finding a Lyapunov function satisfying the estimate (2.2) for this class of systems.

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Weight Selection in Feedback Design With Degree Constraints

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***Abstract*—We present an approach for feedback design which is based on recent developments in analytic interpolation with a degree constraint. Performance is cast as an interpolation problem with bounded analytic functions. Minimizers of a certain weighted-entropy functional provide interpolants having degree less than the number of constraints. The choice of weight parameterizes all such bounded degree solutions. However, the relationship between the weights and the shape of corresponding transfer functions is not direct. Thus, in this paper we develop a formalism that guides weight selection.**

***Index Terms*—Control synthesis, weighted entropy-like functionals.**

I. INTRODUCTION

Modern robust control design focuses on shaping the frequency response of closed-loop transfer functions. Performance is cast as a weighted optimization problem where weights relate to desired frequency responses [1], [2]. A drawback of standard H_∞-based methodologies is that they result in a degree inflation for the controller and the feedback system beyond what is necessary for achieving performance.

This paper is about a new formalism based on recent developments in analytic interpolation with a degree constraint [3]–[6]. Here, interpolants are obtained as minimizers of a weighted entropy-like functional and the choice of weight affects the shape of the optimal closed-loop operator (interpolant). Although this approach allows some handle on the degree of interpolants, the relation between the weighting function and the shape of the corresponding interpolant is not direct. Thus, in this work, building on earlier studies by Nagamune, Blomqvist, and others (see e.g., [7]–[12]), we present an approach to address this issue. We formulate a quasi- convex optimization problem for weight selection based on a desired shape for the closed-loop response. We deal with sensitivity shaping of single-input/single-output systems and demonstrate the efficacy of the new methodology with illustrative examples.

II. EXTREMA OF WEIGHTED ENTROPY FUNCTIONALS

Given a nominal scalar plant model $P(z)$, in discrete-time, internal stability of the closed-loop system with a suitable control $C(z)$ can be expressed via interpolation conditions on the sensitivity function $S(z) = (1 + P(z)C(z))^{-1}$ [13]. The conditions are as follows: first $S(z)$ must be analytic in the complement of the open unit disk $\mathbb{D}^c$, and then

$$S(z_i) = \begin{cases} 0 & \text{when } z_i \text{ is a pole of } P \text{ in } \mathbb{D}^c \\ 1 & \text{when } z_i \text{ is a root of } P \text{ in } \mathbb{D}^c \end{cases} \quad (1)$$

for $i = 0, 1, \ldots, n$, i.e., the number of interpolation conditions is assumed to be $n+1$. Multiple poles and zeros induce interpolation on the

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derivatives of S, however, for simplicity of notation we assume these to be simple.

Feedback control synthesis was formulated by Zames [1] as an H_∞-minimization problem. In this, performance specifications were translated into a desired shape for, e.g., the sensitivity function $S(z)$, and then an admissible sensitivity function was sought which abides by the constraints (1). Herein, we begin with an overall acceptable level $\|S\|_\infty < \gamma$ and a desired shape $|S_d|$ for the sensitivity gain $|S|$. These stem from the interest in noise attenuation and disturbance reduction for the closed-loop system. Then, we search for a low-degree function S, analytic in $\mathbb{D}^c$, which satisfies (1) and whose shape approximates $|S_d(e^{j\theta})|$. This is sought as a maximizer of a weighted entropy functional [3], [4]

$$\int_{-\pi}^{\pi} \Psi(e^{j\theta}) \log(\gamma^2 - |S(e^{j\theta})|^2) d\theta \tag{2}$$

where the weight-function Ψ depends on S_d. Indeed, if Ψ is a positive function with poles at the interpolation points and their conjugates, i.e., $\Psi(e^{j\theta}) =: |\sigma(e^{j\theta})|^2$ with $\sigma(z) \in \mathcal{K}$ and

$$\mathcal{K} := \left\{ \frac{\rho(z)}{\tau(z)} : \tau(z) = \prod_{i=0}^{n} (1 - \bar{z}_i z) \right. \left. \rho(z) \text{ is a polynomial of degree at most } n \right\}$$

then there exists an interpolant (i.e., a sensitivity function $S(e^{j\theta})$) of degree less than or equal to n which maximizes (2) subject to (1) [3]. Further, all interpolants of degree $\leq$n are maximizers of (2) for a suitable weight. Hence, provided the interpolation problem is solvable, the desired shape of the sensitivity function can be obtained with a "suitable" choice of Ψ. In view of this, our approach amounts to first selecting a "suitable" weight and then obtaining a maximizer of (2) subject to (1). We begin by discussing properties of the maximizer which guide the weight selection in the first step.

