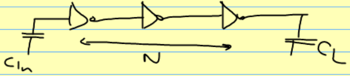
Inverter Chain Sizing Recap



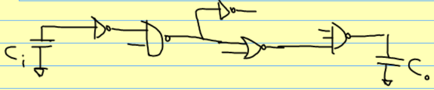
$$F = \frac{C_L}{C_{in}} \qquad f^* = \sqrt[N]{F}$$

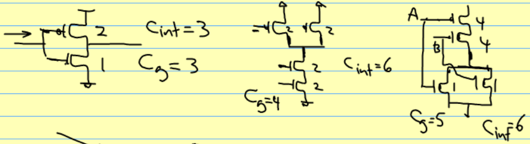
$$\gamma = \frac{C_{int}}{C_g} \qquad \text{for } \gamma = 1 \quad f^* \simeq 3.6$$

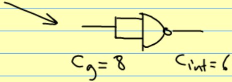
$$C_{int} \propto W$$

$$C_g \propto C_{ox} W$$

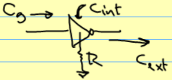
Logical Effort (p252)







Delay



$$C_{int} = \gamma_{inv} C_g \quad (C_g \rightarrow C_{ox}WL)$$

$$t_p = 0.69\, R\,(C_{int} + C_{ext})$$

$$= 0.69\, R\, C_{int}\left(1 + \frac{C_{ext}}{C_{int}}\right)$$

$$= \underbrace{0.69\, R\, C_{int}}\left(1 + \frac{C_{ext}}{\gamma_{inv} C_g}\right)$$

$$= t_{inv_\phi}\left(1 + \frac{1}{\gamma_{inv}} \cdot f\right) \longrightarrow t_{inv_\phi} + \frac{t_{inv\phi}}{\gamma_{inv}} \cdot f$$

$$t_{p_{NAND}} = t_{NAND\phi} + \frac{t_{NAND\phi}}{\gamma_{NAND}} \cdot f$$

Normalize to inv ϕ

$$\frac{t_{inv}}{t_{inv\phi}} = 1 + \frac{1}{\gamma_{inv}} f = p_{inv} + \frac{1}{\gamma_{inv}} g_{inv} f$$

(with $p_{inv} = 1$, $g_{inv} = 1$)

$$\frac{t_{nand}}{t_{inv\phi}} = \frac{t_{nand\phi}}{t_{inv\phi}} + \frac{1}{\gamma_{nand}} \frac{t_{nand\phi}}{t_{inv\phi}} \cdot f$$

$$= \frac{t_{nand\phi}}{t_{inv\phi}} + \frac{1}{\gamma_{inv}} \frac{t_{nand\phi}/\gamma_{nand}}{t_{inv\phi}/\gamma_{inv}} \cdot f$$

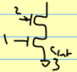
$$= p_{nand} + \frac{1}{\gamma_{inv}} \cdot g_{nand} \cdot f$$

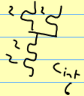
$$\frac{t_{nor}}{t_{inv\phi}} = p_{nor} + \frac{1}{\gamma_{inv}} g_{nor} \cdot f$$

Parasitic term p _ Logical effort g

$p_{inv} = 1$

$$p_{nand} = \frac{t_{nand\phi}}{t_{inv\phi}} = \frac{(R\,C_{int})_{nand}}{(R\,C_{int})_{inv}}$$





$$= \frac{6}{3} = 2$$

$$p_{nor} = \frac{6}{3} = 2$$

Logical effort term

$C_{int} = \gamma \cdot C_g$

$g_{inv} = 1$

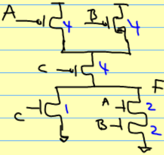
$$g_{nand} = \frac{t_{nand\phi}/\gamma_{nand}}{t_{inv\phi}/\gamma_{inv}} = \frac{(R\,C_{int})_{nand}/\gamma_{nand}}{(R\,C_{int})_{inv}/\gamma_{inv}}$$

