lecture 22

April 24/2014

last time

- Stability of time-varying systems

$$\begin{cases} \dot{x} = f(x,t) \ ; \quad f(\bar{x}=0,t)=0 \\ w_1(x) \le V(x,t) \le w_2(x) \\ \dot{V} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x,t) \le -w_3(x) \quad < 0 \end{cases}$$

Linear Time-varying System

$$\dot{x} = A(t)x$$

$$V(x,t) = x^T P(t) x$$

$$\dot{V} = \dot{x}^T P(t) x + x^T P(t)\dot{x} + x^T \dot{P}(t) x \quad ①$$

$$= x^T \underbrace{[A^T(t)P(t) + P(t)A(t) + \dot{P}(t)]}_{-Q(t)} x = -x^T Q(t) x$$

$$\underbrace{x^T P(t) x \le k_2 \|x\|^2}_{x^T x}, \quad k_1\|x\|^2 \le x^T P(t) x, \quad K_3 I \le Q(t)$$

$$\boxed{\dot{V} = -x^T Q(t) x \le -K_3\|x\|^2 < 0 \quad \forall x \ne 0}$$

$$\boxed{P(t): \text{Solution to}: \quad \frac{dP(t)}{dt} + A^T P(t) + P(t)A(t) + Q(t) = 0 \quad (*)}$$

Recall: $A^T P + PA = -Q$ (Eq. point for $\dot{P} + A^T P + PA + Q = 0$)

$$P = \int_0^\infty e^{A^T t} Q e^{At} dt$$

"Converse" ThM:

Suppose that $\bar{x} = 0$ of $\dot{x} = A(t)x$ is uniformly exponentially Stable, $A(t)$ is Continuous and bounded, and $Q(t)$ is Continuous with: "$0 < k_3 I \leq Q(t) \leq k_4 I$" (*) and

Then, there is a unique $P(t) = P^T(t)$ s.t. it Solves (*) "$0 < k_1 I \leq P(t) \leq k_2 I$".

Estimation of Constant but unknown parameters:

I

$$y(t) = \psi^T(t)\,\theta$$

$y(t)$ → Scalar measurement

$\psi(t)$ → Regressor vector $\psi(t) \in \mathbb{R}^p$ (known function of time)

θ → a vector of unkown parameters $\theta \in \mathbb{R}^p$ $\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$

Ex: $\sum_{i=0}^{n} a_i y^{(i)}(t) = \sum_{i=0}^{m} b_i u^{(i)}(t)$, $a_n = 1$, $n \geq m$

want to estimate unknown vector of parameters θ:

$$\boxed{\hat{\theta}(t) \xrightarrow{t \to \infty} \theta}$$

$\hat{\theta}(t)$ → a vector of estimates of unknown parameters

$\tilde{\theta}(t) = \theta - \hat{\theta}(t)$: estimation error

$$\tilde{\theta}(t) \xrightarrow[t \to \infty]{} 0$$

$$\Rightarrow \quad \dot{\tilde{\theta}}(t) = \underbrace{\cancel{\dot{\theta}}}_{0} - \dot{\hat{\theta}}(t) \quad \boxed{= -\dot{\hat{\theta}}(t)}$$

$$\Rightarrow \quad \boxed{\dot{\tilde{\theta}}(t) = -\dot{\hat{\theta}}(t)}$$

$$\hat{y}(t) = \psi^T(t)\,\hat{\theta}(t) \quad \cdots \text{II}$$

$$\Rightarrow e(t) = y(t) - \hat{y}(t) = \psi^T(t)\,\tilde{\theta}(t) \quad \boxed{= \tilde{\theta}^T(t)\,\psi(t)}$$

<u>objective</u> $\quad \underset{\tilde{\theta}}{\text{minimize}} \; J(\tilde{\theta})$

$$J := \frac{1}{2} e^2(t) = \frac{1}{2} \tilde{\theta}^T(t)\,\psi(t)\,\psi^T(t)\,\tilde{\theta}(t)$$

$$\Rightarrow \quad \dot{\tilde{\theta}}(t) = -\nabla_{\tilde{\theta}} J(\tilde{\theta})$$

$$\Rightarrow \quad \dot{\tilde{\theta}}(t) = -\psi(t)\,\psi^T(t)\,\tilde{\theta}(t)$$

<u>Note:</u> $\quad \dot{\tilde{\theta}}(t) = A(t)\,\tilde{\theta}(t)$

where $\quad A(t) = -\psi(t)\,\psi^T(t)$

Objective: Determine conditions on $\psi(t)$ s.t.

$$\tilde{\theta}(t) \xrightarrow{t\to\infty} 0$$

Propose: $$V(\tilde{\theta}) := \frac{1}{2}\tilde{\theta}^T\tilde{\theta}$$

$$\dot{V} = -\tilde{\theta}^T \underbrace{\psi(t)\psi^T(t)}_{\text{rank one}}\tilde{\theta}$$

rank one, so, $\dot{V}$ is just NSD! not negative definite!

Bottom line: $$\dot{x} = A(t)x$$

$$\dot{V} = -\underbrace{x^T(t)C^T(t)}_{\psi(t)}\overbrace{\underbrace{C(t)}_{\psi^T(t)}x(t)}^{z(t)} = -z^T(t)z(t)$$

$\dot{x} = A(t)x$ $\rightarrow$ A, C should be uniformly observable

$z(t) = C(t)x(t)$ $\rightarrow$ It will result in some rules for ψ!

to have stable system ($\tilde{\theta}\to 0$)