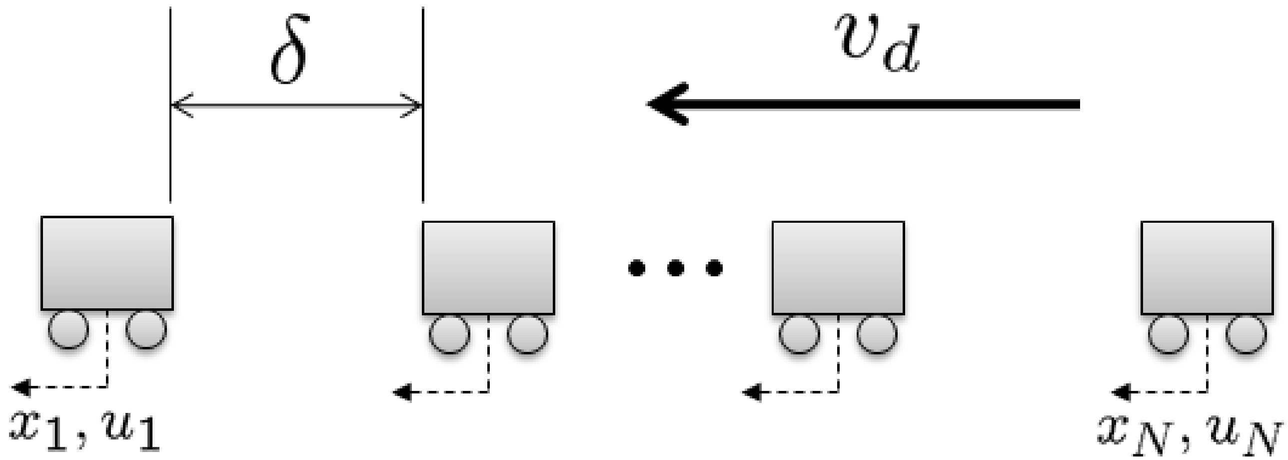
3D:

fish schools

An example: Vehicular strings

AUTOMATED CONTROL OF EACH VEHICLE

tight spacing at highway speeds



KEY ISSUES (**also in: control of swarms, flocks, formation flight**)

⋆ Is it enough to only look at neighbors?

⋆ How does performance scale with size?

⋆ Fundamental limitations?

FUNDAMENTALLY DIFFICULT PROBLEM

⋆ **scales poorly with size**

Problem formulation in 1d

$$\ddot{x}_n = u_n + d_n$$

$u_n \downarrow$ **control** $\quad$ $d_n \downarrow$ **disturbance**

desired trajectory	deviations
$\bar{x}_n(t) := v_d t + n\,\delta$	$p_n(t) := x_n(t) - \bar{x}_n(t)$
constant velocity v_d	$v_n(t) := \dot{x}_n(t) - v_d$

- **OPEN-LOOP DYNAMICS**

$$\begin{bmatrix} \ddot{p}_1 \\ \vdots \\ \ddot{p}_N \end{bmatrix} = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} + \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}$$

- **STATE-FEEDBACK CONTROLLER**

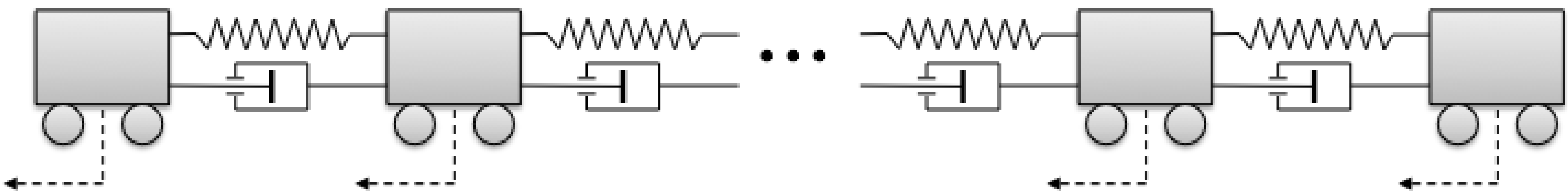
$$u(t) \;=\; -\,K_p\,p(t) \;-\; K_v\,v(t)$$

- **CLOSED-LOOP SYSTEM**

$$\begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} d$$

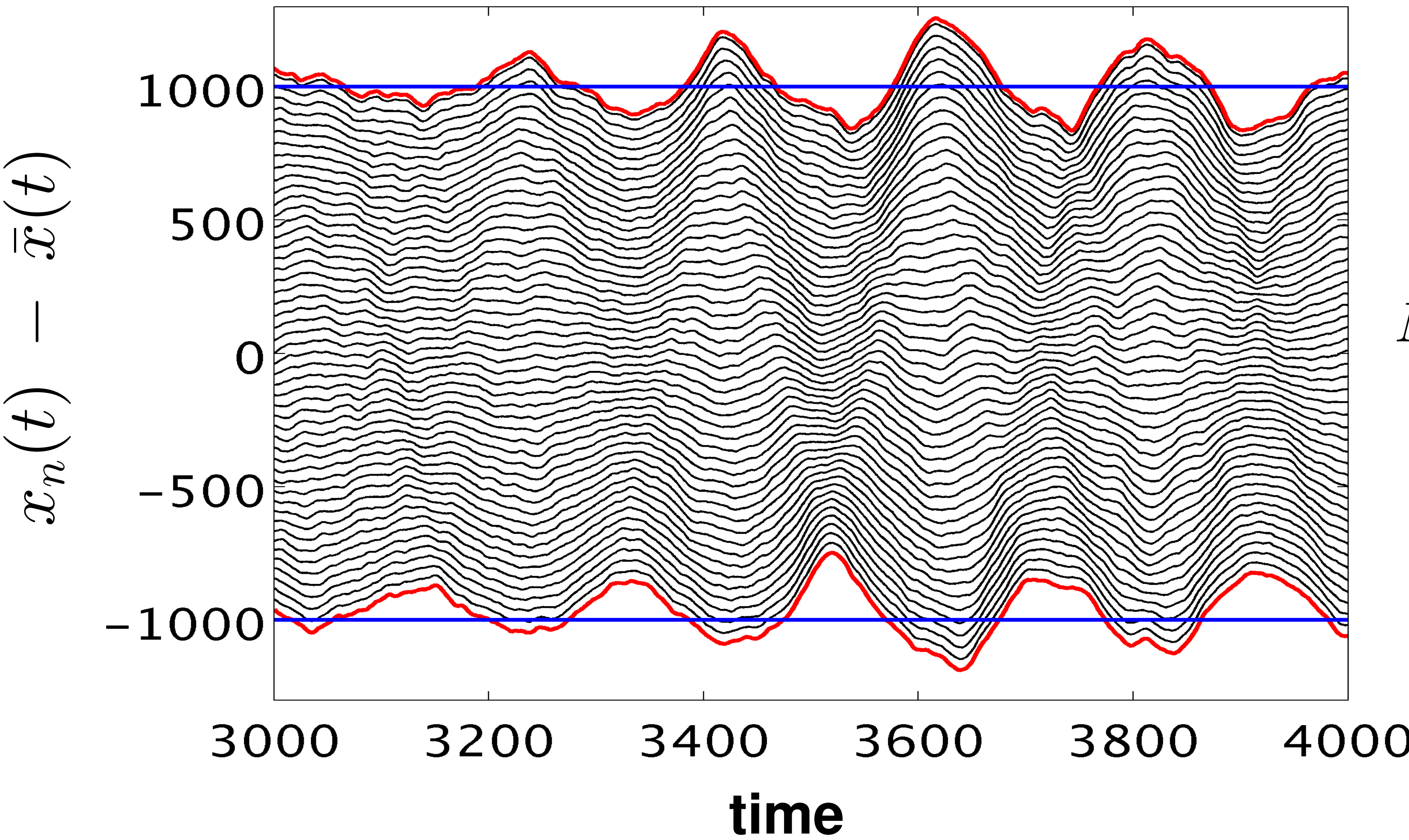
- AN EXAMPLE: { **nearest neighbor feedback** / **relative measurements** }

⋆ **e.g., use a simple strategy:**



Incoherence phenomenon

trajectories of every other vehicle:



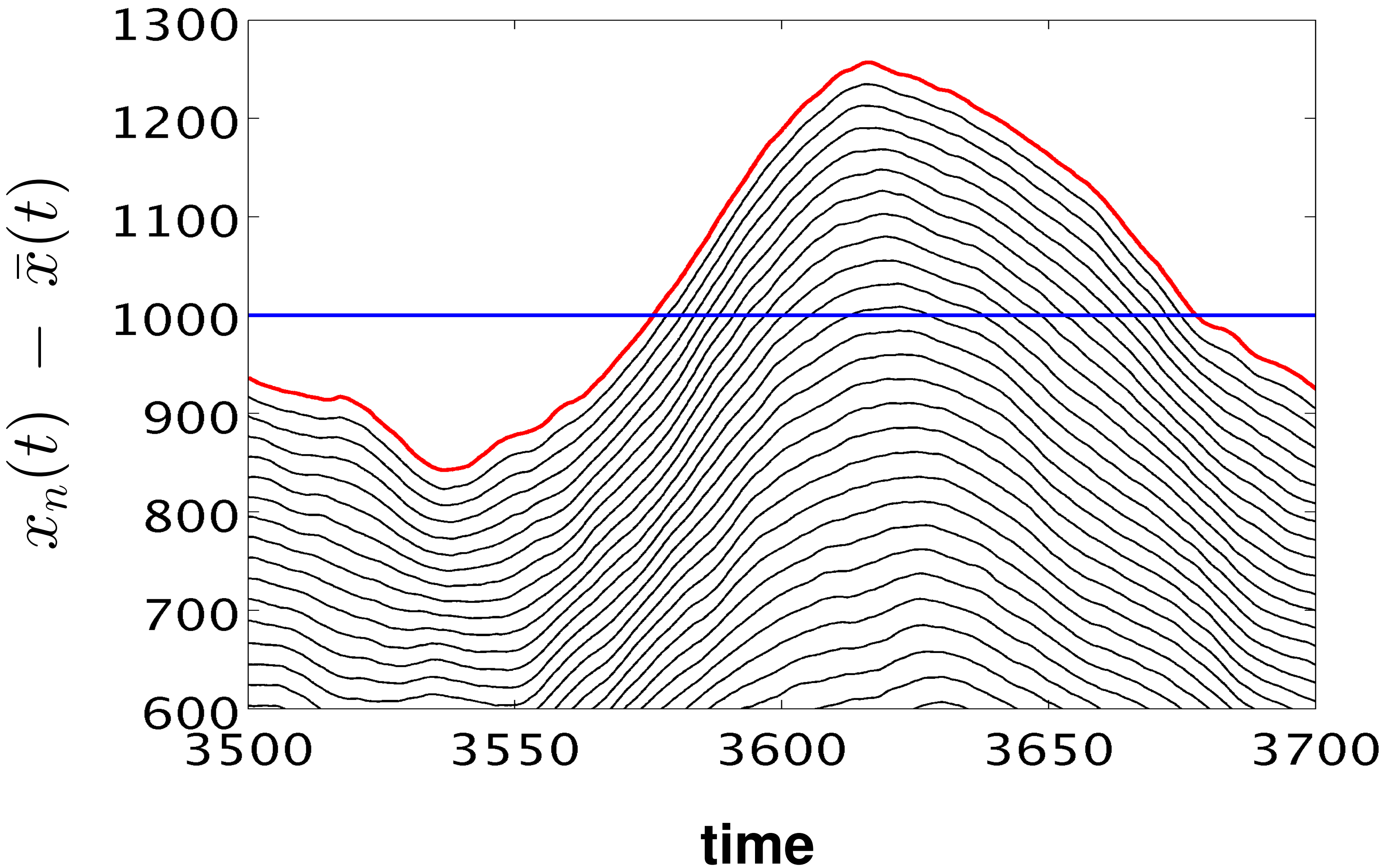
$N = 100$

$\delta = 20$

local feedback
relative measurements } $\Rightarrow$ **meandering large-scale motion**

- **Small-scale behavior** $\approx$ **well-regulated**
  - $\star$ **no collisions**

zoomed in trajectories:



Poor design or fundamental limitation?

