Design of structured optimal feedback gains for interconnected systems

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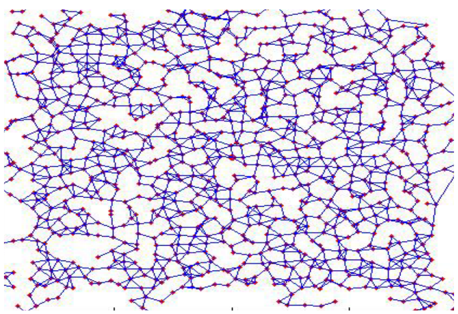
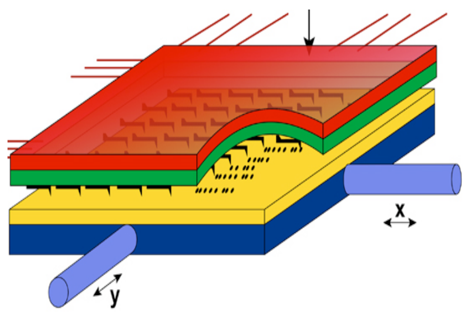
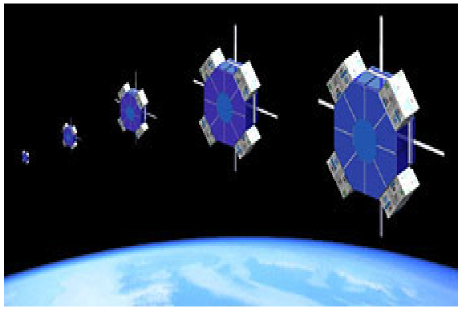
Distributed systems

- OF INCREASING IMPORTANCE IN MODERN TECHNOLOGY

APPLICATIONS:

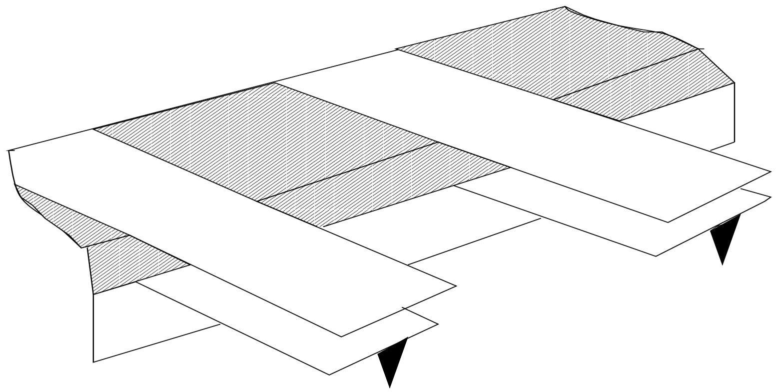
sensor networks	arrays of micro-cantilevers	UAV formations satellite constellations
		

- INTERACTIONS CAUSE COMPLEX BEHAVIOR
  cannot be predicted by analyzing isolated subsystems

- SPECIAL STRUCTURE
  every unit has sensors and actuators

Array of micro-cantilevers

ELECTROSTATICALLY ACTUATED MICRO-CANTILEVERS



POTENTIAL APPLICATION: MASSIVELY PARALLEL DATA STORAGE

problem: slow scans $\equiv$ low throughput

solution: go massively parallel

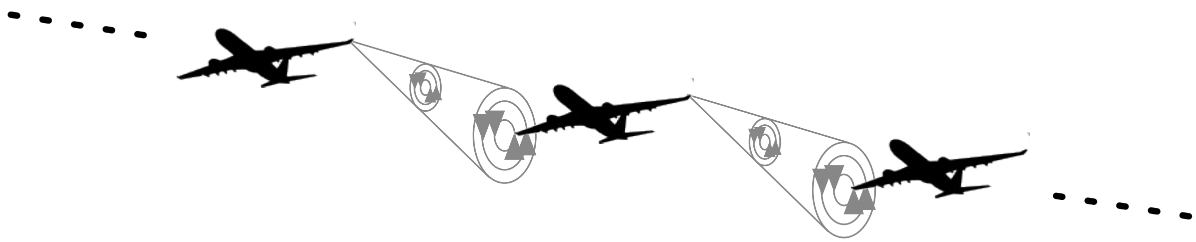
ISSUES:

tightly coupled dynamics	$\Rightarrow$	**spatio-temporal instabilities**
large number of devices	$\Rightarrow$	**localized control imperative**

