# MOS Transistor

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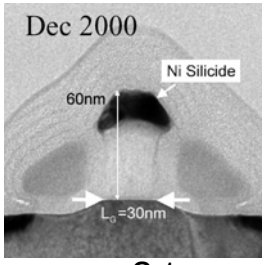
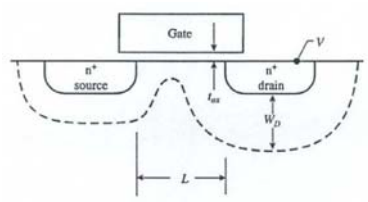
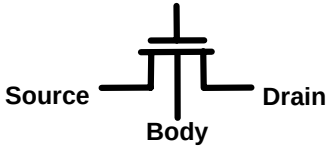
# MOS Transistor





Kuroda, IEDM panel

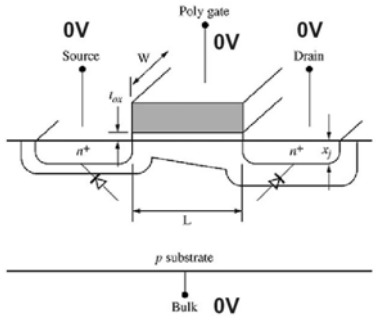




# MOS Transistor Current Equation

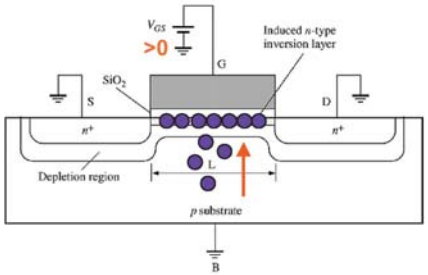


# Basic Operation (1)




- **Device is in cut-off region**
- **Simply, two back-to-back reverse biased pn diodes.**

# Basic Operation (2)

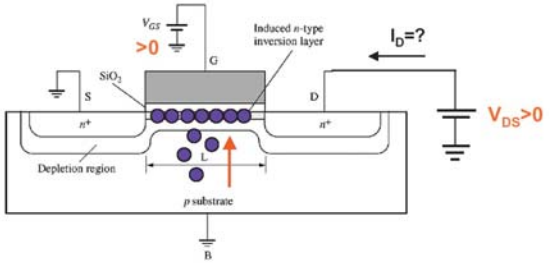



- With a positive gate bias, electrons are pulled toward the positive gate electrode
- Given a large enough bias, the electrons start to "invert" the surface (p→n type), a conductive channel forms
- Threshold voltage $V_t$





# Basic Operation (3)

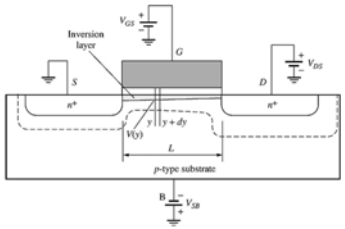



- Current flows from drain to source with a positive drain voltage
- What is current in terms of $V_{gs}$, $V_{ds}$, $V_{bs}$?





# MOS Current




$I_{ds} = 0$

$V_{gs} < V_t$ : cut-off

$I_{ds} = \mu_e C_{ox} W/L \left((V_{gs}-V_t) V_{ds} - 0.5 V_{ds}^2\right)$

$0 < V_{ds} < V_{gs} - V_t$ : triode (linear) mode

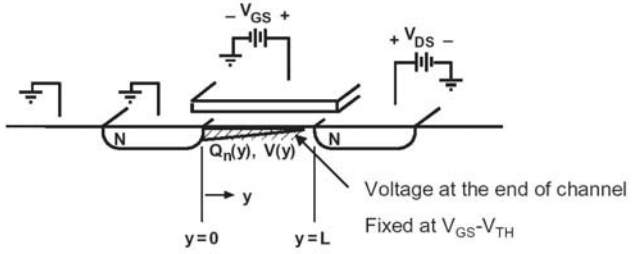
$I_{ds} = \mu_e C_{ox} W/(2L) (V_{gs}-V_t)^2$