The maximization of (2) over a choice of S is equivalent to the minimization of the Kullback-Leibler divergence

$$\mathbb{S}(\Psi, \Phi) := \int_{-\pi}^{\pi} \Psi(e^{j\theta}) \log \frac{\Psi(e^{j\theta})}{\Phi(e^{j\theta})} d\theta$$

between Ψ and $\Phi(e^{j\theta}) = \gamma^2 - |S(e^{j\theta})|^2$. This represents a notion of distance between a desired "shape" provided by Ψ and $\gamma^2 - |S|^2$. It turns out that it is convenient to replace S with a corresponding positive-real function

$$F = \frac{\gamma - S}{\gamma + S}. \tag{3}$$

This has the same McMillan degree as S and the interpolation constraints of S and F are in a direct correspondence. Further, the minimizer of $\mathbb{S}(\Psi, \gamma^2 - |S|^2)$ subject to the interpolation constraints is unique and is attained at the same place as the minimizer of $\mathbb{S}(\Psi, \Re e\, F)$ over positive-real F's which are subject to the corresponding constraints [3, p.972]. The introduction of F has the added advantage of allowing a convenient normalization $\frac{1}{2\pi}\int_{-\pi}^{\pi} \Re e\; F(e^{j\theta})d\theta = 1$ via a conformal mapping that takes one of the interpolation conditions to the origin and scales accordingly the others. Thus, the second step amounts to minimizing

$$\mathbb{S}(\Psi(e^{j\theta}), \Re e\; F(e^{j\theta})) \tag{4}$$

subject to inherited constraints on F.

To compare this formalism with the classical approach of H_∞-control consider

$$f_d(z) := \text{argmin}\{\|f(z)\|_\infty : f(z_i) = w_i W_d(z_i)^{-1}\}$$

where w_i's are interpolation values on the sensitivity function imposed by the stability requirement, W_d is a weighting function and $S(z) = W_d(z)f(z)$ is the resulting sensitivity function. Incorporating $W_d(z)$ in this formalism affects the degree of sensitivity function (being added to the degree of f) and thereby inflates the degree of the resulting controller accordingly [7], [9]. In our formulation on the other hand, the degree of S is the same as that of F, which in turn can be bound by the number of interpolation constraints provided the weight is chosen within a specified class of rational functions. More specifically [3], [14], the optimal F is such that $\Re e\; F = \Psi/Q$ with Q positive and having spectral factors in $\mathcal{K}$. Thus, if Ψ is selected within

$$\mathcal{D} = \left\{ |\sigma(e^{j\theta})|^2 : \sigma(z) \in \mathcal{K} \right\}$$

the degree of $\Re e\; F$ is at most equal to the generic degree of Ψ (since the denominators of Ψ and Q cancel out). Thus, the optimal F, and therefore S as well, have degrees bounded by the number of interpolation constrains.

In view of the above, our basic plan is as follows. Starting from a function $S_d(z)$ which has the desired shape for a sensitivity function but may not necessarily satisfy the interpolation conditions, as step (i), we translate this into a desired shape for the real part of the corresponding positive real function

$$\Phi_d(e^{j\theta}) := \Re e F_d(e^{j\theta}), \text{ where } F_d(z) := \frac{\gamma - S_d(z)}{\gamma + S_d(z)} \tag{5}$$

and seek a pair (Ψ, Q) in $\mathcal{D}$ so that $\frac{\Psi}{Q}$ is "close to" Φ_d. Then in step (ii), we use this particular choice for Ψ and compute the positive-real interpolant F which minimizes (4), always subject to the relevant interpolation constraints. The choice of Ψ represents a compromise on the desired frequency characteristic for F, and therefore S, that permits a bound on the McMillan degree as indicated above.