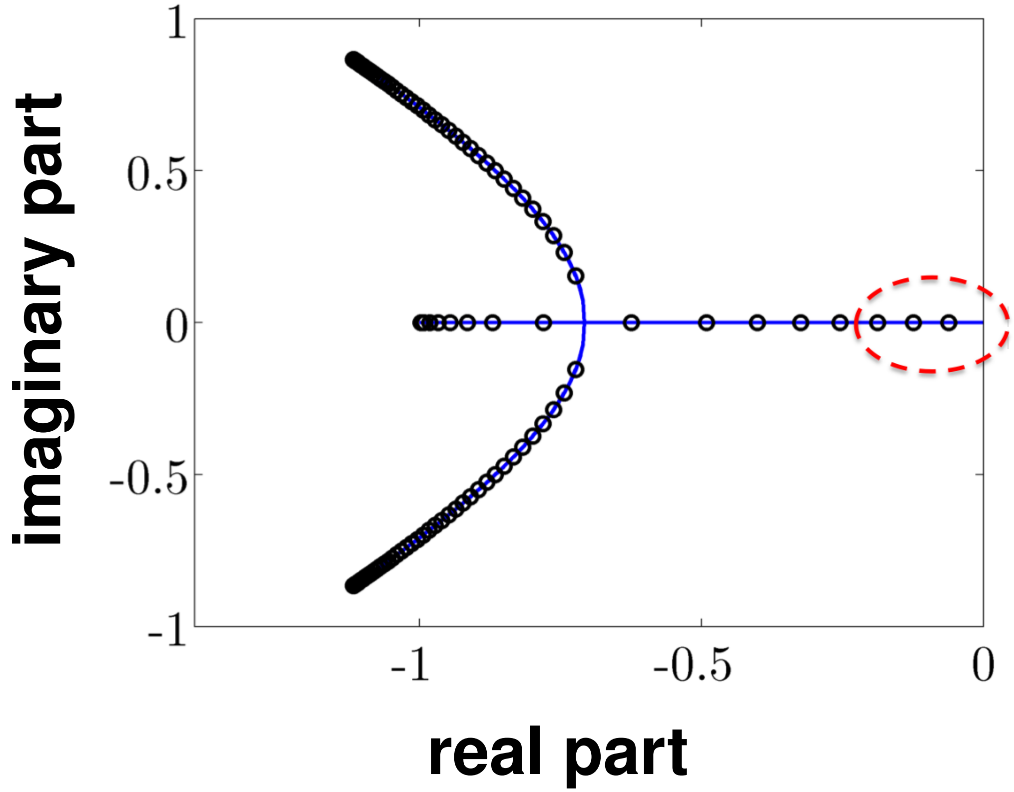
☞ **Optimal centralized design**

- ⋆ **not immune to some of these issues!**

$$\text{minimize} \quad \int_0^\infty \sum_n \Big(\big(p_n(t) \; - \; p_{n-1}(t)\big)^2 \; + \; v_n^2(t) \; + \; u_n^2(t) \Big) \, \mathrm{d}t$$

- ORIGINAL FORMULATIONS
  - ⋆ *Levine & Athans, IEEE TAC '66*
  - ⋆ *Melzer & Kuo, Automatica '71*
- REVISITED
  - ⋆ *Jovanović & Bamieh, IEEE TAC '05*

Closed-loop spectrum with K_{opt}



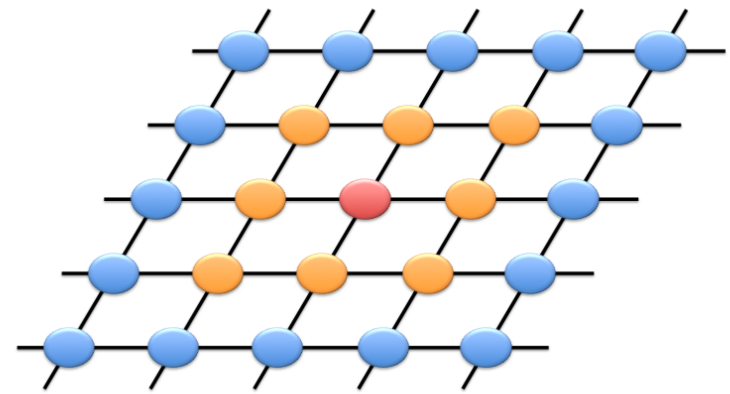
- FOR LARGE FORMATIONS
  - ⋆ **e-values accumulate towards imaginary axis**

PROBLEMATIC MODES

- ⋆ **slow temporal scale**
- ⋆ **long spatial wavelength**

Spatially invariant lattices

- EXPLICIT RESULTS FOR
  - ⋆ **d-dimensional torus $\mathbb{Z}_N^d$ with $M = N^d$ vehicles**



STRUCTURAL FEATURES

- ⋆ **spatially-localized feedback**
- ⋆ **mirror symmetry in feedback gains**

relative vs **absolute measurements:**

$$
\begin{aligned}
u_n(t) \; = \; & - K_p^- \left(p_n(t) \; - \; p_{n-1}(t)\right) \quad - \quad K_p^+ \left(p_n(t) \; - \; p_{n+1}(t)\right) \\
& - K_v^- \left(v_n(t) \; - \; v_{n-1}(t)\right) \quad - \quad K_v^+ \left(v_n(t) \; - \; v_{n+1}(t)\right) \\
& - K_p^0 \, p_n(t) \qquad\qquad\qquad\qquad - \quad K_v^0 \, v_n(t)
\end{aligned}
$$

Performance measures

- **Microscopic: local position deviation**

$$V_{\mathrm{micro}} := \lim_{t \to \infty} \mathcal{E}\left(\sum_n \left(p_n(t) - p_{n-1}(t) \right)^2 \right)$$

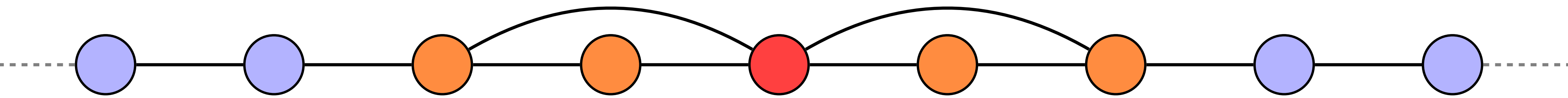
- **Macroscopic: deviation from average**

$$V_{\mathrm{macro}} := \lim_{t \to \infty} \mathcal{E}\left(\sum_n \left(p_n(t) - \bar{p}(t) \right)^2 \right)$$

How does variance per vehicle scale with system size?

$$\frac{V_{\mathrm{micro}}}{M} \quad \text{vs} \quad \frac{V_{\mathrm{macro}}}{M}$$

Performance in 1D



asymptotic scaling (per vehicle):

Feedback type	Microscopic performance	Macroscopic performance
absolute position absolute velocity	bounded	bounded
relative position absolute velocity	bounded	M
relative position relative velocity	M	M^3

$$\left.\begin{array}{c}\textbf{local feedback}\\ + \\ \textbf{relative position}\\ \textbf{measurements}\end{array}\right\} \Rightarrow$$

large coherent formations:

impossible in 1D!

Role of dimensionality: variance per vehicle

- **Lower bounds** for any spatially-invariant stabilizing local feedback with:
  - ⋆ **bounded control effort at each vehicle:** $\mathcal{E}\left(u_n^2\right) \;\leq\; U_{\max}$

asymptotic scaling:

Feedback type	Microscopic performance	Macroscopic performance
absolute position absolute velocity	$\dfrac{1}{U_{\max}}$	$\dfrac{1}{U_{\max}}$
relative position absolute velocity	$\dfrac{1}{U_{\max}}$	$\dfrac{1}{U_{\max}} \begin{cases} M & d = 1 \\ \log(M) & d = 2 \\ 1 & d \geq 3 \end{cases}$
relative position relative velocity	$\dfrac{1}{U_{\max}^2} \begin{cases} M & d = 1 \\ \log(M) & d = 2 \\ 1 & d \geq 3 \end{cases}$	$\dfrac{1}{U_{\max}^2} \begin{cases} M^3 & d = 1 \\ M & d = 2 \\ M^{1/3} & d = 3 \\ \log(M) & d = 4 \\ 1 & d \geq 5 \end{cases}$

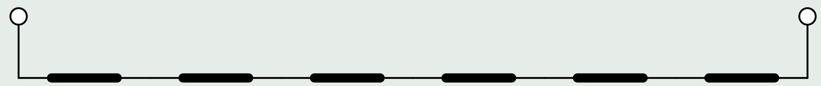
Bamieh, Jovanović, Mitra, Patterson, IEEE Trans. Automat. Control ’12

Connections

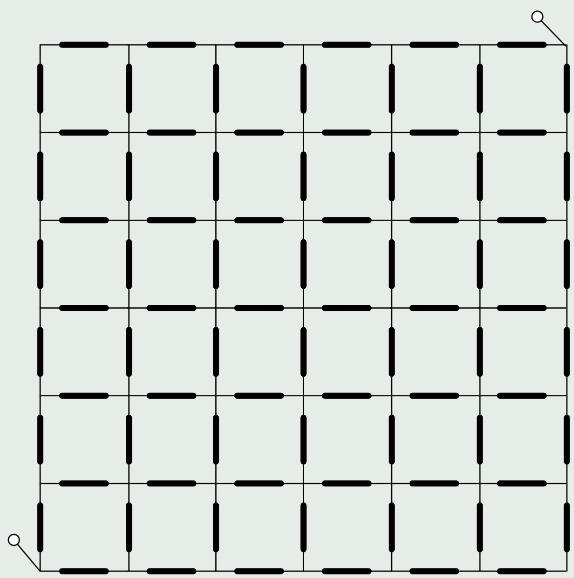
- SIMILAR SCALING TRENDS OBSERVED IN
  - ⋆ distributed estimation from relative measurements
  - ⋆ effective resistance in electrical networks
  - ⋆ global mean first-passage time of random walks
  - ⋆ statistical mechanics of harmonic solids
  - ⋆ Wiener index of a molecule

RESISTIVE NETWORK ANALOGY

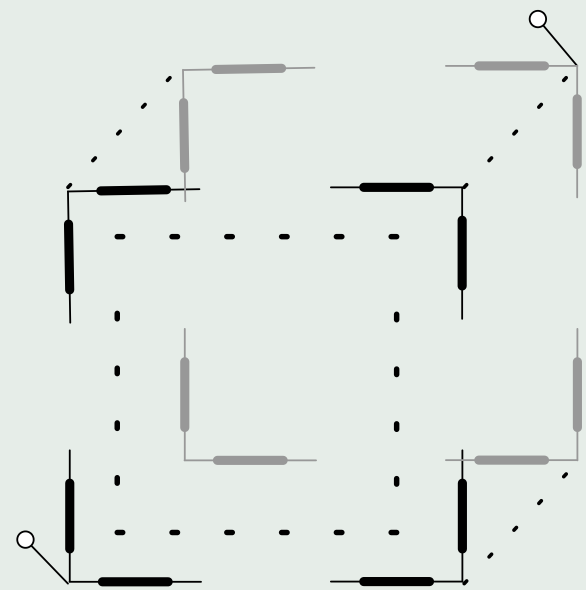
1D :



Net resistance $= R\,M$



2D :

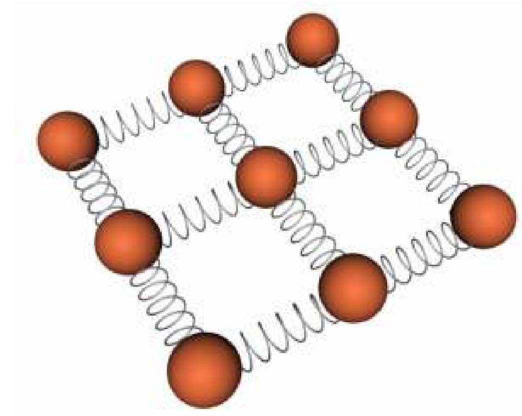


3D :

Net resistance $= O\left(\log(M)\right)$

Net resistance is *bounded*!