$0 < V_{gs} - V_t < V_{ds}$ : saturation mode





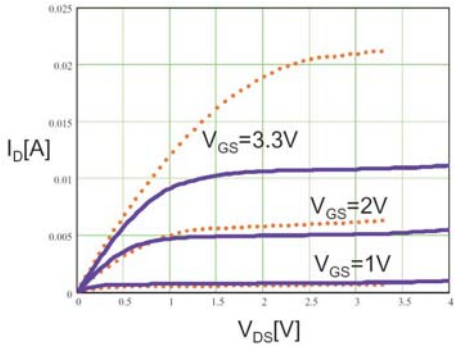
# Channel Length Modulation




- Pinch-off depletion layer width increases as the drain voltage increases
- Extreme case of this is punch-through

$$L = L_o - \zeta V_{ds} \qquad I_{ds} = I_{dsat} \times \frac{L_o}{L_o - \zeta V_{ds}} = I_{dsat} \times (1 + \lambda V_{ds})$$

# Simulation versus Model (NMOS)

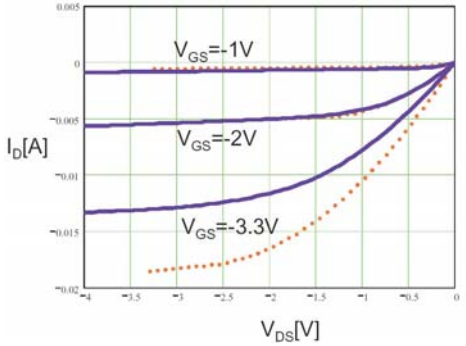



- The square-law model doesn't match well with simulations
- Only fits for low $V_{gs}$, low $V_{ds}$ (low E-field) conditions





# Simulation versus Model (PMOS)

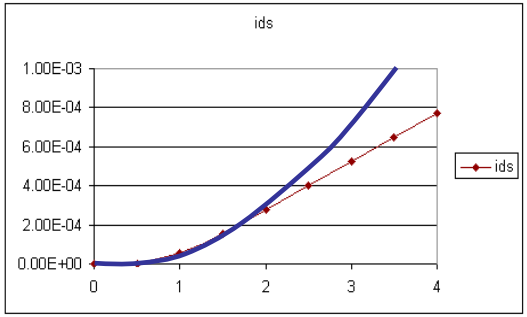



- Not as bad as the NMOS device
- Still large discrepancies at high E-field conditions





# Simulation versus Model ($I_{ds}$ vs. $V_{gs}$)

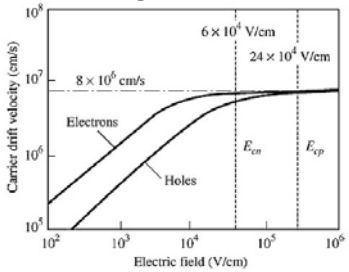



- Saturation current does not increase quadratically
- The simulated curves looks like a straight line
- Main reason for discrepancy: velocity saturation





# Velocity Saturation




- E-fields have gone up as dimensions scale
- Unfortunately, carrier velocity in silicon is limited
- Electron velocity saturates at a lower E-field than holes
- Mobility ($\mu_e$=v/E) degrades at higher E-fields
- Simple piecewise linear model can be used

# Velocity Saturation

$$v = \frac{\mu_e E}{\left(1+\left(E/E_c\right)^n\right)^{1/n}} \quad for\ E < E_c \qquad E_c = \frac{2v_{sat}}{\mu_e}$$

$$= v_{sat} \quad for\ E > E_c$$

[Toh, Ko, Meyer, JSSC, 8/1988]

- **Modeled through a variable mobility**
- **n=1 for PMOS, n=2 for NMOS**
- **To get an analytical expression, let's assume n=1**