III. Weight Selection Via Quasi-Convex Optimization

We begin with a (possibly) high-order $S_d(e^{j\theta})$ which has the desired shape but may not necessarily satisfy the required interpolation conditions. The corresponding F_d and Φ_d are constructed via (5). As indicated earlier, we seek a pair (Ψ, Q) such that $\Psi(e^{j\theta})/Q(e^{j\theta})$ approximates $\Phi_d(e^{j\theta})$, e.g.

$$(\Psi, Q) = \underset{\Psi, Q \in \mathcal{D}}{\text{argmin}} \left\| \frac{\Psi(e^{j\theta})}{Q(e^{j\theta})} - \Phi_d(e^{j\theta}) \right\|_\infty. \tag{6}$$

This is a non-convex problem which, interestingly, can be solved exactly by turning it into a quasi-convex one as follows. Write

$$\Psi(e^{j\theta}) = \frac{\psi(\theta)}{|\tau(e^{j\theta})|^2}, \quad Q(e^{j\theta}) = \frac{q(\theta)}{|\tau(e^{j\theta})|^2}$$

for positive trigonometric polynomials

$$\psi(\theta) := b_0 + b_1 \cos(\theta) + \cdots + b_n \cos(n\theta), \text{ and}$$
$$q(\theta) := 1 + a_1 \cos(\theta) + \cdots + a_n \cos(n\theta)$$

and note that (6) is equivalent to

$$\min\{\delta : | \psi(\theta) - q(\theta)\Phi_d(\theta) | < \delta q(\theta), \psi(\theta) > 0,\ q(\theta) > 0,\ \forall \theta \in [0, \pi]\}. \tag{7}$$

Quasi-convexity of (7) is due to the fact that the positivity of $\psi(\theta)$ and $q(\theta)$ defines intersection of infinitely many half-spaces while the first inequality in (7) leads to more intersection of half-spaces [15].

Solution of (7) will be obtained using a localization/cutting-plane method, and in particular, the ellipsoid algorithm [16]. We now explain how to deal with the constraint $\psi(\theta) > 0$, and the same procedure applies almost verbatim to the other constraints. Briefly, continuity of $\psi(\theta)$ implies that it cannot be both positive and negative in $[0, \pi]$ without crossing zero at some point, i.e., $\exists\, \theta_0 \in [0, \pi] : \psi(\theta_0) = 0$. Therefore, solutions of $\psi(\theta) = 0$ generate positivity cuts for the ellipsoid algorithm. These solutions can readily be obtained by finding roots of

$$\frac{b_n}{2} z^{2n} + \frac{b_{n-1}}{2} z^{2n-1} + \cdots + \frac{b_1}{2} z^{n+1} + b_0 z^n + \frac{b_1}{2} z^{n-1} + \cdots + \frac{b_{n-1}}{2} z + \frac{b_n}{2} = 0$$

with modulus one (i.e., $|z| = 1$). An alternative method to solve (7) is to express all constraints of the problem as linear matrix inequalities and then use bisection on δ. The resulting Ψ will then be used as the weighting function in step (ii) of the loop-shaping algorithm.

IV. Case Studies in Sensitivity Shaping

We now revisit two case studies on loop-shaping. The first one is in [7], [17] and is based on a continuous-time plant with non-minimum phase zeros. For convenience of notation, we convert this problem into the discrete-time domain. The second is in [7] and deals with mixed sensitivity reduction for a discrete-time plant. Compared to the standard H_∞-design techniques (e.g., [17]), the approach we present here, as well as Nagamune's approach in [7], [8] and [11], all give rise to controllers with lower McMillan degree (see also [18] and [19] for additional examples).

1) Example 1: (from [7], [17]): Consider the continuous-time plant

$$\hat{P}(s) = \frac{(s-1)(s-2)}{(s+1)(s^2+s+1)}$$

and the following specifications

$$\begin{cases} |\hat{S}(j\omega)| \le 0.1, & 0 \le \omega < 0.01 \left(\frac{\text{rad}}{\text{s}}\right) \\ \|\hat{S}(j\omega)\|_\infty < 1.3 \end{cases} \tag{8}$$

for the closed-loop sensitivity function. Since we mostly work in the z-domain, we use "ˆ" to denote functions in the s- domain.

