EE5583: Homework . 10/01/2013

Total points: 37. Due date: 10/10/2013 (before class starts)

(1) Linear codes.
   (a) Suppose, the generator matrix of a $q$-ary $[n, k, d]$ linear code is given
       by

       $$G = [I_k \mid A],$$

       where $I_k$ is a $k \times k$ identity matrix and $A$ is an $k \times n - k$ matrix. Provide a
       parity check matrix for this code (remember the code is $q$-ary). Justify. 3
   (b) Consider the parity check matrix of the code $C$ below:

       $$\begin{pmatrix}
       0 & 1 & 1 & 0 & 1 & 1 \\
       0 & 1 & 0 & 1 & 1 & 0 \\
       1 & 1 & 1 & 1 & 0 & 0
       \end{pmatrix}$$

       Provide the parameters $[n, k, d]$ of the code $C$. Is this code a perfect code? 3
   (c) Draw the standard array of the code $C$ above. Draw the syndrome table of it. 3+2
   (d) Assume you receive the vector $(111000)$. Decode it for the code $C$
       with the help of the above syndrome table. 3

(2) Finite fields.
   (a) Show that $f(x) \equiv x^4 + x^3 + 1$ is an irreducible polynomial over $\mathbb{F}_2$. 2
   (b) Assuming the root of $f(x)$ to be $\alpha \in \mathbb{F}_{2^4}$, construct the field $\mathbb{F}_{16}$
       as an algebraic extension of $\mathbb{F}_2$. Namely, provide the table that shows all the
       elements of the field in exponential, polynomial and vector forms. 4
   (c) Find all the cyclotomic cosets and the corresponding minimal polynomials of the field you have constructed above. 4

(3) BCH codes.
   (a) Write the binary parity check matrix of a 2-error correcting binary
       BCH code of length 15 with the help of above problem. 3
   (b) Assume, you receive the vector 111010001000011 as a result of trans-
       mission of a 2-error correcting binary BCH codeword (as above). Decode
       it. 4
   (c) If the received word is 110000000000000 then what was transmitted? 1
   (d) In the above code, find the location of the errors if the syndrome is
       10010110. 2

(4) Reed Solomon Codes.
   Design a 5-ary Reed Solomon code of length 4 and dimension 2. That is,
   write all the codewords in a table. 4