- **STATISTICAL MECHANICS OF HARMONIC SOLIDS**
  - ⋆ a d-dimensional lattice of masses and springs



physics: **short range interactions** vs **long range order**

↓ ↓

networks: **local feedback** vs **network coherence**

WITH SHORT RANGE INTERACTIONS LONG RANGE ORDER IS:	**impossible in 1D and 2D** **achievable in 3D**

DESIGN OF NETWORKS

Sparsity-promoting optimal control

$$\text{minimize} \quad J(K) \quad + \quad \gamma \sum_{i,\,j} W_{ij} \, |K_{ij}|$$

$\downarrow$ **variance amplification**

$\downarrow$ **sparsity-promoting penalty function**

- $\star$ $\gamma > 0$ – performance vs sparsity tradeoff
- $\star$ $W_{ij} \geq 0$ – weights (for additional flexibility)

Lin, Fardad, Jovanović, IEEE TAC '13 (also: arXiv:1111.6188)

Design of undirected networks

dynamics: $\dot{x} \;=\; d \,+\, u$

objective function: $J \;:=\; \lim_{t \,\to\, \infty} \mathcal{E} \left(x^T(t)\, Q\, x(t) \;+\; u^T(t)\, R\, u(t) \right)$

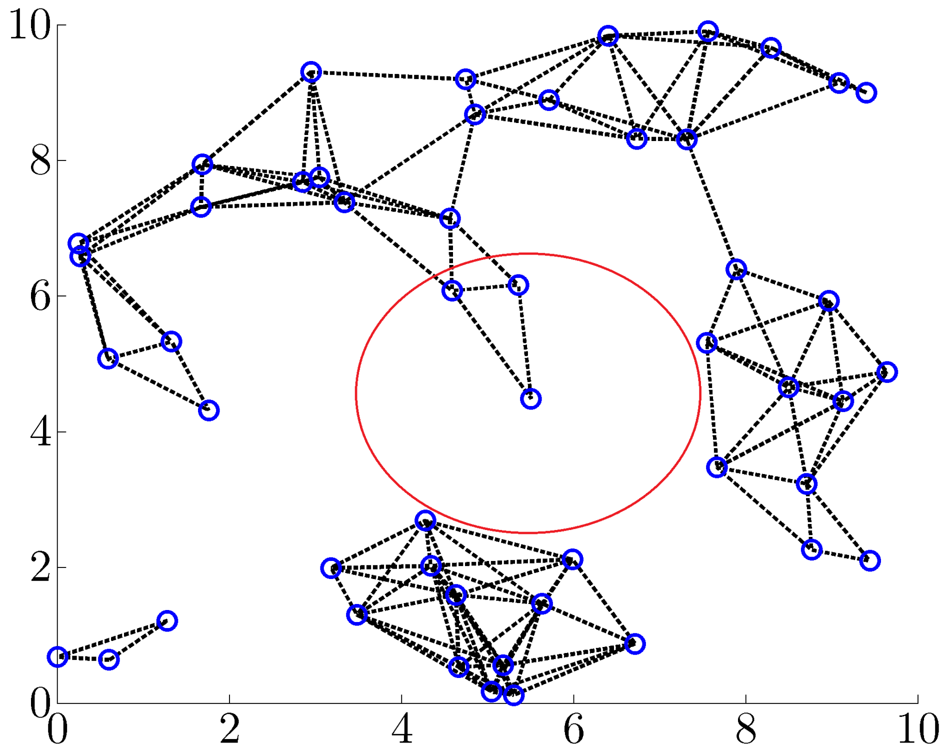
performance weights: $Q \,\succeq\, 0, \quad R \,\succ\, 0$

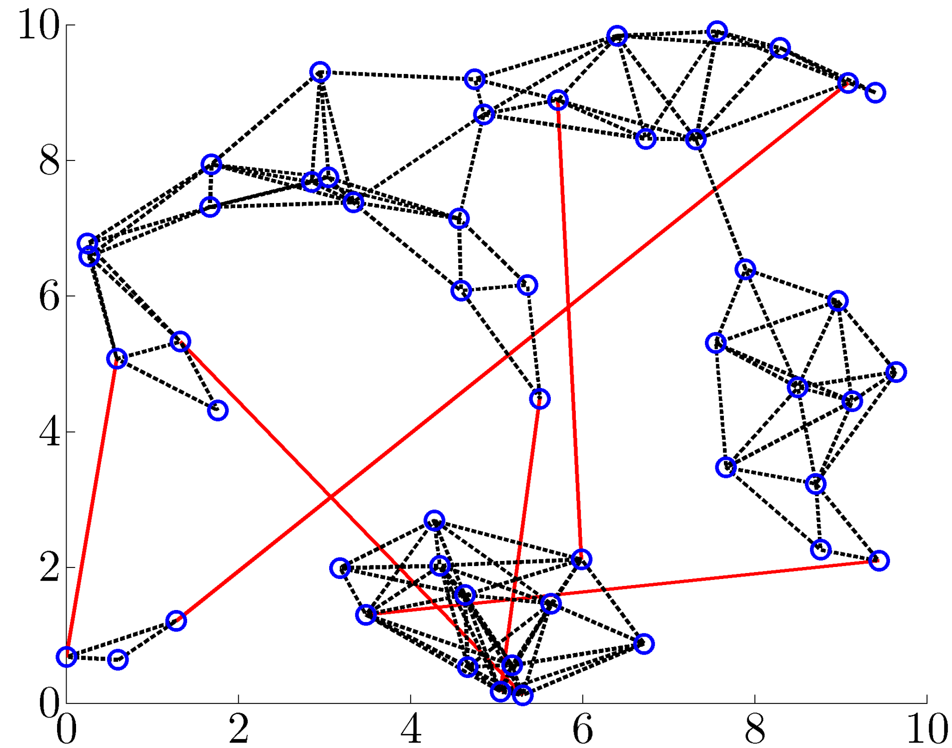
can be formulated as an SDP:

$$
\begin{array}{ll}
\text{minimize} & \operatorname{trace}\,(X \,+\, R\,K) \;+\; \gamma\, \mathbb{1}^T\, Y\, \mathbb{1} \\[1em]
\text{subject to} & \begin{bmatrix} X & Q^{1/2} \\ Q^{1/2} & K + \mathbb{1}\mathbb{1}^T/N \end{bmatrix} \;\succeq\; 0 \\[1em]
& -Y \;\leq\; W \circ K \;\leq\; Y \\[1em]
& K\,\mathbb{1} \;=\; 0
\end{array}
$$

• SPARSITY-PROMOTING CONSENSUS ALGORITHM

local performance graph: **identified communication graph:**





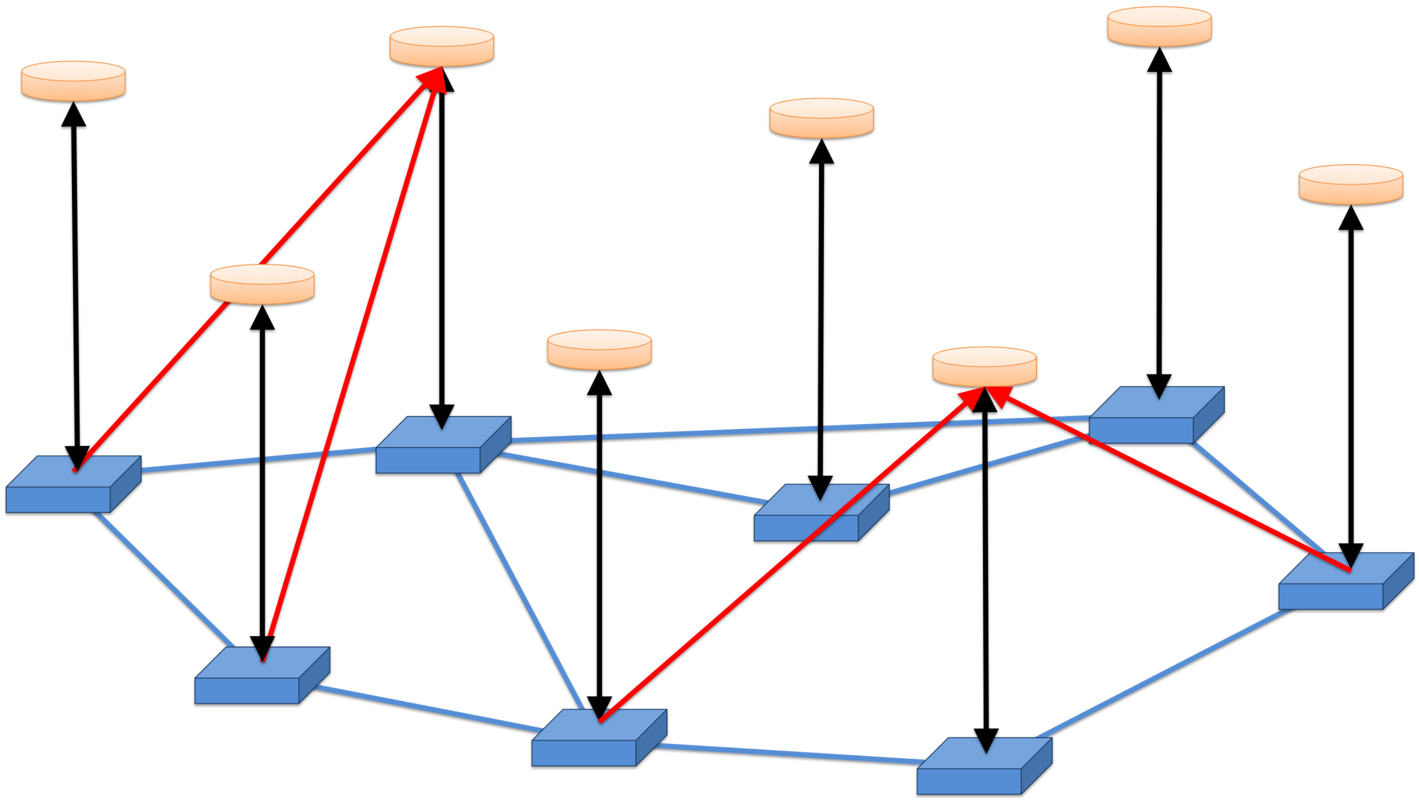
$$Q := Q_{\text{loc}} + \left(I - \frac{1}{N} \mathbb{1}\mathbb{1}^T \right)$$