In the s-domain the system has three non-minimum phase zeros at $s = 1, 2$ and infinity. Therefore, for internal stability [13] the sensitivity function must satisfy

$$\hat{S}(1) = \hat{S}(2) = \hat{S}(\infty) = 1.$$

We translate these to the z-domain using a Möbius transformation. The interpolation constraints on $S(z = \frac{1+s}{1-s}) = \hat{S}(s)$ become

$$S(\infty) = S(-3) = S(-1) = 1. \tag{9}$$

Accordingly, the required specifications (8) change to

$$\begin{cases} |S(e^{j\theta})| \le 0.1, & 0 \le \theta < 0.02 \left(\frac{\text{rad}}{\text{s}}\right) \\ \|S(e^{j\theta})\|_\infty < 1.3. \end{cases} \tag{10}$$

Now, in order to avoid the boundary condition $S(-1) = 1$ and for ease of comparison, we follow Nagamune [7] and define $S_\varepsilon(z) := S(\frac{z}{1+\varepsilon})$, where ε is a small positive number-say, $\varepsilon = 0.005$. Then, the interpolation conditions (9) change to

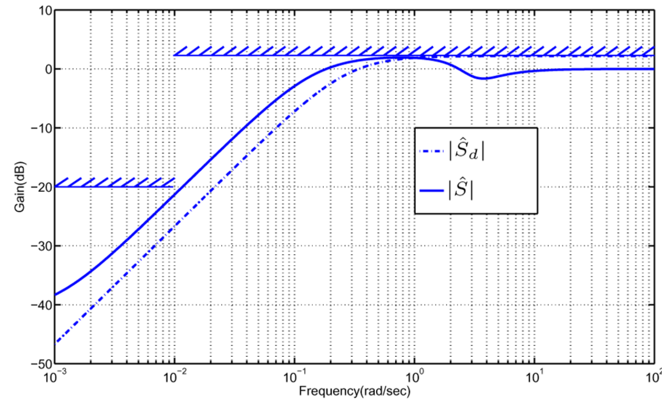

Fig. 1. Plots of $|\hat{S}_d|$, $|\hat{S}|$, and the desired specification.

$$S_\varepsilon(\infty) = S_\varepsilon(-3.015) = S_\varepsilon(-1.005) = 1. \tag{11}$$

These three conditions can be satisfied by interpolants of degree 2. However because of the tight specification at low-frequency given in (10) there is no sensitivity function of degree 2 which meets both (10) and (11) at the same time [7]. To address this issue, following [7] we introduce an extra interpolation condition $S_\varepsilon(1.002) = 0$. This condition increases the degree of interpolant by one and it also helps satisfy low-frequency requirement. In Section V we present a less-ad-hoc approach for pre-specified increase in the degree of sensitivity function. We now apply the two-step algorithm outlined earlier for sensitivity shaping. A first-order transfer function

$$S_d(z) = \frac{1.0171z - 1.0171}{z - 0.5648} \tag{12}$$

is chosen, which satisfies the desired specification (10), but not necessarily the interpolation conditions (11). This is a discrete Chebyshev filter whose band-pass gain is 1.3 (i.e., norm bound constraint on the desired sensitivity). This $S_d(z)$ generates the corresponding $\Phi_d(z)$ via (5). Then solving (7) provides a weighting function $\Psi(z)$ with zeros at $z = \{-0.4692, 0.1114 \pm 0.4011j\}$ (and at their reflections). By using this Ψ in the second step, we minimize (4) subject to the corresponding interpolation conditions. This minimization is established via a homotopy method as in [20] and the optimal F results in (after transformation back to the s-domain)

$$\hat{S}(s) = \frac{s^3 + 3.0464s^2 + 10.3143s - 0.0103}{s^3 + 3.6586s^2 + 8.4911s + 1.2020}$$

and

$$\hat{C}(s) = \frac{0.6122s^3 + 1.2244s^2 + 1.2244s + 0.6122}{s^3 + 3.0464s^2 + 10.3143s - 0.0103}$$

for the sensitivity function and controller, respectively. Fig. 1 shows $|\hat{S}_d(j\omega)|_{dB}$ (equivalent of (12) in the s-domain), $|\hat{S}(j\omega)|_{dB}$, and horizontal lines marking the desired specifications. It is evident that the obtained sensitivity satisfies all the requirements and has McMillan degree equal to 3, whereas the resulting sensitivity in [17] which is based on standard H_∞-design is of degree 4 and does not quite meet the specifications at low frequencies.

