EE 3161

MIDTERM EXAM #2

SOLUTIONS

SPRING 2008
(1) a) \[ \text{I} \quad \text{Reverse Bias} \quad \text{log I} \quad \text{forward bias} \]

Diagram showing:
- Breakdown
- \(-V_{BR}\)
- \(I_R - G\)
- \(R - G\)
- Series Resistance
- High Level Injection (see part b)
- Ideal

b) Breakdown:
\[ V_{BR} = \frac{N_A + N_D}{N_A N_D} \frac{e}{2q} E_{cr}^2 \]

\[ E_{cr} = 4 \times 10^5 \text{ V/cm} \]

\[ V_{BR} = -18.3 \text{ V} \]

\[ I_{R-G \text{ to Ideal}}: \]
\[ qA \frac{n_i^2}{2T} W e^{\frac{V}{2kT}} = q n_i^2 A \left( \frac{D_p}{L P N_0} + \frac{D_n}{L n N_A} \right) e^{\frac{qV}{2kT}} \]

\[ W_n = 1 \mu m \]
\[ D_0 = \frac{kT}{q} \mu_0 (3 \times 10^{16}) = 0.076 \left( 400 \frac{V^2}{cm \cdot s} \right) = 10.4 \frac{cm^2}{s} \]
\[ D_0 = \frac{kT}{q} \mu_0 (5 \times 10^{12}) = 0.026 \left( 300 \frac{V^2}{cm \cdot s} \right) = 10.1 \frac{cm^2}{s} \]
\[ L_n = \sqrt{D_0 \tau_n} = 14.2 \mu m \]
\[ L_p = \sqrt{D_p \tau_p} = 14.4 \mu m \]
\[ V_A = \frac{2kT}{q} \ln \left[ \frac{\frac{n_i}{2\pi}}{W} \right] \]
\[ = \frac{n_i}{2\pi} \frac{W}{n_i^2 \left( \frac{D_0}{\ln(N_0)} + \frac{D_p}{N_0 W_0} \right)} \]
\[ V_{bi} = \frac{kT}{q} \ln \frac{N_0}{n_i^2} = 8.4 V \]
\[ W = \sqrt{\frac{2e}{q} V_b} \frac{N_{A+N_0}}{N_A N_0} \approx 2 \mu m \]
\[ |V_A| = 0.29 V \] for \( IR = 6 \) to \( I_{ideal} \) transition

Ideal to \( I_{HL} \):

\( HLI \) starts at roughly
\[ \frac{N_0 e}{N_0} \approx \frac{1}{10} \]
\[ \frac{n_i^2 \sqrt{V_A} \tau_A}{N_0} \approx \frac{1}{10} \]
\[ V_A = \frac{kT}{q} \ln \left( \frac{N_d}{n_i^2} \right) \]

\[ V_A_{HII} = 71V \]

**Series Resistance**

Note that if \( P = \frac{1}{qM_{Na}} = \frac{1}{qM_{Na} = 5 \times 10^{-12} N_a} \)

series resistance develops, it will be in the p-region

because it is so much larger than the n-region

\[ P = 0.066 \Omega \text{-cm} \]

\[ R = \frac{L}{A} = \frac{(0.066 \times 1 \text{ cm})}{(10 \times 10^{-8} \text{ cm}^2)} \]

\[ R \approx 66 \Omega \]

\[ V_{sh} = IR \]

Let series resistance be appreciable when \( V_{sh} \approx 0.1V \), so \( I \approx 1.5 \mu A \)

\[ qn_i^2 \left( \frac{D_n}{2m_{Na}} + \frac{D_p}{N_a W_a} \right) e^{\frac{V_{sh}}{kT}} = 1.5 \mu A \]
\[ V_{\text{junction}} = \frac{KT}{q} \ln \left( \frac{1.5 \mu A}{q n^2 \left( \frac{D_m}{2 n N_A} + \frac{D_p}{N_0 N_N} \right)} \right) \]

\[ V_{\text{junction}} \approx 0.22 V \]

Series Resistance starts to be a factor long before Ideal or Full even starts. So these two regions are absent in this diode.
(2) For this problem, I will assume a small area IC transistor ($A \sim 4 \mu m^2$). In this case, if a concentration gradient develops between the two regions of the base, it will be eliminated by diffusion in a time short compared to $\tau_B$. ($t \sim \frac{x^2}{2D} \sim \frac{(2\mu m)^2}{2(100 \mu m^2)} \sim 60 \mu s \ll \tau_B$). Therefore $Q_B$ in both halves of the transistor is about the same.

In steady state, $I_{BB} = \frac{Q_B}{\tau_B}$

$$I_{BB} = \frac{q(\frac{A}{2}) W}{2 \tau_B} \Delta P_B(0, t) + \frac{q(\frac{A}{2}) W \Delta P_B(0, t)}{2 \tau_B}$$

$$I_{BB} = \frac{qA W \Delta P_B(0, t)}{2} \left[ \frac{2}{2 \left( \frac{1}{\tau_B} + \frac{1}{\tau_{B1}} \right)} \right]$$

$$\therefore \frac{1}{\tau_{eff}} = \frac{1}{2} \left( \frac{1}{\tau_B} + \frac{1}{\tau_{B1}} \right)$$

$$\tau_{eff} \approx 18 \mu s$$
\[ \tau_r = \tau_{eff} \ln \left( \frac{1 - \frac{I_{cc} T_t}{I_{BB} T_{eff}}}{} \right) \]

\[ \beta = \frac{T_{eff}}{T_t} \quad \beta = \frac{18 \mu s}{T_t} \quad T_t = \frac{W^2}{2D} \]

\[ W = 0.5 \mu m \quad D = 32 \text{ cm}^2 \]

\[ T_t \approx 39.95 \text{ ps} \]

\[ \beta \approx 4600 \]
b.) Jeff replaces T3

c.) A relative voltage drop would appear across the base, so the most transistor action would occur for the base region nearest the contact. Further regions would have a more weakly biased BE junction.
2. For a moderate to large area transistor, the minority carrier diffusion time will be too slow to redistribute minority carriers in the base \( \frac{L^2}{2D} \) to keep \( Q(x,t) \) the same in both halves of the BJT. However, the majority carriers can redistribute extremely quickly \( \tau \ll \tau_{GS} \), so the biases in each region are always constant.

The two BJTs can be treated separately but will enter saturation simultaneously.

\[ Q \] vs. \( t \) to long

\[ Q \text{ to short} \]
\[ \frac{V_{cc}}{R} \]

\[ \geq Z_{r1} = \tau_{r1} \ln \left[ \frac{1}{1 - \frac{V_c/t}{\tau_{cc}/t}} \right] \]