# Velocity Saturation

- **Plug it into the original current equation**

$$\therefore\ I_{ds} = \begin{cases} \mu_e C_{ox}\frac{W}{L}\left((V_{gs}-V_t)V_{ds}-\frac{V_{ds}^2}{2}\right)\times\frac{1}{1+\frac{V_{ds}}{E_c L}} & (V_{ds} < V_{dsat}) \\ C_{ox}Wv_{sat}(V_{gs}-V_t-V_{dsat}) & (V_{ds} > V_{dsat}) \end{cases}$$

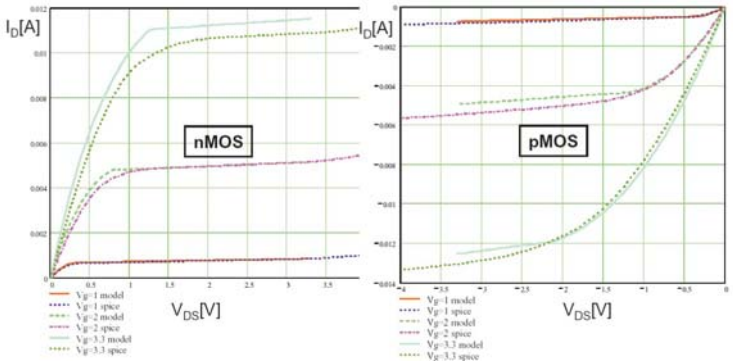
**Equate the two expressions to get**

$$V_{dsat} = \frac{(V_{gs}-V_t)E_c L}{(V_{gs}-V_t)+E_c L}$$





# Simulation versus Model

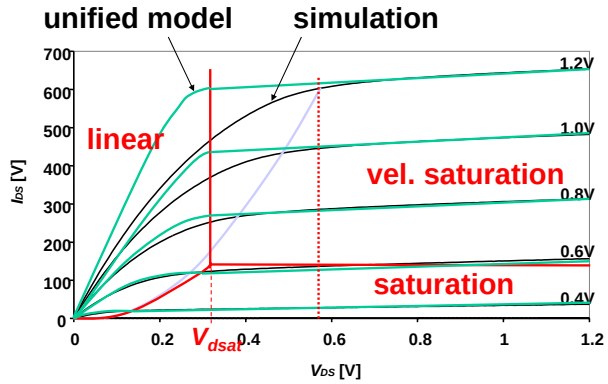



- **Model incorporating velocity saturation matches fairly well with simulation**

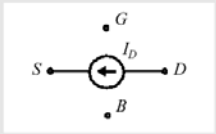




# Unified MOS Model



# Unified MOS Model Equations




$$I_D = 0 \text{ for } V_{GT} \le 0$$

$$I_D = k'\frac{W}{L}\left(V_{GT}V_{min} - \frac{V_{min}^2}{2}\right)(1+\lambda V_{DS}) \text{ for } V_{GT} \ge 0$$

with $V_{min} = \min(V_{GT}, V_{DS}, V_{DSAT})$,

$V_{GT} = V_{GS} - V_T$,

and $V_T = V_{T0} + \gamma(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|})$

$\gamma$ - body effect parameter

- **Model presented is compact and suitable for hand analysis.**
- **Still have to keep in mind the main approximation: that $V_{DSat}$ is constant.**
- **But the model still works fairly well.**





# Alpha Power Law

- **Simple empirical model for short channel MOS**

$$I_{ds} = \frac{W}{2L}\mu_e C_{ox}(V_{gs} - V_t)^{\alpha}$$

[Sakurai and Newton, JSSC 1990]

- **Parameter α is between 1 and 2**
- **α=1-1.2 for short channel devices**
- **Parameters α and $V_t$ are fitted to measured data for minimum square error → fitted $V_t$ can be different from physical $V_t$**