2) Example 2: (from [7]): Consider the discrete-time plant

$$P(z) = \frac{1}{z - 1.05}$$

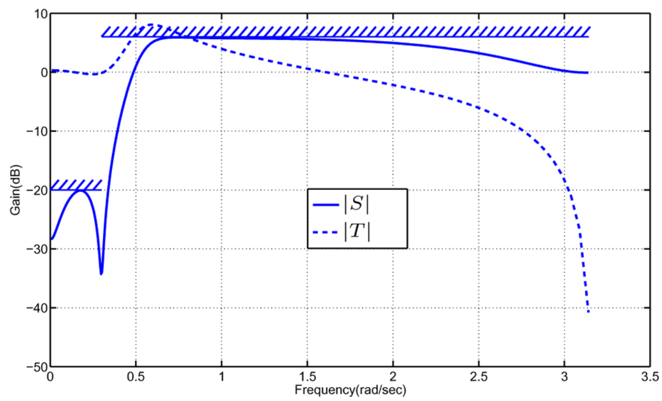

Fig. 2. Plots of $|S|$, $|T|$ and the desired specification.

and let $T := \frac{PC}{1+PC}$ denote the closed-loop complementary sensitivity function. The goal is to design a sensitivity function which gives rise to a stable closed-loop system and satisfies the following constraints:

$$\begin{cases} |S(e^{j\theta})| < 0.1 (= -20 \text{ dB}), & \theta \in [0, 0.3] \\ |T(e^{j\theta})| = |1 - S(e^{j\theta})| < 0.5 (\approx -6.02 \text{ dB}), & \theta \in [2.5, \pi] \\ |S(e^{j\theta})| < 2 (\approx 6.02 \text{ dB}), & \theta \in [0, \pi]. \end{cases} \quad (13)$$

This plant has one unstable pole at $z = 1.05$ and one non-minimum phase zero at infinity. Thus, internal stability of the closed-loop system requires that

$$S(1.05) = 0, \quad S(\infty) = 1 \quad (14)$$

and of course that S in analytic in $\mathbb{D}^c$. Furthermore, we represent all the nominal performance conditions in (13) in terms of only S or T. Since according to [7] no sensitivity function of degree 1 can satisfy the conditions (13) and (14) at the same time, again following [7], we augment the interpolation conditions by imposing

$$S(-1.01) = 1, S(1.01e^{\pm 0.3j}) = 0$$

so as to allow interpolants of a suitably high degree that can meet the performance objectives.

We continue with the choice

$$S_d(z) = \frac{0.3664z^4 - 1.4656z^3 + 2.1984z^2 - 1.4656z + 0.3664}{z^4 - 1.8408z^3 + 1.7064z^2 - 0.7234z + 0.2109}$$

which satisfies the frequency requirements, but not the interpolation conditions. This S_d has been constructed via Matlab filter design command (cheb1ord) simply on the basis of the performance bounds. We use the corresponding $\Phi_d(e^{j\theta})$ in the optimization problem (7) to obtain $\Psi = \psi/|\tau|^2$. This has roots at $\{0.6980 \pm 0.6025j, 0.1880 \pm 0.4894j\}$. Using this choice of Ψ we minimize (4) subject to the relevant interpolation conditions to obtain $F(z)$. This minimizer, via (3), gives

$$S(z) = \frac{z^4 - 2.3426z^3 + 1.1478z^2 + 0.8699z - 0.6825}{z^4 - 1.5613z^3 + 0.6973z^2 + 0.1770z - 0.1166}.$$

This is a fourth-order sensitivity function and corresponds to the third-order controller

$$C(z) = \frac{0.7813z^3 - 0.4506z^2 - 0.6929z + 0.5658}{z^3 - 1.2926z^2 - 0.2094z + 0.6500}.$$

Fig. 2 shows that the resulting sensitivity and complementary sensitivity functions satisfy all design requirements.

Using a more classical H_∞-design technique for this example one could seek

$$\inf_C \left\| \begin{bmatrix} W_1 S \\ W_2 T \end{bmatrix} \right\|_\infty$$

subject to the internal stability conditions, where W_1 and W_2 are appropriate weights. This formalism which has been used in [7] does not lead to a satisfactory result.