Coordinated control of formations

FORMATION FLIGHT FOR AERODYNAMIC ADVANTAGE

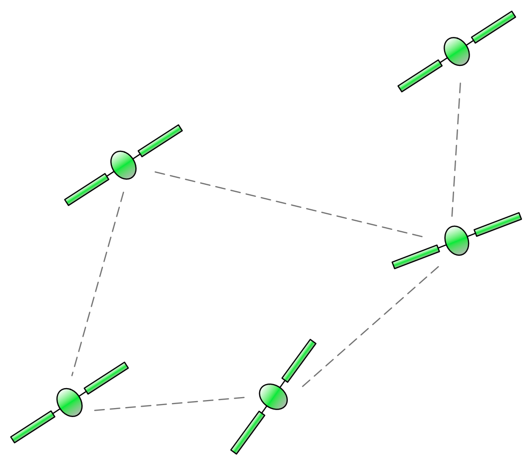
e.g. additional lift in V-formations



precise control needed

MICRO-SATELLITE FORMATIONS

e.g. for synthetic aperture

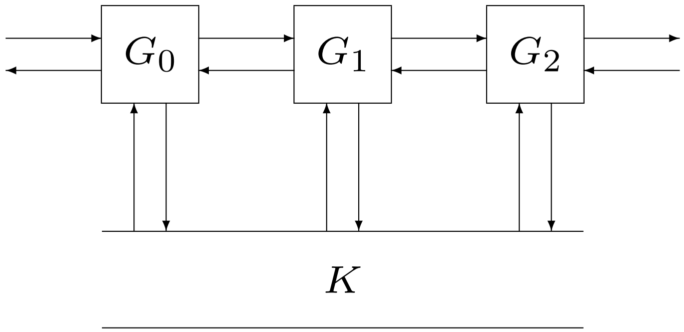


MAKE VEHICLES SMALLER AND CHEAPER $\Rightarrow$ USE MANY

cooperative control becomes a major issue

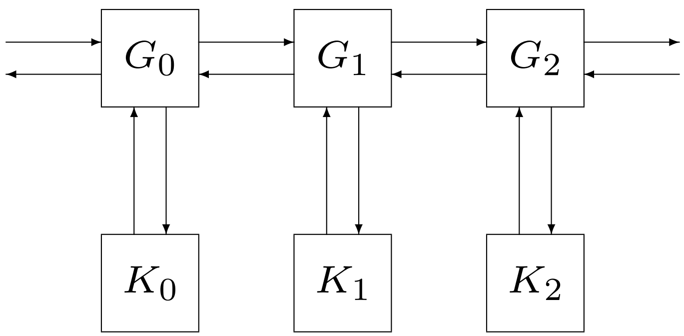
Controller architectures

CENTRALIZED



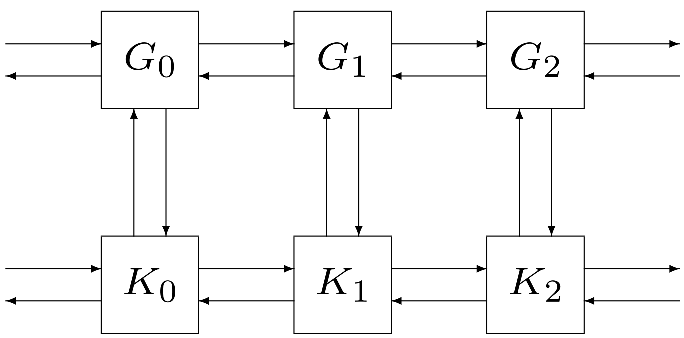
best performance
excessive communication

FULLY DECENTRALIZED



worst performance
no communication

LOCALIZED



many possible architectures

Outline

❶ STRUCTURED OPTIMAL DESIGN

⋆ sparsity constraints on feedback gains

❷ OPTIMAL LOCALIZED CONTROL OF VEHICULAR PLATOONS

⋆ design of spatially-varying feedback gains

❸ PERFORMANCE VS. SIZE

⋆ coherence of formation

❹ PARTING THOUGHTS

STRUCTURED OPTIMAL DESIGN

Structured H_2 problem

$$\begin{aligned} \dot{x} &= A\,x + B_1\,d + B_2\,u \\ z &= \begin{bmatrix} Q^{1/2} \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ R^{1/2} \end{bmatrix} u \\ u &= -K\,x \qquad\qquad K \in \mathcal{S} \end{aligned}$$

- STRUCTURAL CONSTRAINTS $K \in \mathcal{S}$

centralized

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

fully decentralized

$$\begin{bmatrix} \diamond & & & \\ & \star & & \\ & & \triangle & \\ & & & \divideontimes \end{bmatrix}$$

localized

$$\begin{bmatrix} * & * & & \\ * & * & * & \\ & * & * & * \\ & & * & * \end{bmatrix}$$

OBJECTIVE:

design stabilizing $K \in \mathcal{S}$ that minimizes $\| d \to z \|_2^2$

- CLOSED-LOOP SYSTEM

$$\begin{aligned} \dot{x} &= (A - B_2 K)\, x + B_1\, d \\ z &= \begin{bmatrix} Q^{1/2} \\ -R^{1/2} K \end{bmatrix} x, \qquad K \in \mathcal{S} \end{aligned}$$

STRUCTURED H_2 PROBLEM:

minimize	$J(K) := \operatorname{trace}\left(P(K) B_1 B_1^T\right)$
subject to	$\begin{cases} (A - B_2 K)^T P + P\,(A - B_2 K) = -\left(Q + K^T R K\right) \\ K \in \mathcal{S} \end{cases}$

$J(K)$ – nonconvex function of K

- RELATED PROBLEMS:
  - ⋆ **static output feedback:** Levine & Athans, IEEE TAC'70
  - ⋆ **structured dynamic controller:** Wenk & Knapp, IEEE TAC'80

Necessary conditions for optimality

$$\begin{aligned}
(A - B_2 K)^T P \, + \, P\,(A - B_2 K) &= -\left(Q \, + \, K^T R K\right) \\
(A - B_2 K)\, L \, + \, L\,(A - B_2 K)^T &= -B_1 B_1^T \\
\left[\left(RK - B_2^T P\right) L\right] \circ I_{\mathcal{S}} &= 0
\end{aligned}$$

$I_{\mathcal{S}}$ - structural identity

$$K \;=\; \begin{bmatrix} * & * & & \\ * & * & * & \\ & * & * & * \\ & & * & * \end{bmatrix} \qquad \Rightarrow \qquad I_{\mathcal{S}} \;=\; \begin{bmatrix} 1 & 1 & & \\ 1 & 1 & 1 & \\ & 1 & 1 & 1 \\ & & 1 & 1 \end{bmatrix}$$

- SPECIAL CASES:

  ⋆ **no constraints**

$$\begin{aligned}
A^T P \, + \, PA \, - \, PB_2 R^{-1} B_2^T P \, + \, Q &= 0 \\
K_c &= R^{-1} B_2^T P
\end{aligned}$$

  ⋆ **expensive control of stable open-loop systems**

  perturbation analysis: Fardad, Lin, Jovanović, CDC'09

Perturbation analysis

- EXPENSIVE CONTROL: $R = (1/\varepsilon)\, I, \quad 0 < \varepsilon \ll 1$

$$P = \sum_{n=0}^{\infty} \varepsilon^n P_n, \quad L = \sum_{n=0}^{\infty} \varepsilon^n L_n, \quad K = \sum_{n=0}^{\infty} \varepsilon^n K_n$$

$$O(1): \quad K_0 = 0$$

$$O(\varepsilon): \begin{cases} A^T P_0 + P_0 A = -Q \\ A L_0 + L_0 A^T = -B_1 B_1^T \\ [K_1 L_0] \circ I_{\mathcal{S}} = [B_2^T P_0 L_0] \circ I_{\mathcal{S}} \end{cases}$$

$$O(\varepsilon^2): \begin{cases} A^T P_1 + P_1 A = (\text{matrix function of } K_1 \text{ and } P_0) \\ A L_1 + L_1 A^T = (\text{matrix function of } K_1 \text{ and } L_0) \\ [K_2 L_0] \circ I_{\mathcal{S}} = [B_2^T (P_0 L_1 + P_1 L_0) - K_1 L_1] \circ I_{\mathcal{S}} \end{cases}$$

followed by homotopy

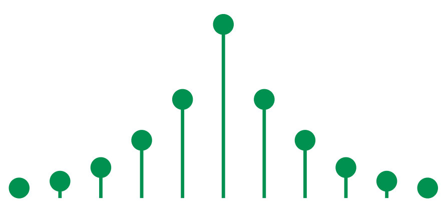
Numerical computation

$$\begin{aligned}(A - B_2K)^T P + P(A - B_2K) &= -\left(Q + K^T R K\right)\\ (A - B_2K)L + L(A - B_2K)^T &= -B_1 B_1^T\\ \left[\left(RK - B_2^T P\right)L\right] \circ I_{\mathcal{S}} &= 0\end{aligned}$$