$$\frac{J - J_{\text{c}}}{J_{\text{c}}} \approx 11\%$$

Structured distributed control

- **Blue layer: distributed plant and its interaction links**

memoryless structured controller



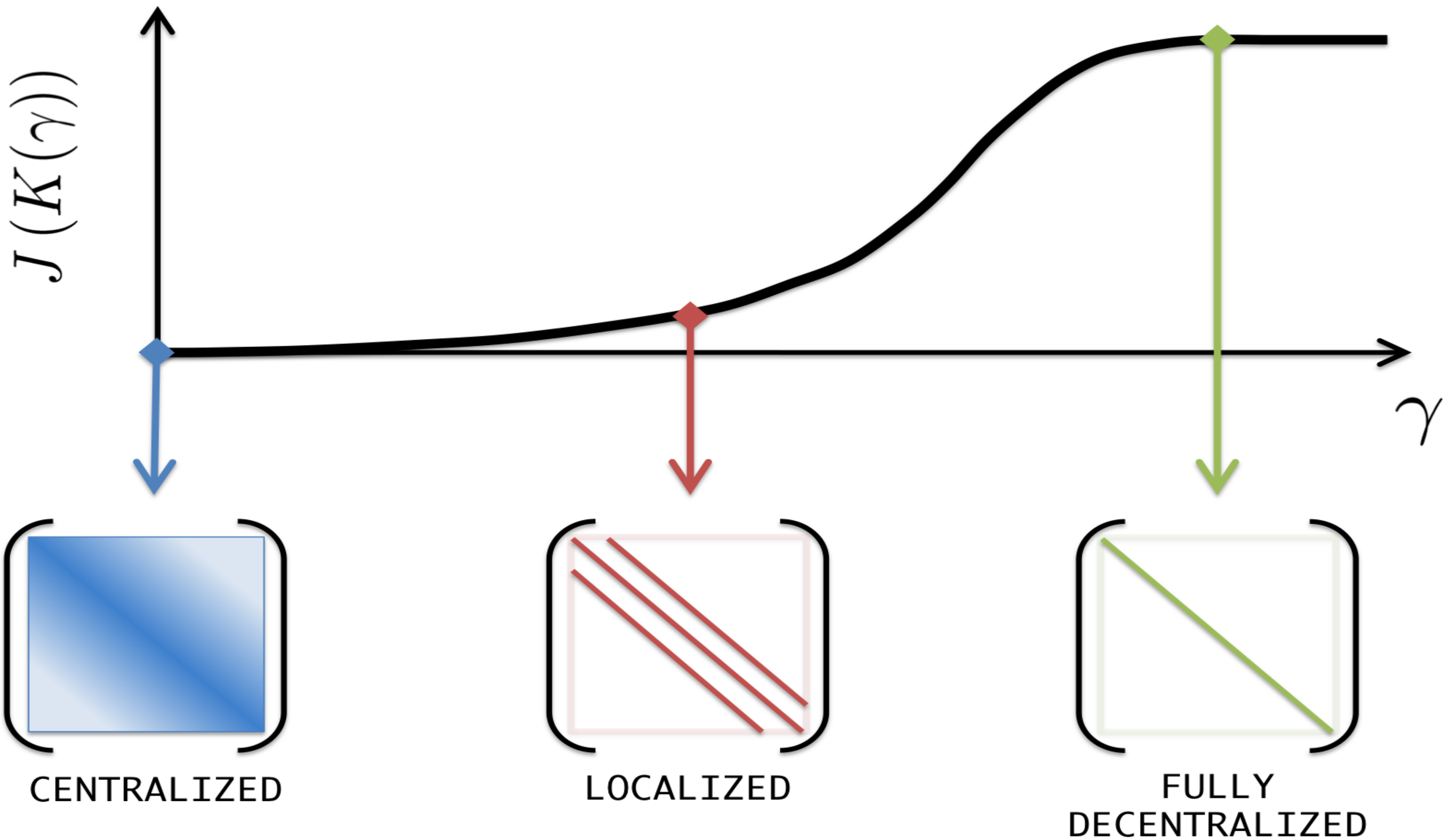
KEY CHALLENGE:

identification of a signal exchange network

performance vs sparsity

Parameterized family of feedback gains

$$K(\gamma) \; := \; \underset{K}{\arg\min} \left(J(K) \; + \; \gamma \, g(K) \right)$$



ALGORITHM: **alternating direction method of multipliers**

Boyd et al., Foundations and Trends in Machine Learning '11

Dynamics and control of fluids

- **Objective**
  - ⋆ **controlling the onset of turbulence**

- **Transition initiated by**
  - ⋆ **high flow sensitivity**

- **Control strategy**
  - ⋆ **reduce flow sensitivity**

Jovanović & Bamieh, J. Fluid Mech. '05

Moarref & Jovanović, J. Fluid Mech. '10

Lieu, Moarref, Jovanović, J. Fluid Mech. '10

Moarref & Jovanović, J. Fluid Mech. '12

Transition to turbulence

- LINEAR STABILITY

  ⋆ $Re \geq Re_c \;\Rightarrow\;$ exp. growing normal modes

  corresponding e-functions
  (**Tollmien-Schlichting waves**) } **exp. growing flow structures**

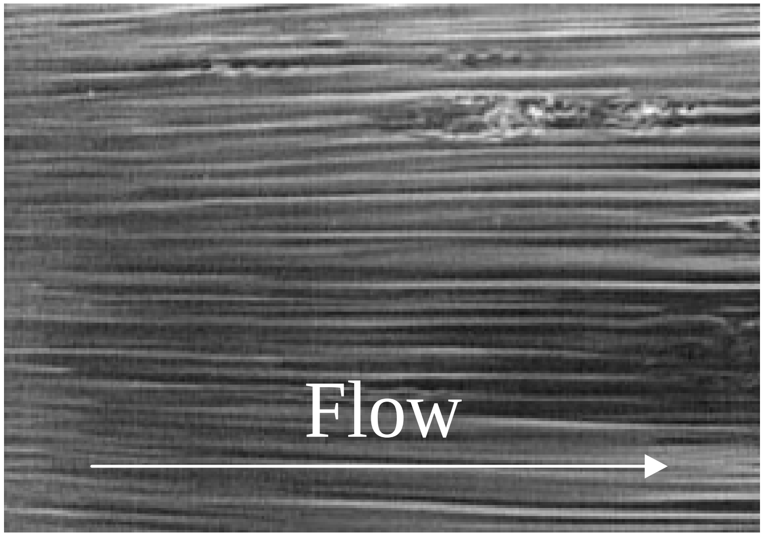
- EXPERIMENTAL ONSET OF TURBULENCE

  ⋆ **much before instability**

  ⋆ **no sharp value for** Re_c

streamwise streaks

spanwise direction



streamwise direction

Matsubara & Alfredsson, J. Fluid Mech. '01

Bypass transition

- **Triggered by high flow sensitivity**
  - ⋆ large transient responses
  - ⋆ large noise amplification
  - ⋆ small stability margins

TO COUNTER THIS SENSITIVITY: **must account for modeling imperfections**

TRANSITION ≈ STABILITY + RECEPTIVITY + ROBUSTNESS

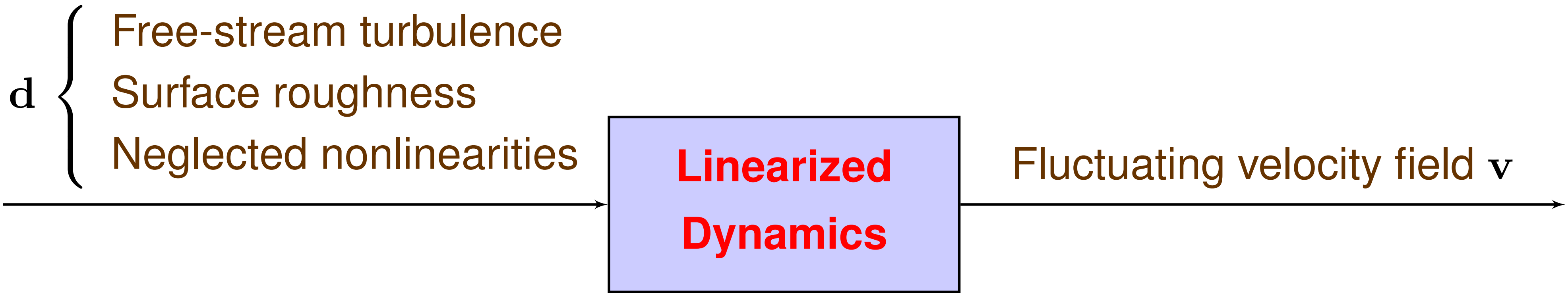
RECEPTIVITY ↓ **flow disturbances**

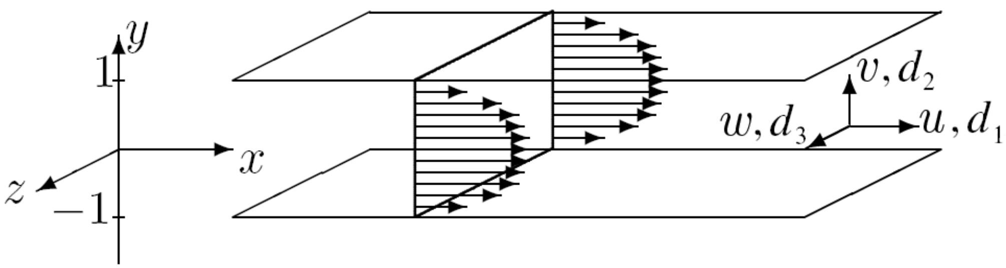
ROBUSTNESS ↓ **unmodeled dynamics**

Farrell, Ioannou, Trefethen, Henningson, Schmid, Kim, Bewley, Bamieh, etc.

Tools for quantifying sensitivity

- INPUT-OUTPUT ANALYSIS: **spatio-temporal frequency responses**





$$\underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{\text{amplification}} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\mathbf{v}}$$

IMPLICATIONS FOR:

transition: insight into mechanisms

control: control-oriented modeling

Response to stochastic forcing

$$\left.\begin{array}{rl}\textbf{white} & \text{in } t \text{ and } y \\ \textbf{harmonic} & \text{in } x \text{ and } z\end{array}\right\} \Rightarrow \quad \mathbf{d}(x,y,z,t) \;=\; \hat{\mathbf{d}}(k_x,y,k_z,t)\,\mathrm{e}^{\mathrm{i}k_x x}\,\mathrm{e}^{\mathrm{i}k_z z}$$

- LYAPUNOV EQUATION
  - ⋆ **propagates white correlation of $\hat{\mathrm{d}}$ into colored statistics of $\hat{\mathrm{v}}$**

$$\mathbf{A}(\boldsymbol{\kappa})\,\mathbf{X}(\boldsymbol{\kappa}) \;+\; \mathbf{X}(\boldsymbol{\kappa})\,\mathbf{A}^*(\boldsymbol{\kappa}) \;=\; -\,\mathbf{I}$$

