Nonlinear Systems

Lecture 28 05/09/13

Last few lectures,

- Input-output linearization
- Relative degree
- Zero dynamics
- Normal form

make z-d. transparent

$$\dot{z} = f_0(z, \xi) \quad : \text{zero dynamics}$$

$$\dot{\xi} = A_\xi \xi$$

after input-output linearization is done.

Linearization :

$$A = \begin{bmatrix} \frac{\partial f_0}{\partial z}\Big|_0 & \frac{\partial f_0}{\partial \xi} \\ 0 & A_\xi \end{bmatrix}$$

$$\text{LAS} \Leftrightarrow \left.\begin{array}{c} \frac{\partial f_0}{\partial z}\Big|_0 \\ A_\xi \end{array}\right\} \text{Hurwitz}$$

Q. How about global asymptotic stability

After I/O linearization

$$\boxed{\dot{\xi} = A_\xi \xi} \xrightarrow{\xi} \boxed{\dot{z} = f_0(z,\xi)} \xrightarrow{z}$$

We need some additional assumptions on f_0

(e.g. Input to state stability, from input ξ to state z)

<u>Ex</u>

$$\dot{z} = -z + z^2 \xi$$
$$\dot{\xi} = A_\xi \xi$$

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z-subsystem is not ISS (input to state stable)
In fact, finite escape time can happen for large enough initial conditions (inspite of local asymptotic stability)

$\downarrow$

strength of nonlinearity can beat decay of linear term

Recall example from last time:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \alpha x_3 + u \\ \dot{x}_3 &= \beta x_3 - u \\ \hline y &= x_1 \end{aligned}\right\} \qquad \begin{aligned} \dot{z} &= (\alpha+\beta) z - (\alpha+\beta)\xi_2 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \alpha z - \alpha\xi + u \\ \hline z &:= x_2 + x_3 = \xi_2 + x_3 \end{aligned}$$

transfer function (from u to y):

$$H(s) = \frac{s-(\alpha+\beta)}{s^2(s-\beta)}$$

Note! zero @ $s = \alpha+\beta$

2 poles at zero $\longrightarrow$ unstable system

A quick note: system is feedback linearizable if there is an output s.t. relative degree is equal to n (order of your system: # of states)

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