$$g_{nand} = \frac{(R\,C_g)_{nand}}{(R\,C_g)_{inv}} = 4/3$$

$g_{nor} = 5/3$

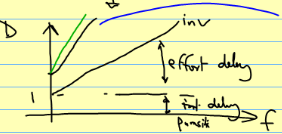
p_{AOI} , g_{AOI}

$F = \overline{A \cdot B + C}$



$$g_{AOI} = \begin{cases} 5/3 & C \\ 2 & A/B \end{cases}$$

$$p_{AOI} = 7/3$$



$d_{nand} = 2 + \frac{4}{3} f$

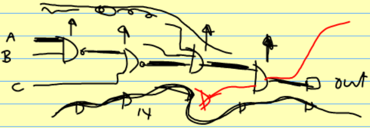
$d_{nor} = 2 + \frac{5}{3} f$

Optimal Sizes

$$d_x = p_x + g_x \frac{1}{\gamma_{inv}} f \rightarrow \frac{C_{j+1}}{C_j}$$

$$t_p = t_{p0} \cdot \sum_j \left(p_j + \frac{g_j}{\gamma_{inv}} \cdot f_j\right)$$

$$= t_{p0} \sum_j \left(p_j + \frac{g_j}{\gamma_{inv}} * \frac{C_{j+1}}{C_j}\right)$$



$\frac{1}{15}$ $\frac{1}{14}$ $\frac{1}{13}$

Opt Sizes

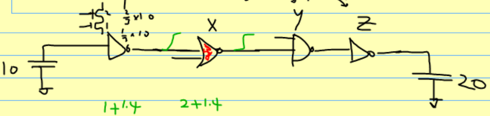
$$\frac{\partial t_p}{\partial C_j} = t_{p_0}\left(\frac{g_j}{\gamma_{inv}}\left(-\frac{C_{j+1}}{C_j^2}\right) + \frac{g_{j-1}}{\gamma_{inv}}\frac{1}{C_{j-1}}\right) = 0$$

$$g_j \cdot \frac{C_{j+1}}{C_j} = g_{j-1}\frac{C_j}{C_{j-1}}$$

$$g_j \cdot f_j = g_{j-1} \cdot f_{j-1} = h^*$$

(gate effort)

Example



$p =$	1	2	2	1
$g =$	1	$5/3$	$4/3$	1
$f =$	$\frac{x}{10}$	$\frac{y}{x}$	$\frac{z}{y}$	$\frac{20}{z}$

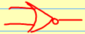
$= h^*$

$$h^{*4} = \frac{x}{10} \cdot \frac{y}{x} \cdot \frac{z}{y} \cdot \frac{20}{z} \cdot \frac{5}{3} \cdot \frac{4}{3} = \frac{40}{9} \Rightarrow h^* = \sqrt[4]{\frac{40}{9}} \simeq 1.4$$

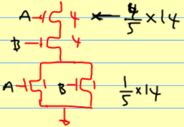
Gate Sizing Example (Cont.)

1. $\frac{X}{10} = h^* = 1.4 \Rightarrow X = 14$

g ↗ f ↗



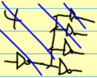
X=14



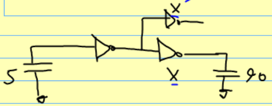
$D = 6 + 1.4 \times 4 = \underline{11.6}$ ←

FO4

$\frac{11.6}{5} = 2.32$ FO4



Branches



$p =$	1	1
$g =$	1	1
$f =$	$\frac{\not{2}x}{5}$	$\frac{90}{x}$
$b =$	2	1

branching effort

$$\frac{C_{on\text{-}path} + C_{off\text{-}path}}{C_{on\text{-}path}}$$

branching Example (cont)

$$h^* = \sqrt{\frac{2}{5} \cdot 90} = 6$$

$$\frac{2X}{5} = 6 \Rightarrow X = 15$$

$$D = (1+6) + (1+6) = 14$$

Method of Logical Effort

Vars on *PATH*

① $G = \prod_j g_j$ path logical effort

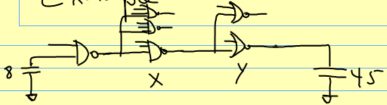
$F = \prod_j f_j$ effective fanout

$B = \prod_j b_j$ branching effort

$H = GBF$ ② $h^* = \sqrt[n]{H} = g_i b_i f_i$ ③ $f_i \rightarrow \frac{C_{i+1}}{C_i}$

④ $D = \sum p_i + \frac{N h^*}{\gamma_{inv}}$

Example



p =	2	3	2
g =	4/3	5/3	5/3
f =	x/8	y/x	45/y
b =	3	2	1

$$H = \frac{4}{3} \times \frac{5}{3} \times \frac{5}{3} \times 3 \times 2 \times \frac{45}{8} = 5^3$$

$$h^* = \sqrt[3]{5^3} = 5$$

$$\frac{4}{3} \times \frac{x}{8} \times 3 = 5 \Rightarrow x = 10$$

$$D = 7 + 3 \times 5 = 22$$

Example – 8-input AND