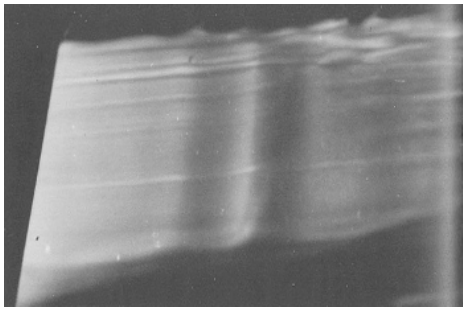
  - ⋆ **variance amplification**

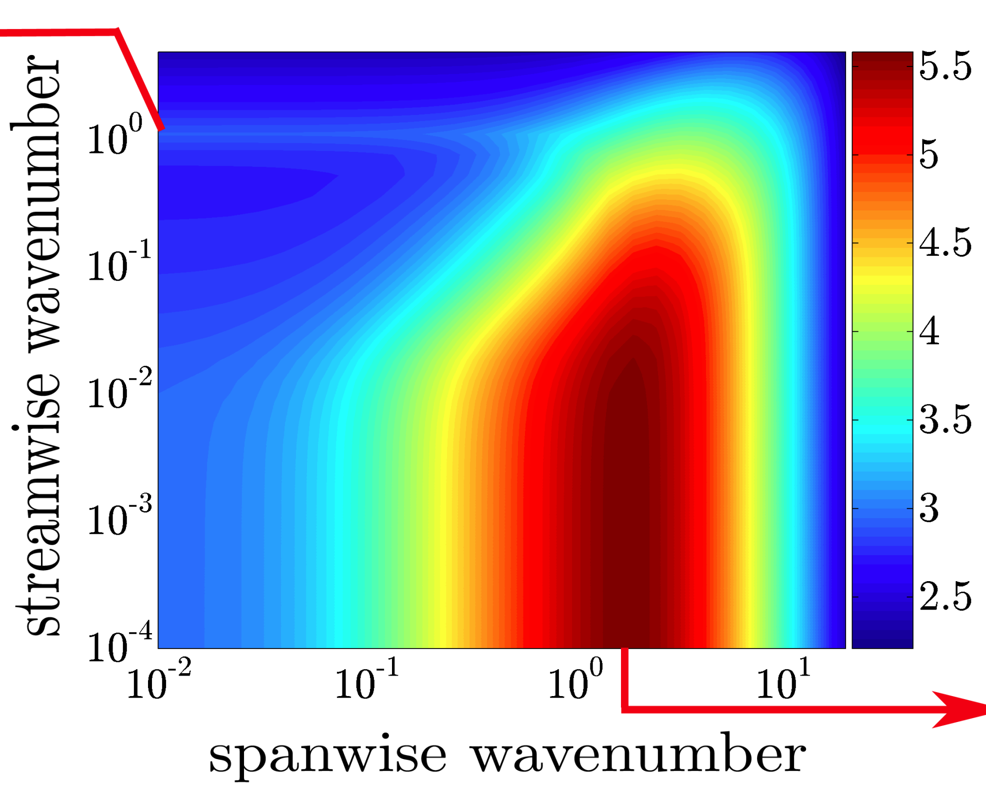
$$\begin{aligned} E\left(\boldsymbol{\kappa}\right) \;&:=\; \lim_{t\,\to\,\infty}\int_{-1}^{1}\mathcal{E}\left(\hat{\mathbf{v}}^*(\boldsymbol{\kappa},y,t)\,\hat{\mathbf{v}}(\boldsymbol{\kappa},y,t)\right)\mathrm{d}y \\ &=\; \mathrm{trace}\left(\mathbf{X}(\boldsymbol{\kappa})\right)\end{aligned}$$

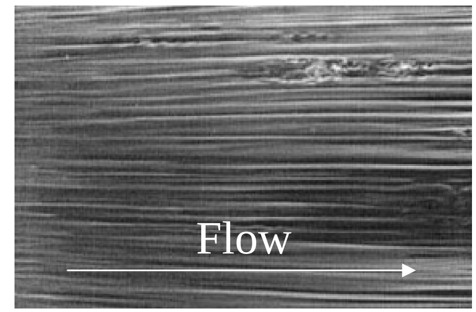
$$\boldsymbol{\kappa} \;:=\; (k_x,k_z)$$

Variance amplification

channel flow with $Re = 2000$:







- **Dominance of streamwise elongated structures**

  streamwise streaks!

Jovanović & Bamieh, J. Fluid Mech. '05

Amplification mechanism

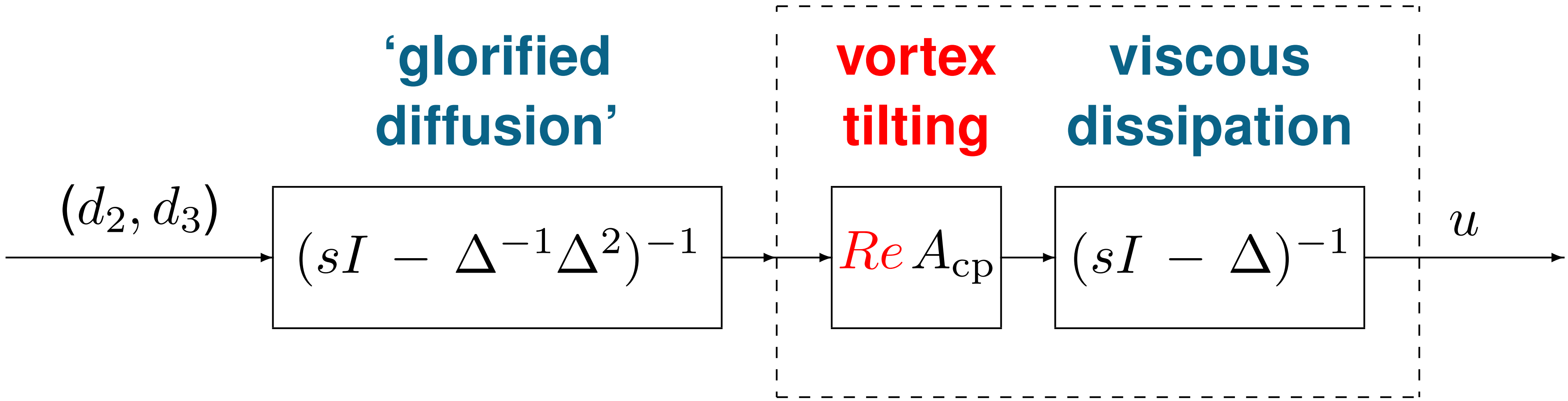
- STREAMWISE-CONSTANT MODEL

$$\begin{bmatrix} \psi_{1t} \\ \psi_{2t} \end{bmatrix} = \overbrace{\begin{bmatrix} A_{\text{os}} & 0 \\ Re\, A_{\text{cp}} & A_{\text{sq}} \end{bmatrix}}^{\text{non-normal}} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} + \begin{bmatrix} 0 & B_2 & B_3 \\ B_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 & C_u \\ C_v & 0 \\ C_w & 0 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

E-VALUES: **misleading measure of**
- **transient responses**
- **noise amplification**
- **stability margins**

- HIGHEST AMPLIFICATION: $(d_2, d_3) \to u$



⋆ **dynamics of normal vorticity ψ_2**

$$\psi_{2t} = \Delta\psi_2 + Re\, A_{cp}\, \psi_1 \qquad A_{cp} = -(\mathrm{i}k_z)\, U'(y)$$

↓

source

spanwise variations

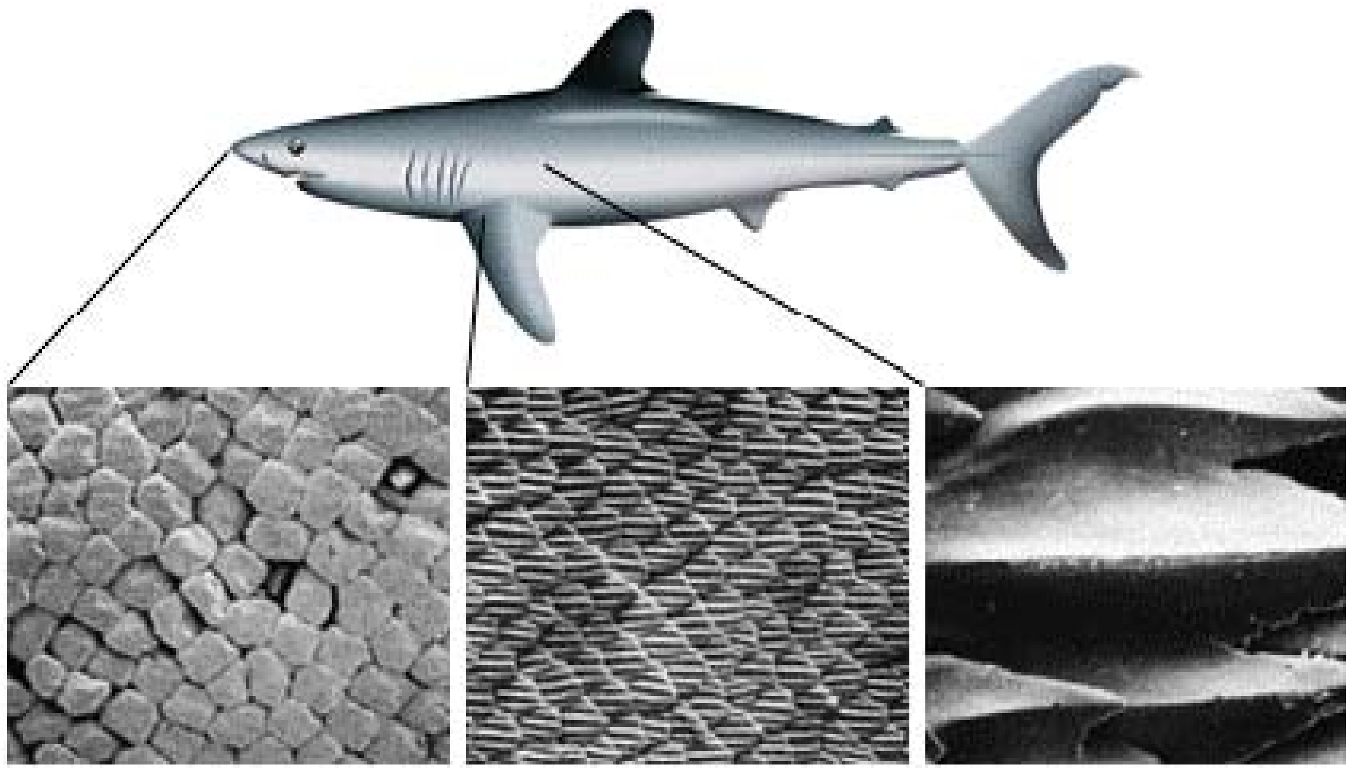
background shear

Jovanović & Bamieh, J. Fluid Mech. '05

FLOW CONTROL

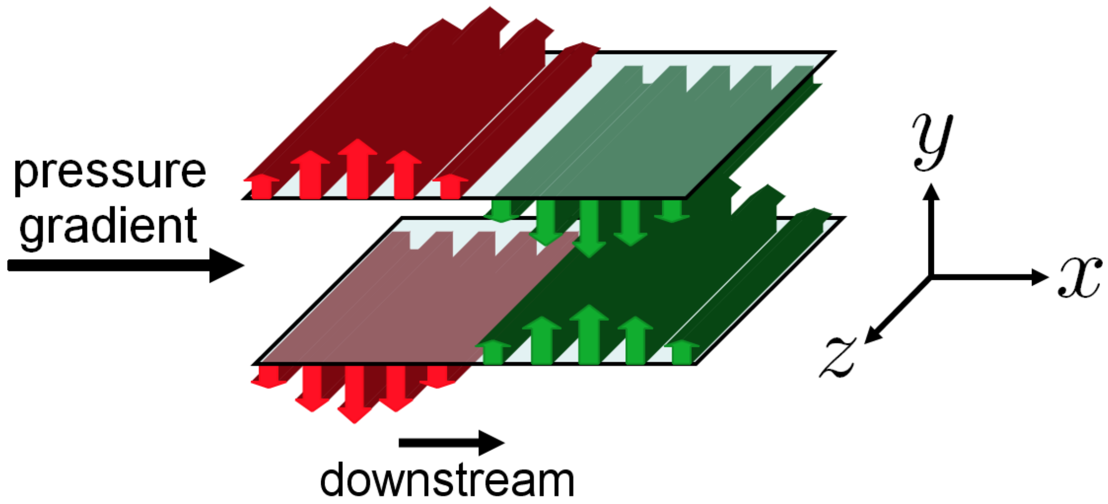
Sensor-free flow control

Geometry modifications	Wall oscillations	Body forces
riblets super-hydrophobic surfaces	transverse oscillations	oscillatory forces traveling waves