- NEWTON'S METHOD

$$K_{i+1} = K_i + s_i \tilde{K}_i \text{ until } \|\nabla J(K_i)\| < \text{tolerance}$$

- FEATURES:
  - ⋆ Newton direction $\tilde{K}_i$: conjugate-gradient method
    sparsity utilized
  - ⋆ step size s_i: backtracking line search
    stability guaranteed
  - ⋆ initial condition: truncated centralized gain $K_c \circ I_{\mathcal{S}}$
    exponential decay
    Bamieh, Paganini, Dahleh, IEEE TAC'02; Motee & Jadbabaie, IEEE TAC'08



Augmented Lagrangian

$$K = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix}$$

$$\downarrow$$

$$0 = k_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \text{trace}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \right)$$

$$\downarrow$$

$$K \in \mathcal{S} \;\Leftrightarrow\; \text{trace}\left(\Lambda^T K\right) = 0, \quad \Lambda \in \mathcal{S}^c$$

$$K = \begin{bmatrix} \diamond & \\ & \star \end{bmatrix} \;\Rightarrow\; \Lambda = \begin{bmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{bmatrix}, \quad I_{\mathcal{S}}^c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

AUGMENTED LAGRANGIAN:

$$\boxed{\mathcal{L}_\gamma(K, \Lambda) = J(K) + \text{trace}\left(\Lambda^T K\right) + \gamma \, \|K \circ I_{\mathcal{S}}^c\|_F^2}$$

Augmented Lagrangian minimization

$$\min_{K} \mathcal{L}_\gamma(K, \Lambda)$$

OPTIMALITY OF $\mathcal{L}_\gamma$: NECESSARY CONDITIONS

$$\begin{aligned}
(A - B_2 K)^T P + P (A - B_2 K) &= -\left(Q + K^T R K\right) \\
(A - B_2 K) L + L (A - B_2 K)^T &= -B_1 B_1^T \\
2 \left(R K - B_2^T P\right) L + \Lambda + \gamma (K \circ I_{\mathcal{S}}^c) &= 0
\end{aligned}$$

- NEWTON'S METHOD WITH LINE SEARCH
  - ⋆ Stability guaranteed
  - ⋆ Deals with ill-conditioning
- UPDATE RULES

$$\Lambda_{i+1} = \Lambda_i + \gamma_i (K_i \circ I_{\mathcal{S}}^c), \quad \gamma_{i+1} = c\,\gamma_i, \quad c > 1$$

- STOPPING CRITERION

$$\|K_i \circ I_{\mathcal{S}}^c\|_F < \text{tolerance}$$

Part 1: summary

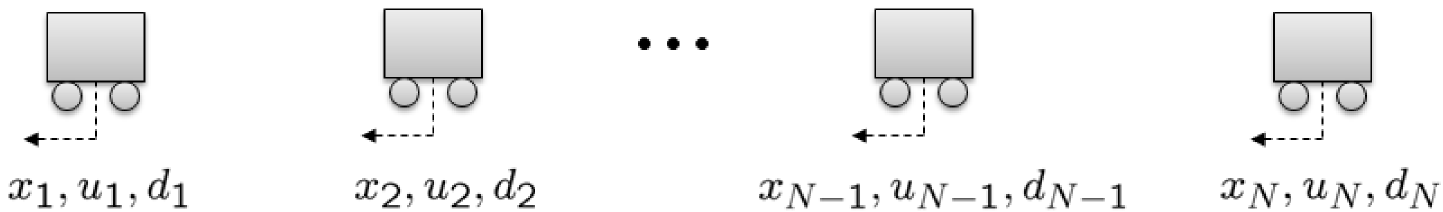
- Structured H_2 problem
  - ⋆ Primal formulation (perturbation analysis & homotopy)
  - ⋆ Augmented Lagrangian
- Computation of optimal structured gains
  - ⋆ Possible
- Challenges
  - ⋆ Understand limitations of proposed methods
  - ⋆ Understand structure of augmented Lagrangian formulation
  - ⋆ Perturbation analysis/homotopy for unstable open-loop systems
  - ⋆ Hidden convexity?

Control of vehicular formations

Vehicular platoons

AUTOMATED CONTROL OF EACH VEHICLE

tight spacing at highway speeds



KEY ISSUES (**also in: control of swarms, flocks, formation flight**)

- ⋆ Is it enough to only look at neighbors?
- ⋆ How does performance scale with size?
- ⋆ Are there any fundamental limitations?

FUNDAMENTALLY DIFFICULT PROBLEM (**scales badly**)

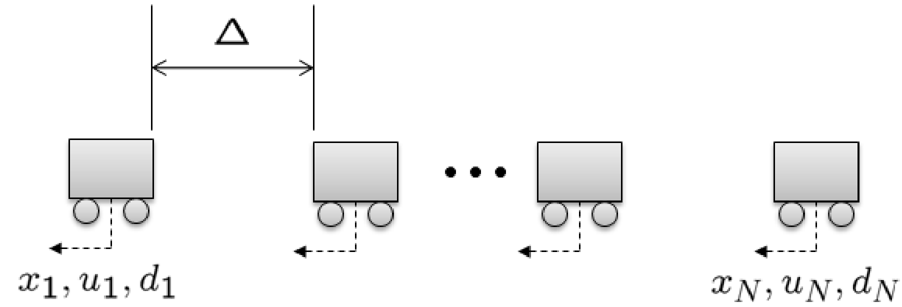
Jovanović & Bamieh, IEEE TAC '05

Bamieh, Jovanović, Mitra, Patterson, IEEE TAC '10 (conditionally accepted)