V. Double Weighted Entropy Functional

Following [6], we now explore a more versatile functional which again allows a bound on the degree of interpolants obtained via optimization. An added weight W can be introduced in (2) as follows:

$$\underset{S}{\operatorname{argmax}} \int_{-\pi}^{\pi} \Psi(e^{j\theta}) \log(\gamma^2 - |W(e^{j\theta})S(e^{j\theta})|^2) d\theta \quad (15)$$

and the optimization carried out subject to the usual interpolation constraints. Both weights Ψ and W in (15) play the role of "tuning parameters" for control design [6]. Incorporating the extra weight $W(z)$ leads to an increase in the order of $S(z)$. In cases where the constraints in the sensitivity shaping problem are too stringent, the resulting increase in the degree of $S(z)$ may be necessary, and W provides a convenient tuning parameter. We demonstrate the efficacy of this formalism by reworking Example 1.

1) Example 1 (Continued): The number of interpolation conditions for internal stability of the closed-loop system is $n = 3$. Hence, the natural generic degree of interpolants is $n - 1 = 2$, which is not sufficient as indicated earlier for meeting the performance objectives. Here, instead of following [7] in imposing added ad-hoc interpolation constraints, we explore the flexibility afforded by a choice of a (stably invertible) weighting function W in (15). More specifically, we choose

$$W(z) = \frac{1.2z - 0.75}{z - 0.99}.$$

This is a first-order low-pass filter, consistent with the high-pass characteristic of the desired sensitivity function. Incorporating this $W(z)$, dictates that $W(s)S_\varepsilon(s)$ satisfies

$$\begin{aligned} W(\infty)S_\varepsilon(\infty) &= 1.2, \\ W(-3.015)S_\varepsilon(-3.015) &= 1.1281, \\ W(-1.005)S_\varepsilon(-1.005) &= 1.0556. \end{aligned}$$

We now repeat the procedure of the previous section starting with $S_d(z)$ in (12). At the end, the minimizer needs to be scaled by $W^{-1}(z)$. The final result in the s-domain is

$$\hat{S}(s) = \frac{s^3 + 2.2394s^2 + 22.2859s + 0.1119}{s^3 + 3.4884s^2 + 18.5675s + 2.5842}$$

for the sensitivity function and

$$\hat{C}(s) = \frac{1.2490s^3 + 2.4980s^2 + 2.4980s + 1.2490}{s^3 + 2.2394s^2 + 22.2859s + 0.1119}$$

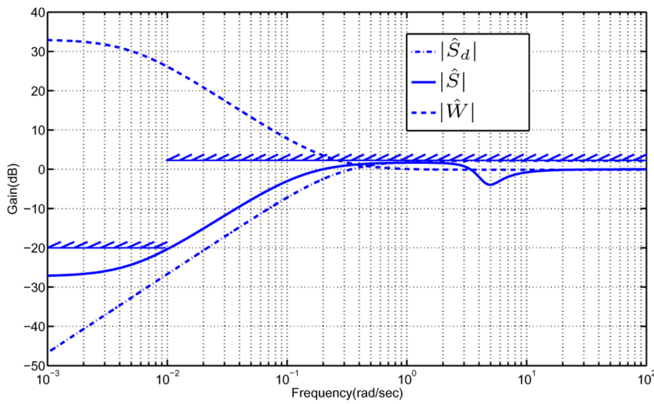

Fig. 3. Plots of $|\hat{S}_d|$, $|\hat{S}|$, $|\hat{W}|$, and the desired specification.

for the corresponding controller. Fig. 3 shows the modulus of $\hat{S}_d(j\omega)$, $\hat{W}(j\omega)$ (continuous equivalent of $W(z)$), as well as the modulus of $\hat{S}(j\omega)$ in dB over frequency. It is seen that all desired requirements are met. Furthermore, comparison of the plots in Figs. 1 and 3 reveal the role of weighting function $W(z)$ in sensitivity shaping. In the latter case the modulus of the resulting sensitivity is closer to the original step-like specification.

VI. Conclusion

We considered issues of weight selection for control synthesis via an approach which is based on minimization of weighted entropy-like functionals. The approach originated in [4] (see also [3], [5], and [6]) and was introduced to handle the McMillan degree of analytic interpolants—as these interpolants represent closed-loop operators in control synthesis problems. For additional exposition of the approach and comparison with an alternative viewpoint due to Gahinet, Apkarian, Skelton, Grigoriades, and Iwasaki we refer to [6] (see also [21]). In the present paper, we focused on the viewpoint in the work by Nagamune, Blomqvist, and others [7]–[12] and cast weight selection for control synthesis as a quasi-convex problem.

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