COMMON THEME: **PDEs with spatially or temporally periodic coefficients**

Blowing and suction along the walls



$c > 0$ downstream

$c < 0$ upstream

BOUNDARY CONDITION: $V(y = \pm 1) \; = \; \mp \, \alpha \cos\big(\omega_x (x \, - \, ct)\big)$

NOMINAL VELOCITY: $(U(\bar{x}, y), \; V(\bar{x}, y), \; 0)$

steady in a traveling wave frame

periodic in $\bar{x} \; := \; x \, - \, ct$

Min, Kang, Speyer, Kim, J. Fluid Mech. '06

Hœpffner & Fukagata, J. Fluid Mech. '09

- NOMINAL VELOCITY

  ⋆ **small amplitude blowing/suction**

  $\alpha \ll 1 \quad \Rightarrow$ weakly-nonlinear analysis

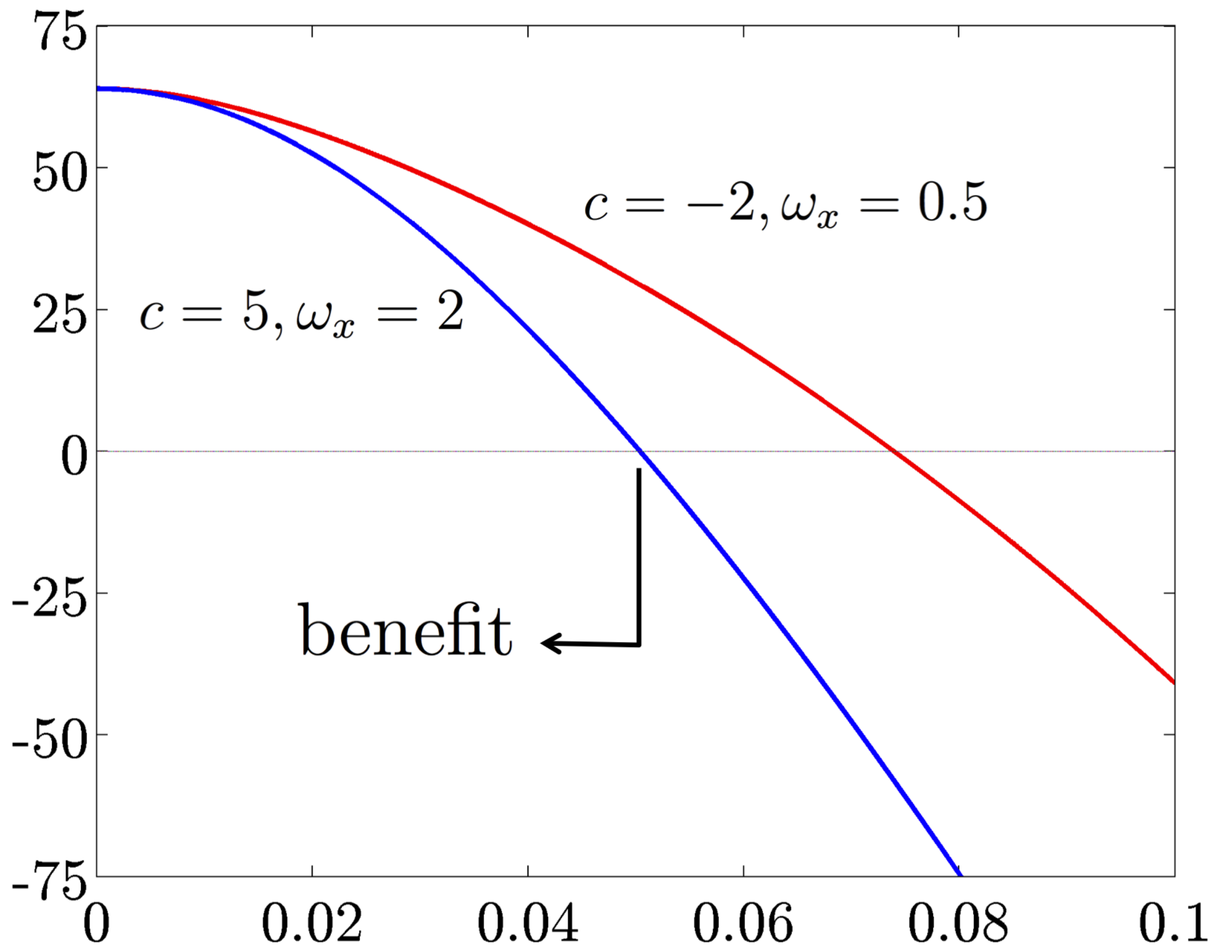
$$U(\bar{x}, y) = \overbrace{U_0(y)}^{\text{parabola}} + \alpha^2 \overbrace{U_{20}(y)}^{\text{mean drift}} + \alpha \overbrace{\big(U_{1c}(y)\cos(\omega_x \bar{x}) + U_{1s}(y)\sin(\omega_x \bar{x})\big)}^{\text{oscillatory: no mean drift}}$$

$$+ \alpha^2 \big(U_{2c}(y)\cos(2\omega_x \bar{x}) + U_{2s}(y)\sin(2\omega_x \bar{x})\big)$$

$$+ O(\alpha^3)$$

- DESIRED EFFECTS OF CONTROL
  - ⋆ **net efficiency** ↗
  - ⋆ **fluctuations' energy** ↘

net efficiency %



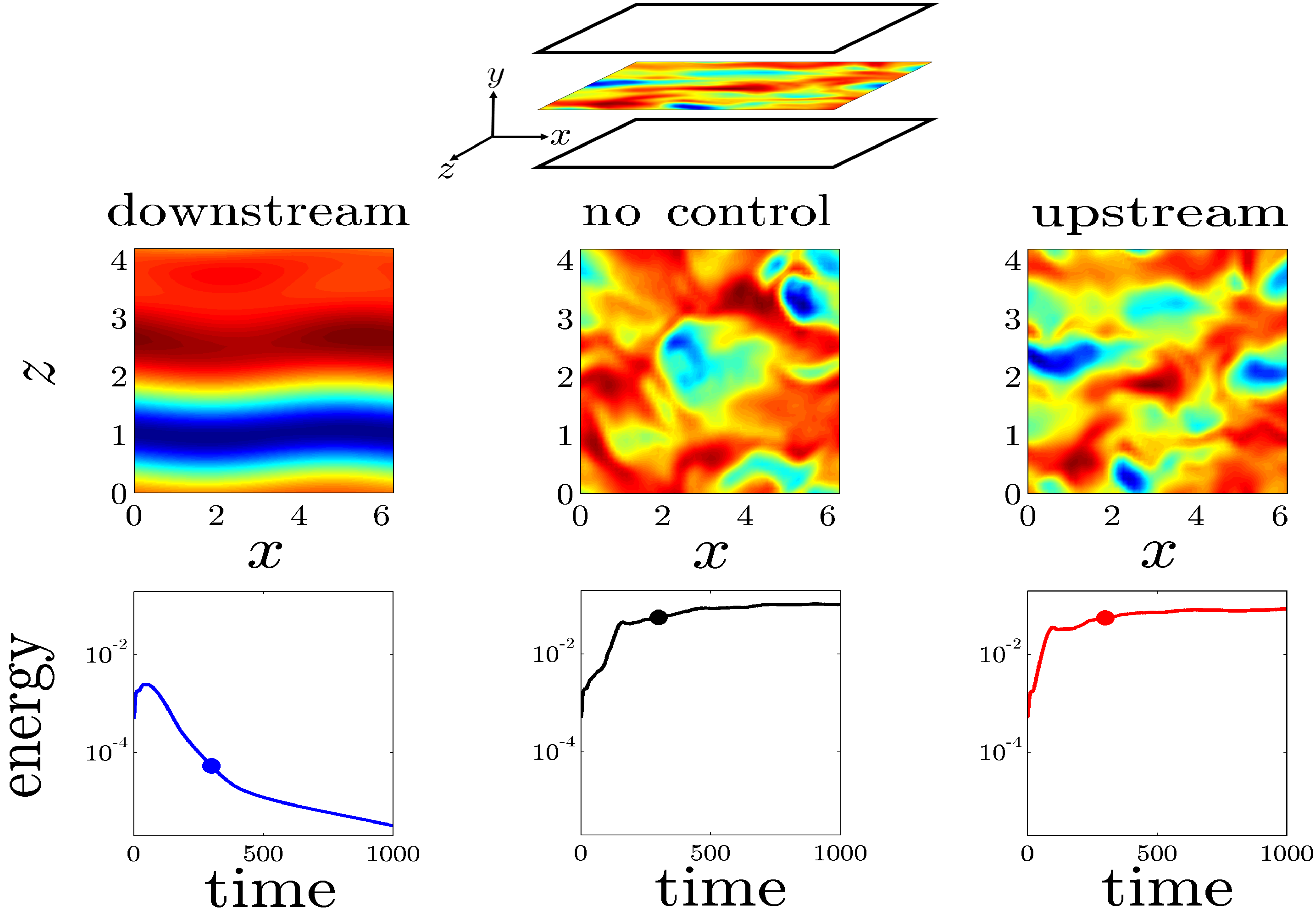
wave amplitude

RELATIVE TO:

uncontrolled

turbulent flow

Nonlinear simulations: avoidance/promotion of turbulence



Lieu, Moarref, Jovanović, J. Fluid Mech. '10

Frequency representation of controlled flow

LINEARIZATION AROUND

$(U(\bar{x}, y),\ V(\bar{x}, y),\ 0)$ $\Rightarrow$ **periodic coefficients in $\bar{x} := x - ct$**

forcing:
- **white** in t and y
- **harmonic** in z
- **Bloch wave** in $\bar{x}$

$$\mathbf{d}(\bar{x}, y, z, t) \;=\; \mathrm{e}^{\mathrm{i}k_z z} \;\times\; \mathrm{e}^{\mathrm{i}\theta\bar{x}} \;\times\; \sum_{n=-\infty}^{\infty} \mathbf{d}_n(\theta, y, k_z, t)\, \mathrm{e}^{\mathrm{i}n\omega_x\bar{x}}$$

$\downarrow$ **exponential modulation** (in $\bar{x}$)

$\downarrow$ **periodic function** (in $\bar{x}$)

Odeh & Keller '64, J. Math. Phys.