Problem formulation

$$\dot{x}_n = u_n + \color{red}{d_n}$$

u_n ↑ control $\quad$ d_n ↑ disturbance

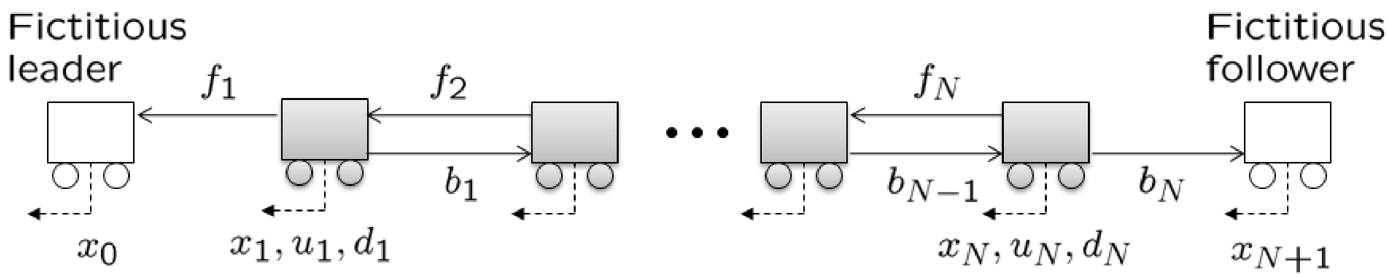


- DESIRED TRAJECTORY: $\begin{cases} \bar{x}_n := vt + n\Delta \\ \text{constant velocity} \end{cases}$

- DEVIATIONS: $\left. \begin{array}{l} \tilde{x}_n := x_n - \bar{x}_n \\ \tilde{u}_n := u_n - v \end{array} \right\} \Rightarrow \dot{\tilde{x}}_n = \tilde{u}_n + \color{red}{d_n}$

- CONTROL: $\tilde{u} = K\tilde{x}, \quad K$: feedback gain

DESIGN K TO USE NEAREST NEIGHBOR FEEDBACK



RELATIVE POSITION FEEDBACK:

$$\begin{aligned} \tilde{u}_n &= -f_n\,(\tilde{x}_n - \tilde{x}_{n-1}) \; - \; b_n\,(\tilde{x}_n - \tilde{x}_{n+1}) \\ \tilde{u} &= -\begin{bmatrix} F_f & F_b \end{bmatrix} \begin{bmatrix} C_f \\ C_b \end{bmatrix} \tilde{x} \end{aligned}$$

$$K \sim \underbrace{\begin{bmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{bmatrix}}_{F_f} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{C_f} + \underbrace{\begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}}_{F_b} \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}_{C_b}$$

Structured feedback design

$$\begin{aligned} \dot{\tilde{x}} &= d + \tilde{u} \\ z &= \begin{bmatrix} Q^{1/2} \\ 0 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ I \end{bmatrix} \tilde{u} \\ \tilde{u} &= -\begin{bmatrix} F_f & F_b \end{bmatrix} \begin{bmatrix} C_f \\ C_b \end{bmatrix} \tilde{x}, \quad F_f,\, F_b \; - \; \text{diagonal} \end{aligned}$$

spatially uniform: $F_f = F_b = I$

- PERFORMANCE MEASURES
  - MICROSCOPIC: local position deviation $(\tilde{x}_n - \tilde{x}_{n-1})$

$$Q_l \sim \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

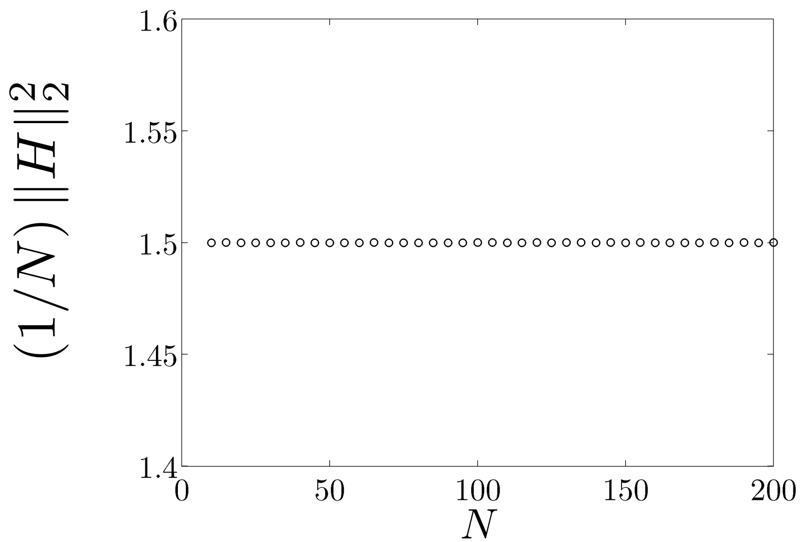
  - MACROSCOPIC: global position deviation $\tilde{x}_n$

$$Q_g = I$$

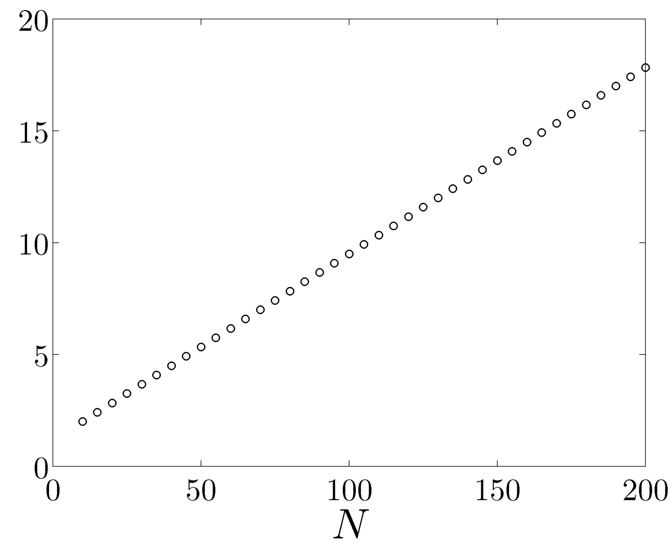
Performance vs. size

- SPATIALLY UNIFORM

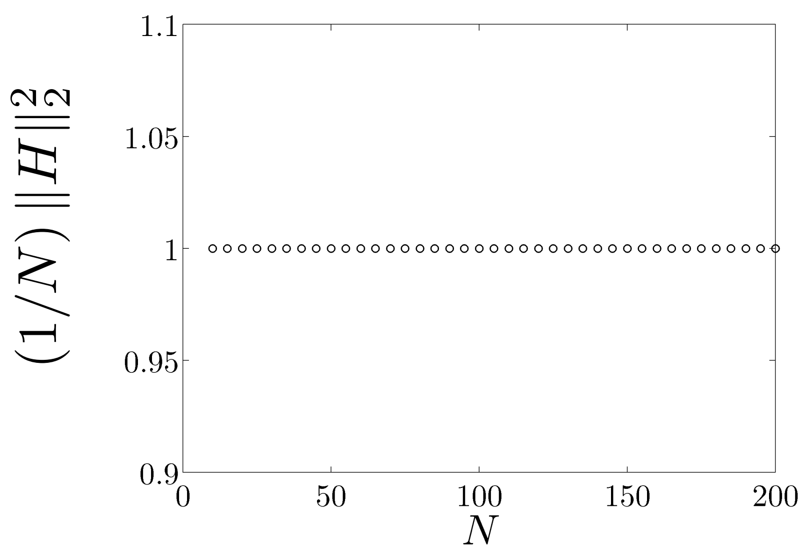
MICROSCOPIC:



