

# Nonlinear Systems

Lecture 28

05/09/13

Last few lectures,

- Input-output linearization
  - Relative degree
  - Zero dynamics
  - Normal form
- ← make z-d. transparent

$$\dot{z} = f_0(z, \xi) \quad : \text{zerodynamics}$$

$$\dot{\xi} = A_{\xi} \xi$$

→ after input-output linearization is done.

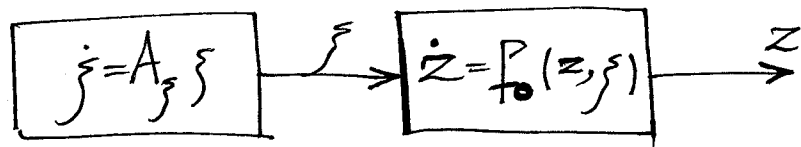
Linearization :

$$A = \begin{bmatrix} \frac{\partial f_0}{\partial z} \Big|_0 & \frac{\partial f_0}{\partial \xi} \\ 0 & A_{\xi} \end{bmatrix}$$

$$\text{LAS} \iff \left. \begin{array}{l} \frac{\partial f_0}{\partial z} \Big|_0 \\ A_{\xi} \end{array} \right\} \text{Hurwitz}$$

Q. How about global asymptotic stability

After I/O linearization



We need some additional assumptions on  $f_0$   
(e.g. Input to state stability, from input  $\xi$  to state  $z$ )

Ex

$$\dot{z} = -z + z^2 \xi$$
$$\dot{\xi} = A_{\xi} \xi$$

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$z$ -subsystem is not ISS (input to state stable)

In fact, finite escape time can happen for large enough initial conditions (inspite of local asymptotic stability)

↓  
strength of nonlinearity can beat decay of linear term

Recall example from last time:

$$\left. \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \alpha x_3 + u \\ \dot{x}_3 = \beta x_3 - u \\ \hline y = x_1 \end{array} \right\} \begin{array}{l} \dot{z} = (\alpha + \beta)z - (\alpha + \beta)\xi_2 \\ \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \alpha z - \alpha \xi_2 + u \\ \hline z := x_2 + x_3 = \xi_2 + x_3 \end{array}$$

transfer function (from  $u$  to  $y$ ):

$$H(s) = \frac{s - (\alpha + \beta)}{s^2(s - \beta)}$$

Note! zero @  $s = \alpha + \beta$

2 poles at zero  $\rightarrow$  unstable system

A quick note: system is feedback linearizable if there is an output s.t. relative degree is equal to  $n$  (order of your system: # of states)

More info: Khalil