- **Evolution model**
  - ⋆ parameterized by spatial wavenumbers $\boldsymbol{\kappa} := (\theta, k_z)$

$$\tilde{\boldsymbol{\psi}}_t(\boldsymbol{\kappa}, y, t) \;=\; \Big[\mathcal{A}(\boldsymbol{\kappa})\, \tilde{\boldsymbol{\psi}}(\boldsymbol{\kappa}, \,\cdot\,, t)\Big](y) \;+\; \tilde{\mathbf{d}}(\boldsymbol{\kappa}, y, t)$$

$$\mathcal{A} \;:=\; \begin{bmatrix} \ddots & & \vdots & & \iddots \\ & A_{-1,-1} & A_{-1,0} & A_{-1,1} & \\ \cdots & A_{0,-1} & A_{0,0} & A_{0,1} & \cdots \\ & A_{1,-1} & A_{1,0} & A_{1,1} & \\ \iddots & & \vdots & & \ddots \end{bmatrix}, \qquad \tilde{\boldsymbol{\psi}} \;:=\; \begin{bmatrix} \vdots \\ \boldsymbol{\psi}_{-1} \\ \boldsymbol{\psi}_0 \\ \boldsymbol{\psi}_1 \\ \vdots \end{bmatrix}$$

bi-infinite

(periodicity in $\bar{x}$)

operator-valued

(in y)

- **Simulation-free approach to determining energy density**

  ⋆ **Lyapunov equation**

$$\mathcal{A}(\boldsymbol{\kappa})\,\mathcal{X}(\boldsymbol{\kappa}) \;+\; \mathcal{X}(\boldsymbol{\kappa})\,\mathcal{A}^{*}(\boldsymbol{\kappa}) \;=\; -\,\mathcal{I}$$

$$E\,(\boldsymbol{\kappa}) \;=\; \mathrm{trace}\,(\mathcal{X}(\boldsymbol{\kappa}))$$

  ⋆ **effect of small wave amplitude**

$$\frac{\textbf{energy density with control}}{\textbf{energy density w/o control}} \;=\; 1 \;+\; \underbrace{\alpha^{2}}_{\text{small}}\, g_2(\boldsymbol{\kappa}; Re;\, \omega_x, c) \;+\; O(\alpha^{4})$$

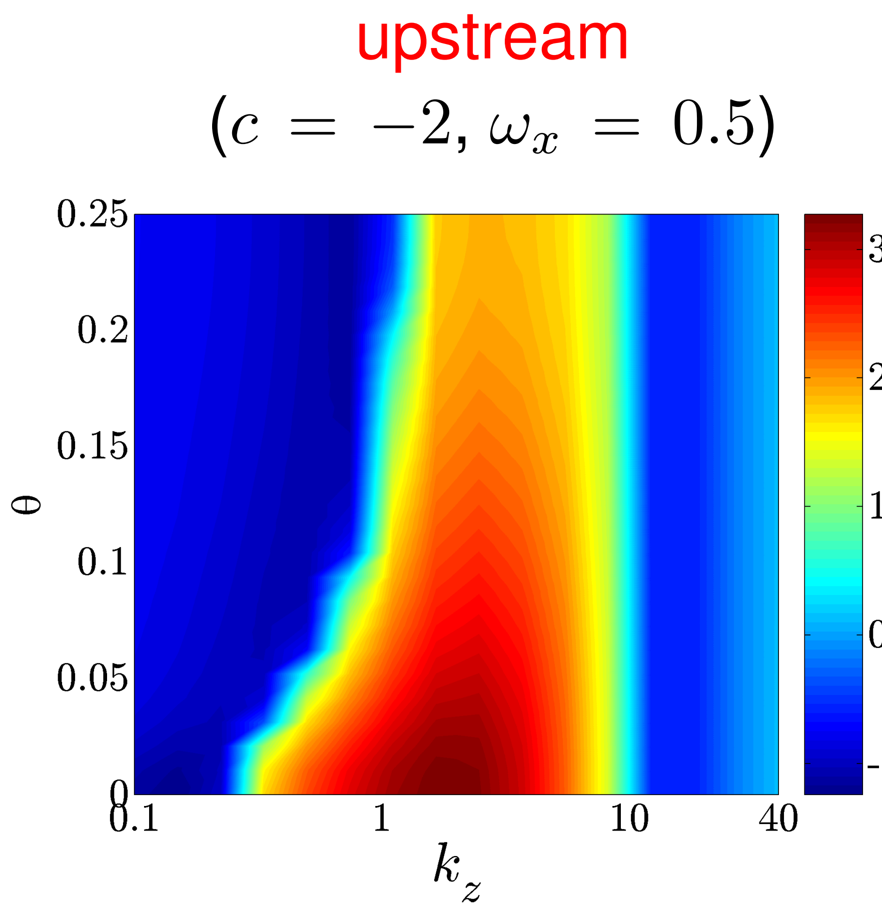
  ⋆ **computationally efficient way for determining** g_2

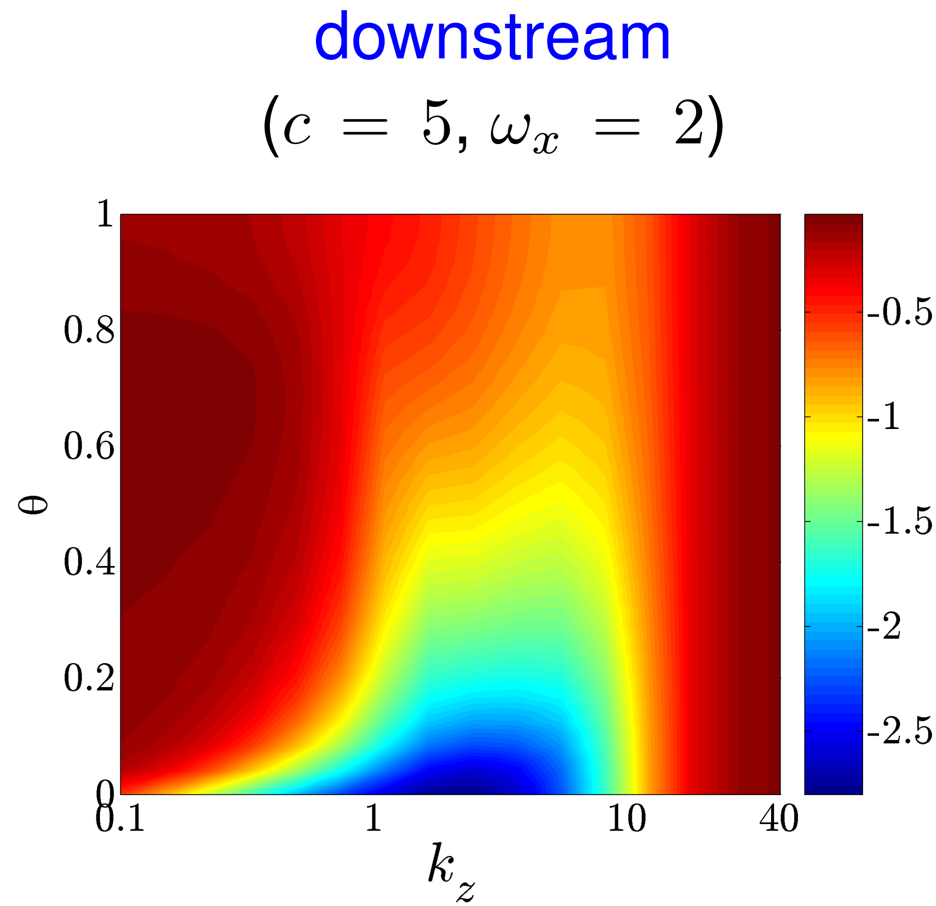
Moarref & Jovanović, J. Fluid Mech. '10

Variance amplification: controlled flow with $Re = 2000$

explicit formula:

$$\frac{\text{energy density with control}}{\text{energy density w/o control}} \approx 1 + \alpha^2 g_2(\theta, k_z;\, \omega_x, c)$$





Summary

- CONTROLLING THE ONSET OF TURBULENCE

facts revealed by perturbation analysis:

DOWNSTREAM WAVES:	**reduce** variance amplification ✓
UPSTREAM WAVES:	**promote** variance amplification

- POWERFUL SIMULATION-FREE APPROACH TO PREDICTING FULL-SCALE RESULTS

  ⋆ **verification in simulations of nonlinear flow dynamics**

Moarref & Jovanović, J. Fluid Mech. '10

Lieu, Moarref, Jovanović, J. Fluid Mech. '10

Summary and Outlook

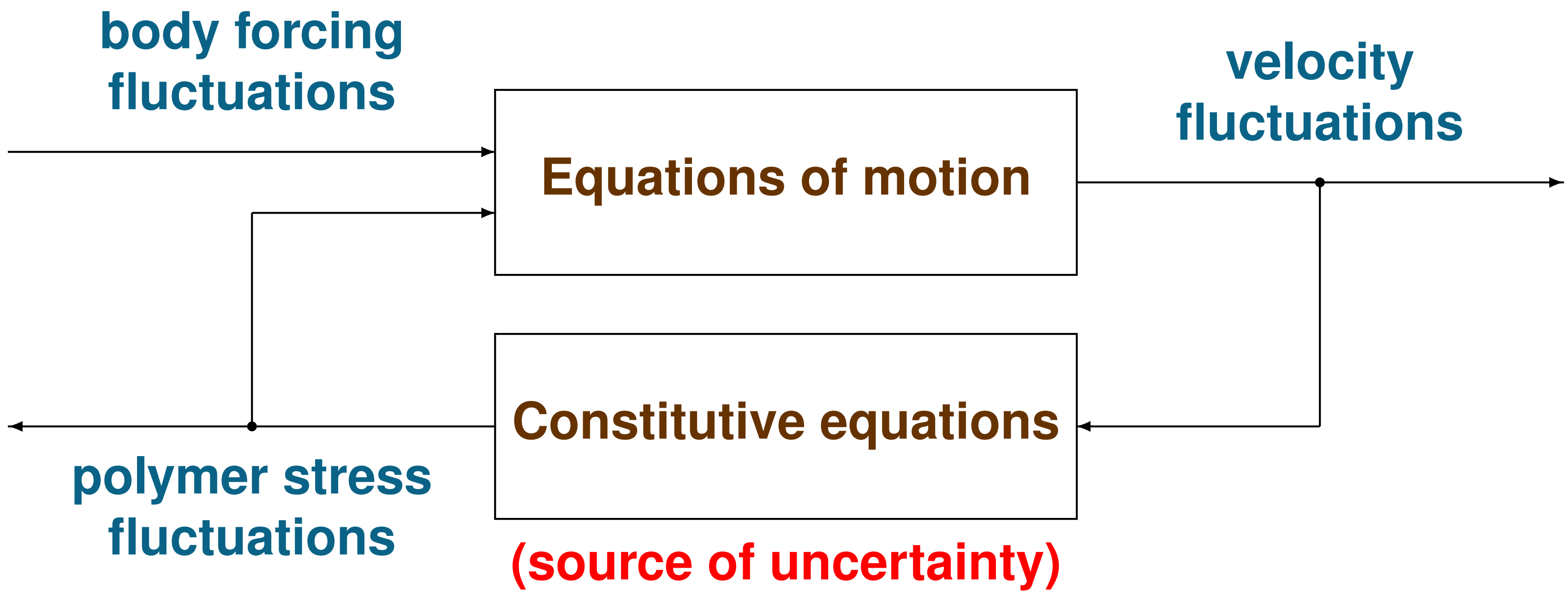
Summary: Early stages of transition

STABILITY	AMPLIFICATION
$\psi_t = \mathbf{A}\,\psi$	$\mathbf{v} = \mathbf{H}\,\mathbf{d}$
e-values of $\mathbf{A}$	singular values of $\mathbf{H}$

- CHALLENGES
  - ⋆ **Complex fluids**
  - ⋆ **Complex geometries**
  - ⋆ **Later stages of transition**
  - ⋆ **Control-oriented modeling of turbulent flows**