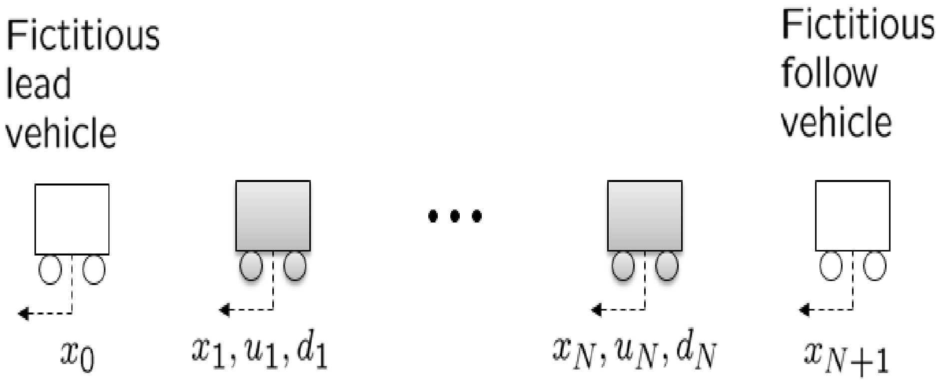
MACROSCOPIC:



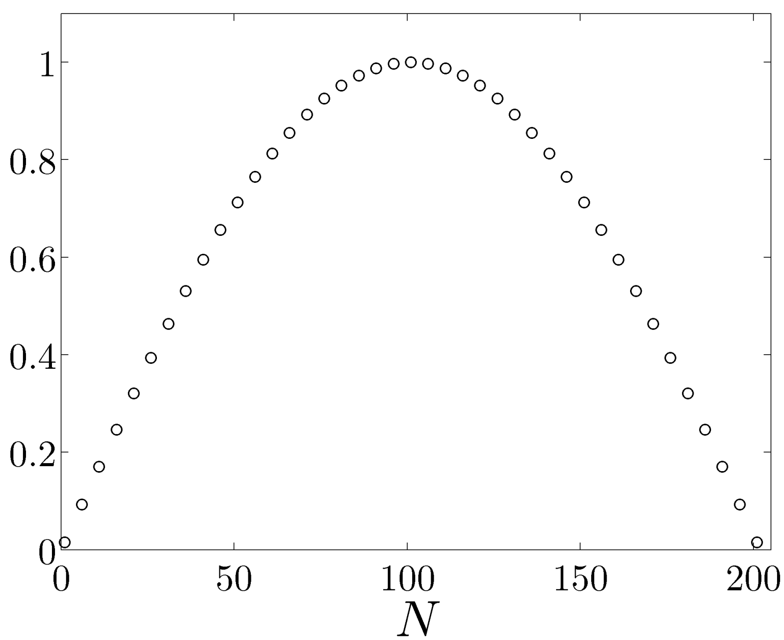
CONTROL:

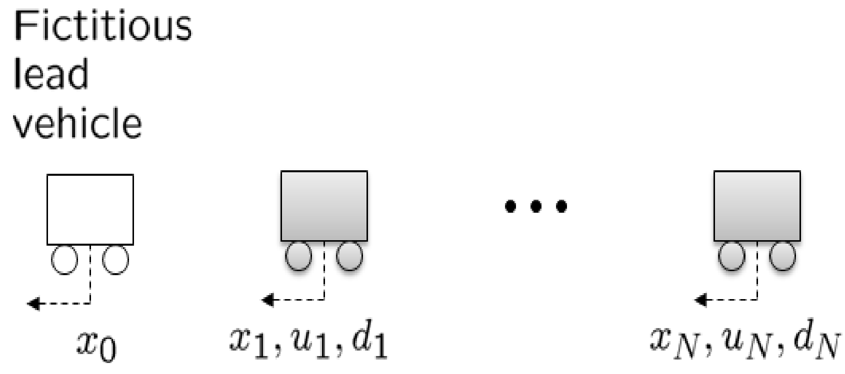


Most energetic spatial profiles

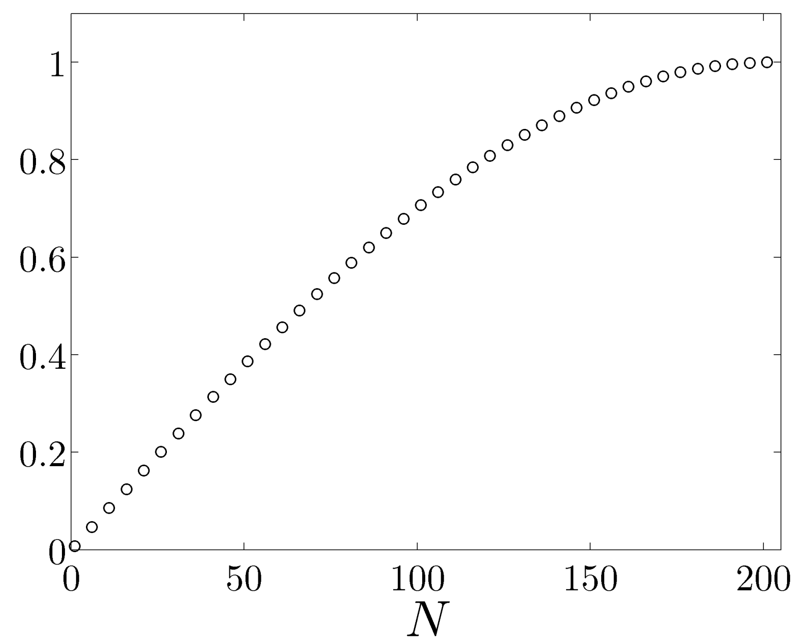


$\sin\left(\pi n/(N+1)\right)$:



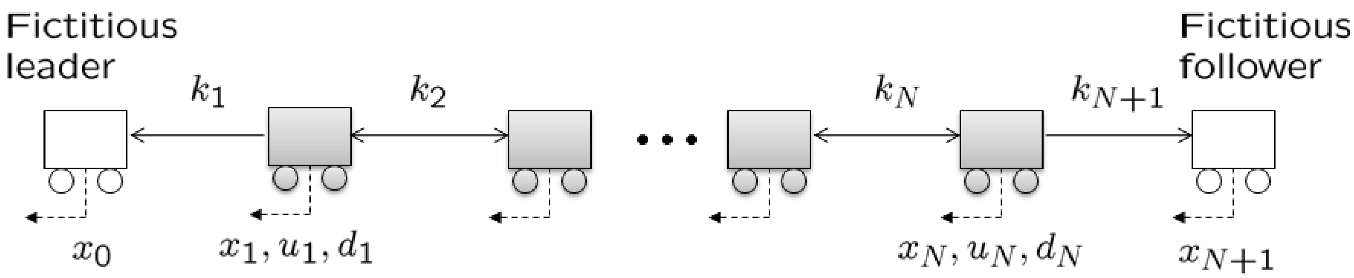


$\cos\left(0.5\pi(n/(N+1)-1)\right)$:



Structured feedback design

Convex structured design



SYMMETRIC GAIN:

$$\tilde{u}_n = -k_n \left(\tilde{x}_n - \tilde{x}_{n-1}\right) - k_{n+1} \left(\tilde{x}_n - \tilde{x}_{n+1}\right)$$

$$\tilde{u} = -K \, \tilde{x}, \quad K = K^T > 0$$

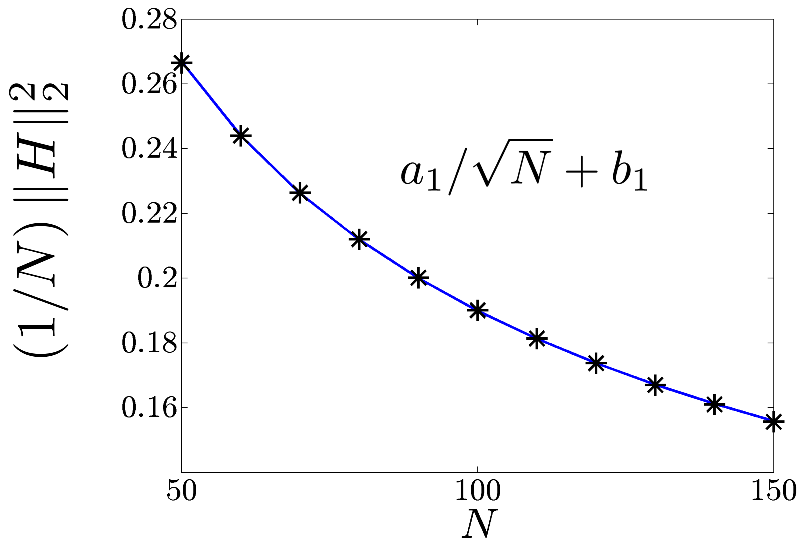
CONVEX PROBLEM:

$$\begin{array}{ll} \text{minimize} & \operatorname{trace}\left(K + Q \, K^{-1}\right) \\ \text{subject to} & 0 < K^T = K \in \mathcal{S} \end{array}$$

Performance vs. size

- SYMMETRIC STRUCTURED OPTIMAL (WITH $Q_g = I$)

MICROSCOPIC:



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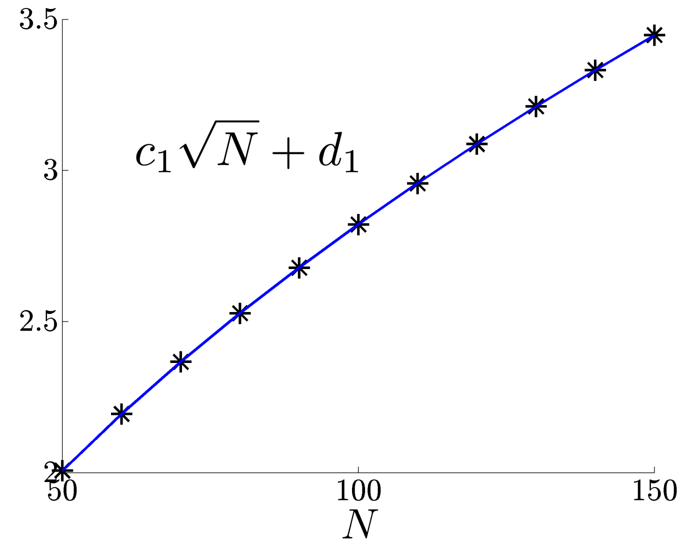

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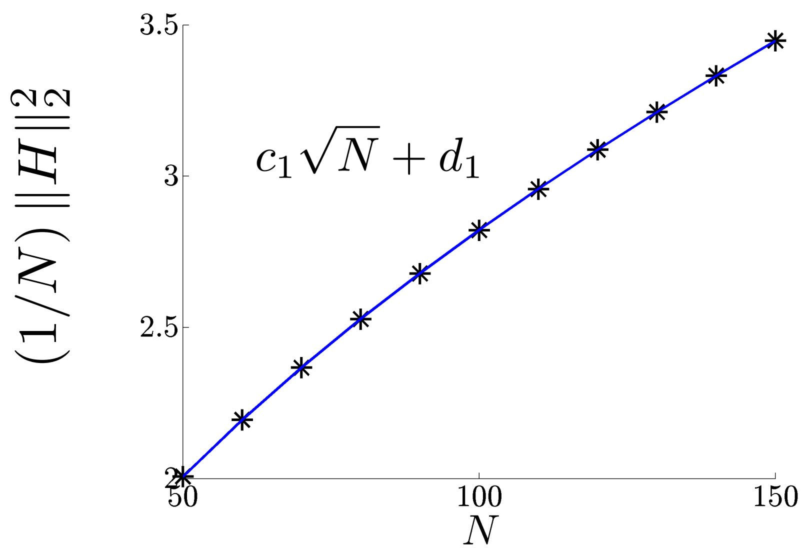


Optimal design of non-symmetric gains

- SPATIALLY UNIFORM $(K_0 = C_f + C_b)$

  inversely optimal wrt $Q_0 = K_0^2$

$$\left. \begin{array}{rcl} -P_0^2 + Q_0 & = & 0 \\ K_0 & = & P_0 \in \mathcal{S} \end{array} \right\} \Rightarrow$$

do design with:

$$Q = Q_0 + \varepsilon (Q_d - Q_0)$$

$$P = \sum_{n=0}^{\infty} \varepsilon^n P_n, \quad L = \sum_{n=0}^{\infty} \varepsilon^n L_n, \quad K = \sum_{n=0}^{\infty} \varepsilon^n K_n$$

