

last time

- Stability of time-varying systems

$$\begin{cases} \dot{x} = f(x, t) ; f(\bar{x}=0, t) = 0 \\ \omega_1(x) \leq V(x, t) \leq \omega_2(x) \\ V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(x, t) \leq -\omega_3(x) < 0 \end{cases}$$

Linear Time-varying System

$$\dot{x} = A(t)x$$

$$V(x, t) = x^T P(t)x$$

$$\begin{aligned} \dot{V} &= \dot{x}^T P(t)x + x^T P(t) \dot{x} + x^T \dot{P}(t)x \quad \textcircled{1} \\ &= x^T \left[\underbrace{A^T(t)P(t) + P(t)A(t) + \dot{P}(t)}_{-Q(t)} \right] x = -x^T Q(t)x \end{aligned}$$

$$x^T P(t)x \leq k_2 \frac{\|x\|^2}{x^T x}, \quad k_1 \|x\|^2 \leq x^T P(t)x, \quad k_3 I \leq Q(t)$$

$$\boxed{\dot{V} = -x^T Q(t)x \leq -k_3 \|x\|^2 < 0 \quad \forall x \neq 0}$$

$$\boxed{P(t): \text{Solution to: } \frac{dP(t)}{dt} + A^T(t)P(t) + P(t)A(t) + Q(t) = 0 \quad \textcircled{*}}$$

Recall: $A^T P + P A = -Q$ (Eq. point for $\dot{P} + A^T P + P A + Q = 0$)

$$P = \int_0^{\infty} e^{A^T t} Q e^{A t} dt$$

"Converse" ThM:

Suppose that $\bar{x} = 0$ of $\dot{x} = A(t)x$ is uniformly exponentially stable, $A(t)$ is continuous and bounded, and $Q(t)$ is

Continuous with: " $0 < k_3 I \leq Q(t) \leq k_4 I$ "

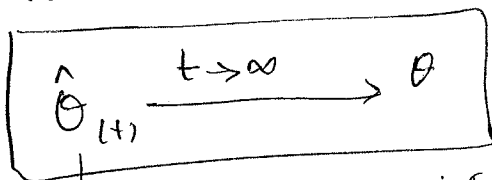
Then, there is a unique $P(t) = P^T(t)$ s.t. it solves $(*)$ and " $0 < k_1 I \leq P(t) \leq k_2 I$ ".

Estimation of constant but unknown parameters

$y(t) = \psi^T(t) \theta$ (I)
↓ a vector of unknown parameters
 Scalar measurement ↓ Regressor vector $\theta \in \mathbb{R}^p$
 $\psi(t) \in \mathbb{R}^p$
(known function of time) $\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$

Ex: $\sum_{i=0}^n a_i^{(i)} y(t) = \sum_{i=0}^m b_i^{(i)} u(t)$, $a_n = 1, n > m$

want to estimate unknown vector of parameters θ :



→ a vector of estimates of unknown parameters

$$\tilde{\theta}(t) = \theta - \hat{\theta}(t) \quad : \text{estimation error}$$

$$\tilde{\theta}(t) \xrightarrow{t \rightarrow \infty} \theta$$

$$\Rightarrow \dot{\tilde{\theta}}(t) = \dot{\theta} - \dot{\hat{\theta}}(t) = -\dot{\hat{\theta}}(t)$$

$$\Rightarrow \dot{\tilde{\theta}}(t) = -\dot{\hat{\theta}}(t)$$

$$\hat{y}(t) = \psi^T(t) \hat{\theta}(t) \quad \text{--- (II)}$$

$$\Rightarrow e(t) = y(t) - \hat{y}(t) = \psi^T(t) \tilde{\theta}(t) = \tilde{\theta}^T(t) \psi(t)$$

objective minimize $J(\tilde{\theta})$

$$J := \frac{1}{2} e^2(t) = \frac{1}{2} \tilde{\theta}^T(t) \psi(t) \psi^T(t) \tilde{\theta}(t)$$

$$\Rightarrow \dot{\tilde{\theta}}(t) = -\nabla_{\tilde{\theta}} J(\tilde{\theta})$$

$$\Rightarrow \dot{\tilde{\theta}}(t) = -\psi(t) \psi^T(t) \tilde{\theta}(t)$$

Note: $\dot{\tilde{\theta}}(t) = A(t) \tilde{\theta}(t)$

where $A(t) = -\psi(t) \psi^T(t)$

Objective: Determine conditions on $\Psi(t)$ s.t.

$$\tilde{\theta}(t) \xrightarrow{t \rightarrow \infty} 0$$

Propose: $V(\tilde{\theta}) := \frac{1}{2} \tilde{\theta}^T \tilde{\theta}$

$$\dot{V} = - \tilde{\theta}^T \underbrace{\Psi(t) \Psi^T(t)}_{\text{rank one}} \tilde{\theta}$$

rank one, so, \dot{V} is just NSD! not negative definite!

Bottomline:

$$\begin{aligned} \dot{x} &= A(t)x \\ \dot{V} &= - x^T(t) \underbrace{C^T(t)}_{\Psi(t)} \underbrace{C(t)}_{\Psi^T(t)} x(t) = - \underbrace{Z^T(t)}_{z(t)} z(t) \end{aligned}$$

$\dot{x} = A(t)x$
 $z(t) = C(t)x(t)$ \rightarrow A, C should be uniformly observable
 \rightarrow it will result in some rules for $\Psi!$

to have stable system ($\tilde{\theta} \rightarrow 0$)