- COMPLEX FLUIDS

⋆ **dynamics of viscoelastic fluids**



⋆ *Lieu, Jovanović, Kumar, J. Fluid Mech. ’13*

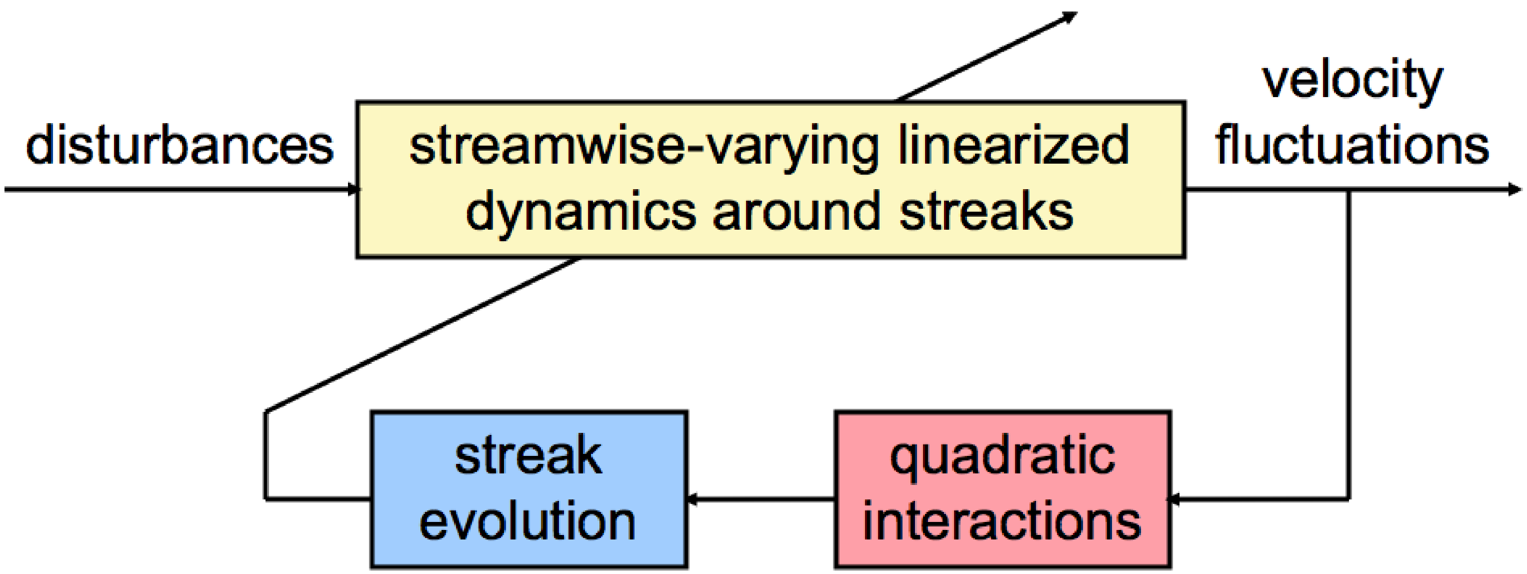
⋆ *Jovanović & Kumar, JNNFM ’11*

⋆ *Jovanović & Kumar, Phys. Fluids ’10*

⋆ *Hoda, Jovanović, Kumar, J. Fluid Mech. ’08, ’09*

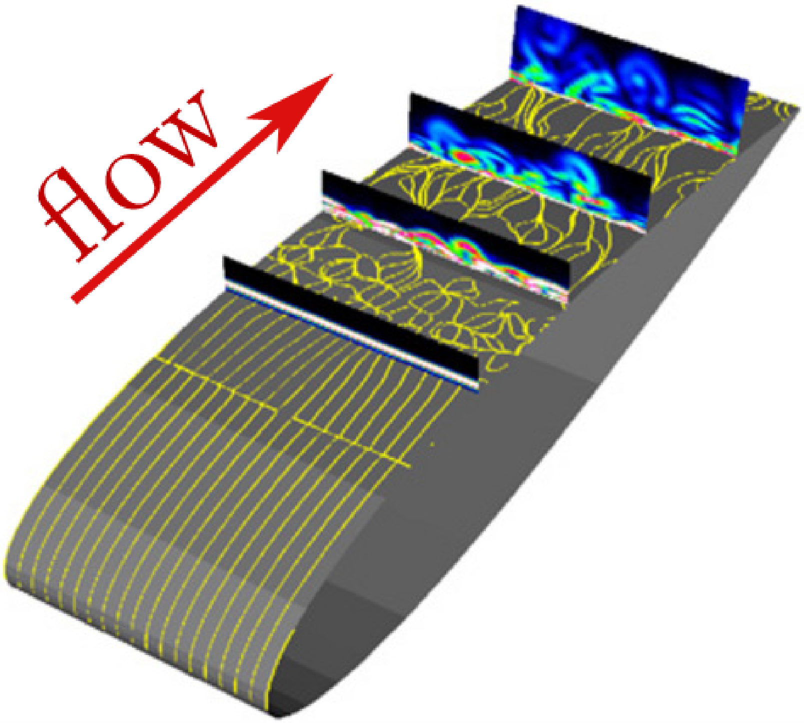
- **COMPLEX GEOMETRIES**
  - ⋆ **iterative schemes for computing singular values**

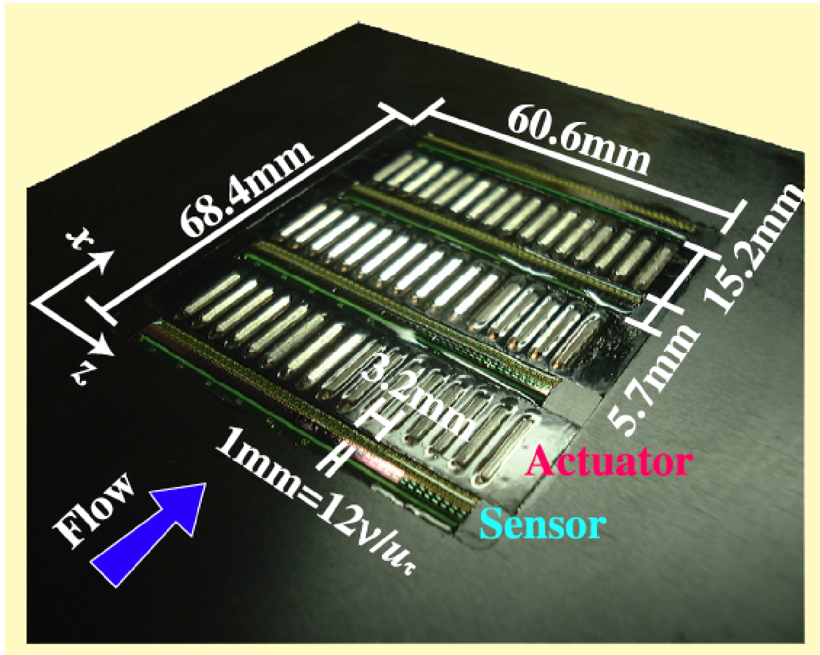
- **LATER STAGES OF TRANSITION**
  - ⋆ **interplay between flow sensitivity and nonlinearity**



- **CONTROL-ORIENTED MODELING OF TURBULENT FLOWS**
  - ⋆ **reproduce turbulent statistics by shaping forcing statistics**

Outlook: Feedback flow control





technology: **shear-stress sensors**; **surface-deformation actuators**

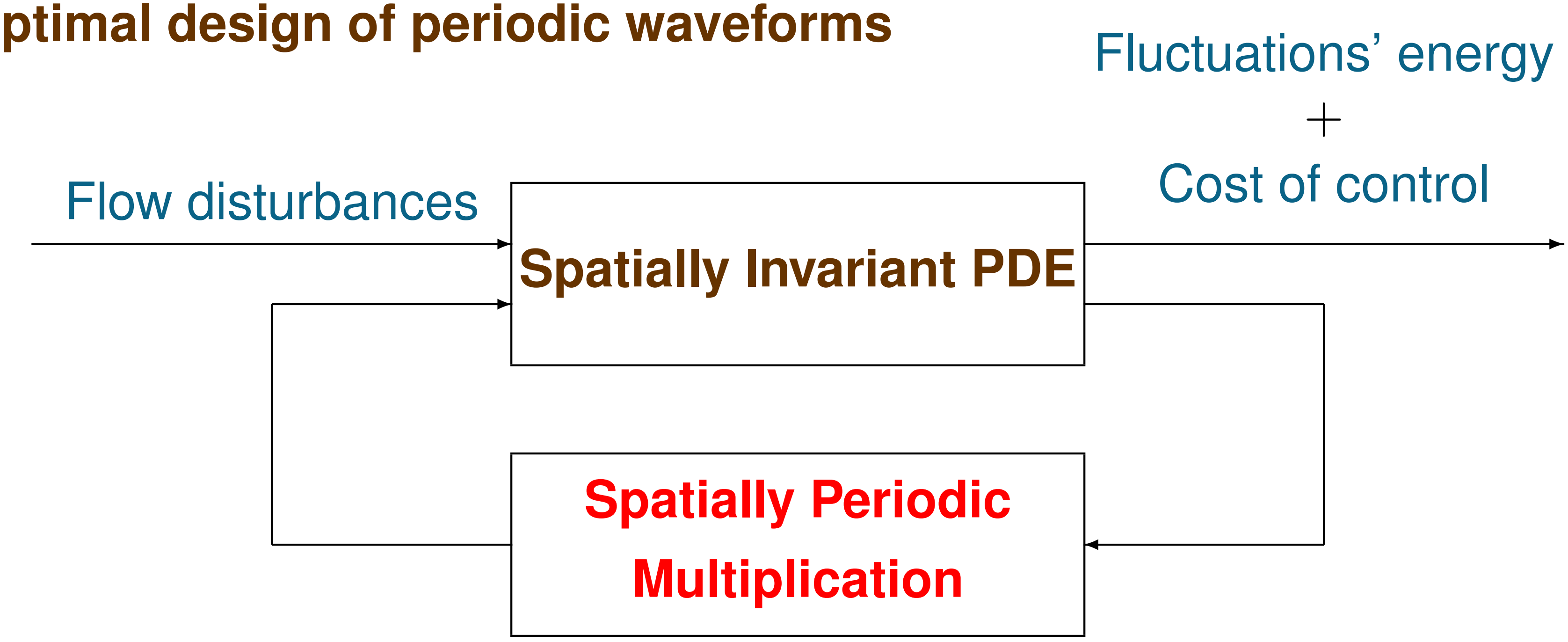
application: **turbulence suppression**; **skin-friction drag reduction**

challenge: **distributed controller design for complex flow dynamics**

Outlook: Model-based sensor-free flow control

Geometry modifications	Wall oscillations	Body forces
riblets super-hydrophobic surfaces	transverse oscillations	oscillatory forces traveling waves

- USE DEVELOPED THEORY TO DESIGN GEOMETRIES AND WAVEFORMS FOR
  - ⋆ control of transition/skin-friction drag reduction
- CHALLENGE
  - ⋆ **optimal design of periodic waveforms**



Outlook: Network design

- SPARSITY-PROMOTING OPTIMAL CONTROL
  - ⋆ Performance vs sparsity tradeoff

    Lin, Fardad, Jovanović, IEEE TAC '13 (also: arXiv:1111.6188)

  - ⋆ Software

    www.umn.edu/∼mihailo/software/lqrsp/

- ONGOING EFFORT
  - ⋆ Leader selection in large dynamic networks

    Lin, Fardad, Jovanović, IEEE TAC '13 (conditionally accepted; arXiv:1302.0450)

  - ⋆ Optimal synchronization of sparse oscillator networks

    Fardad, Lin, Jovanović, IEEE TAC '13 (submitted; arXiv:1302.0449)

  - ⋆ Optimal dissemination of information in social networks

    Fardad, Zhang, Lin, Jovanović, CDC '12

  - ⋆ Wide-area control of power networks

    Dörfler, Jovanović, Chertkov, Bullo, ACC '13

Outlook: Performance of large-scale networks

- OPEN QUESTION: **fundamental limitations for networks with**

$$\left.\begin{array}{r}\textbf{spatially-varying}\\ \textbf{dynamic}\\ \textbf{nonlinear}\end{array}\right\}\ \textbf{controllers}$$

$$\left.\begin{array}{c}\textbf{formations with a leader}\\ +\\ \textbf{spatially-varying}\\ \textbf{nearest-neighbor feedback}\end{array}\right\}\ \Rightarrow$$

improved scaling trends in 1D!

$$O\left(\sqrt{M}\right) \ \text{vs} \ O\left(M\right)$$

Lin, Fardad, Jovanović, IEEE TAC '12

VEHICULAR STRINGS

⋆ need global interactions to address coherence

⋆ even then, convergence of Merge & Split Maneuvers scales poorly with size

Jovanović, Fowler, Bamieh, D'Andrea, Syst. Control Lett. '08