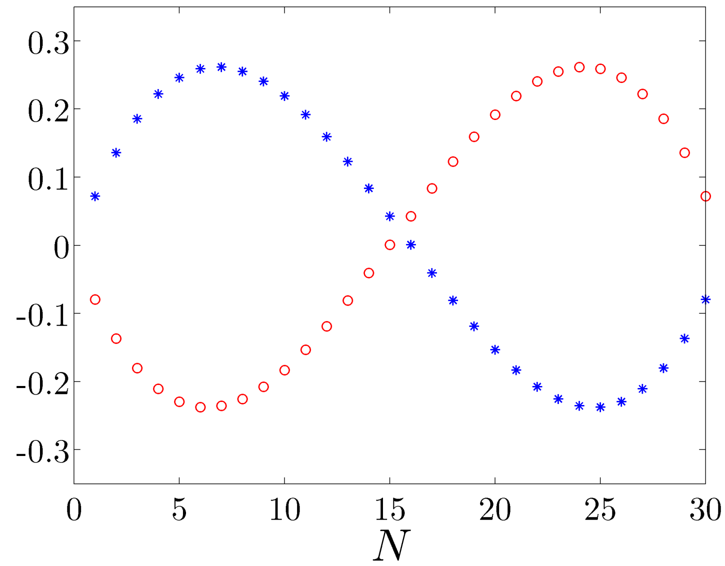
$$O(1): \left\{ \begin{array}{rcl} -P_0^2 + Q_0 & = & 0 \\ K_0 & = & P_0 \\ -K_0 L_0 - L_0 K_0 & = & -I \end{array} \right.$$

$$O(\varepsilon): \left\{ \begin{array}{l} -K_0 P_1 - P_1 K_0 = -(Q_d - Q_0) \\ \left[\left(\left[\begin{array}{cc} F_{f1} & F_{b1} \end{array} \right] \left[\begin{array}{c} C_f \\ C_b \end{array} \right] - P_1 \right) L_0 \left[\begin{array}{cc} C_f^T & C_b^T \end{array} \right] \right] \circ \left[\begin{array}{cc} I & I \end{array} \right] = 0 \end{array} \right.$$

followed by homotopy

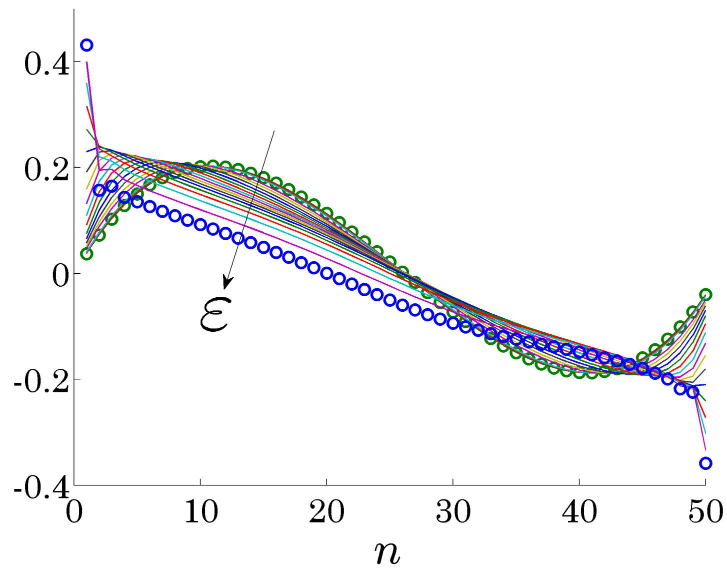
- PERTURBATION ANALYSIS (WITH $Q_g = I$)

FORWARD/BACKWARD GAINS:



- HOMOTOPY

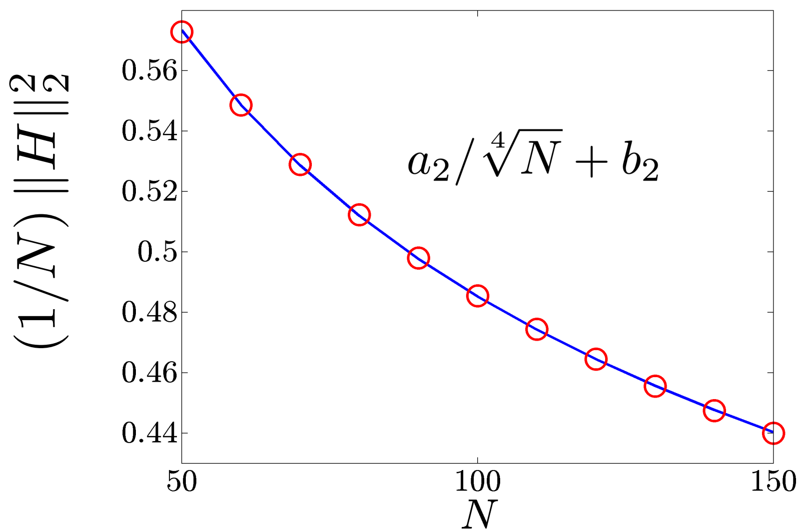
FORWARD GAINS:



Performance vs. size

- STRUCTURED OPTIMAL (WITH $Q_d = I$)

MICROSCOPIC:



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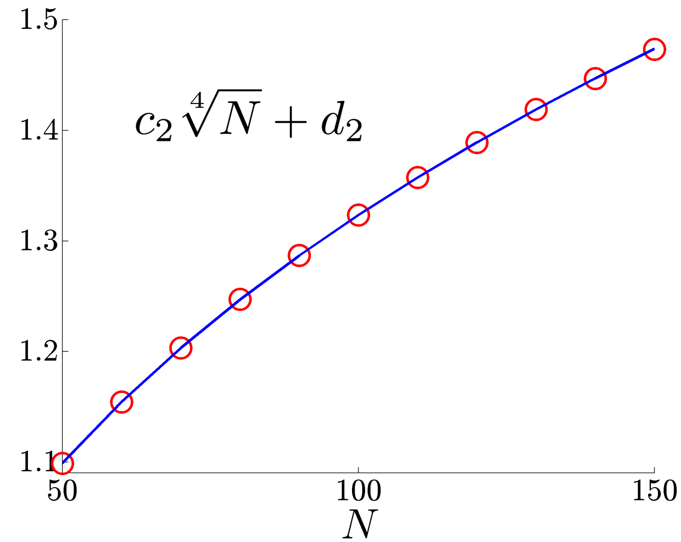
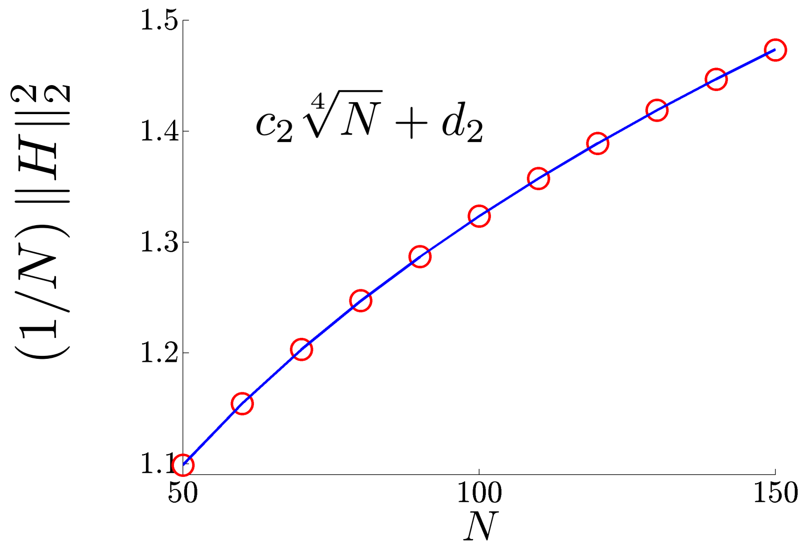


CONTROL:



- **spatial uniform** vs. **symmetric structured optimal** vs. **structured optimal** vs. **centralized optimal**

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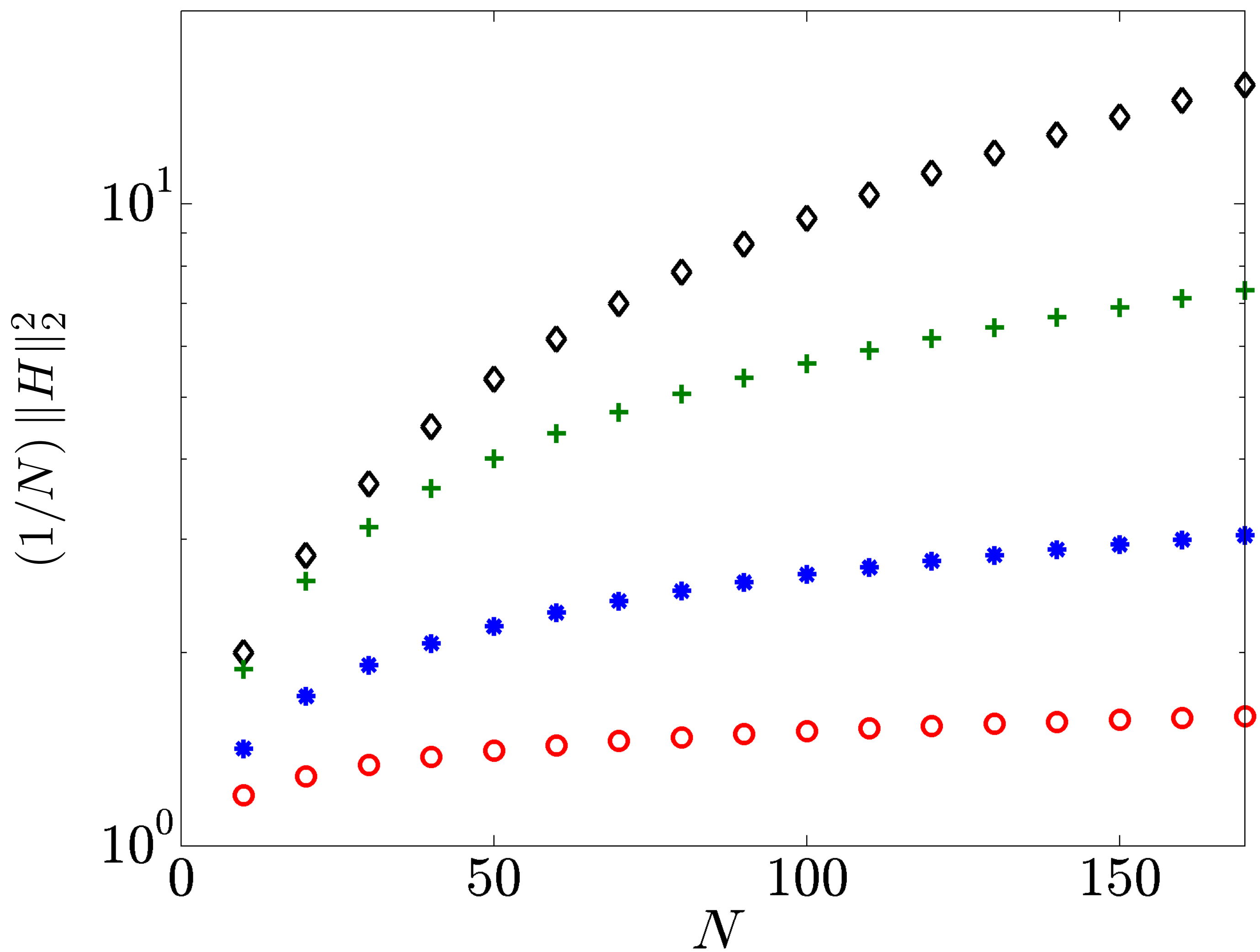

Part 2: summary

- STRUCTURED OPTIMAL CONTROL OF VEHICULAR FORMATIONS
  - ⋆ Optimal gains are spatially non-uniform

- PLATOONS:
  - ⋆ Limitations due to chaining of open-loop integrators
    Jovanović & Bamieh, IEEE TAC '05
  - ⋆ Must have global interactions to address tightness problem
  - ⋆ Even then, convergence of Merge & Split Maneuvers scales badly with N
    Jovanović, Fowler, Bamieh, D'Andrea, SCL '08

- FUNDAMENTAL LIMITATIONS FOR SPATIALLY INVARIANT LATTICES
  - ⋆ Bassam's talk
    Bamieh, Jovanović, Mitra, Patterson, IEEE TAC '10 (accepted)

Acknowledgments

TEAM:

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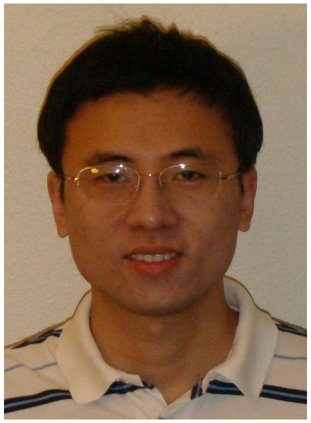

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SUPPORT:

NSF CAREER Award CMMI-06-44793

NSF Award CMMI-09-27720