A New Performance Evaluation Technique for Iteratively Decoded Magnetic Recording Systems

Naveen Mysore[1], Jan Bajcsy[1], Mehmet Akçakaya[1], and Hisashi Kobayashi[2]

[1]Department of Electrical and Computer Engineering, McGill University, Montreal, Quebec H3A 2A7, Canada
[2]Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA

This paper proposes a performance evaluation technique for iteratively (turbo) equalized magnetic recoding systems. The technique decomposes the bit-error rate (BER) function into an integral of two functions. The first function has a closed form expression and is determined by the characteristics of the noise. The second function depends on the parameters of the concatenated system and can be approximated using an interpolation between sample points that rely on a small number of simulations. We present numerical and simulated BER results for coded PR4 and EPR4 channels with turbo equalization. In the "waterfall region" of the BER curves, the calculated and simulated results are within about 0.1 dB.

***Index Terms*—Iterative equalization, magnetic recording systems, partial response channels, performance evaluation.**

I. Introduction

CODING AND advanced signal processing in data storage systems are used to achieve significant increases in linear recording density. As a result, turbo codes and/or iterative equalization have been considered for magnetic recording channels [1]–[4]. Monte Carlo simulations are often used to determine the bit-error rate (BER) performance for different values of signal-to-noise ratio (SNR). In contrast, analytical or approximate performance evaluation techniques can provide significant savings in terms of simulation time and can quantify the impact of various code parameters on the BER.

Performance evaluation of turbo codes and iteratively equalized magnetic channels is difficult, especially in the waterfall region (at low and moderate SNRs) because of the interleaver. The union bound technique provides good BER approximations in the high SNR region but does not converge for moderate and low SNRs [5]–[7]. Density evolution methods [8] and EXIT charts [9] allow prediction of the start of the waterfall region but do not provide BER versus SNR curves.

In this paper, we consider a serially concatenated coded system, where the outer code is a convolutional code and the inner code is a precoded partial response (PR) channel as shown in Fig. 1. We propose a performance evaluation method that approximates the BER in the waterfall region. The method decomposes the BER function into an integral of two functions, one that has a closed form expression and the other which is a property of the code and can be approximated from a small number of samples. Section II describes the setup of the coded digital recording system and presents the proposed technique. Numerical and simulation results are presented in Section III, and we conclude in Section IV.

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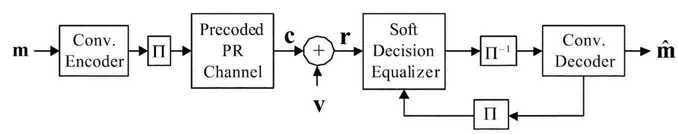

Fig. 1. Schematic block diagram of the considered coded partial response system.

II. Proposed Performance Evaluation Technique

Consider a convolutionally coded partial response system depicted in Fig. 1. Let vector $\mathbf{c}$ (of length n) be the output of the precoded PR channel. It can be expressed as a function of the k bit message sequence $\mathbf{m}$

$$\mathbf{c} = g_2(g_1(\mathbf{m})) \tag{1}$$

where g_1 represents the outer encoder including the interleaver Π and g_2 represents the precoded PR channel. The sequence $\mathbf{c}$ is corrupted with additive Gaussian noise and the output of the channel is given by

$$\mathbf{r} = \mathbf{c} + \mathbf{v}$$

where the elements of $\mathbf{v}$ are independent and identically distributed (i.i.d.) zero mean Gaussian random variables with variance σ^2. The read-back device performs a fixed number of iterations when extrinsic information on the convolutionally coded bits is exchanged between the soft decision equalizer and the convolutional decoder.

We express the BER at the output of the iterative equalizer as

$$\mathrm{BER}(\sigma) = \int_0^\infty f(u) p_\sigma(u) \mathrm{d}u \tag{2}$$

where $p_\sigma(u)$ is a chi density function with n degrees of freedom and is given by

$$p_\sigma(u) = \frac{u^{n-1} e^{-u^2/2\sigma^2}}{2^{(n-2)/2}\sigma^n \Gamma(n/2)}. \tag{3}$$

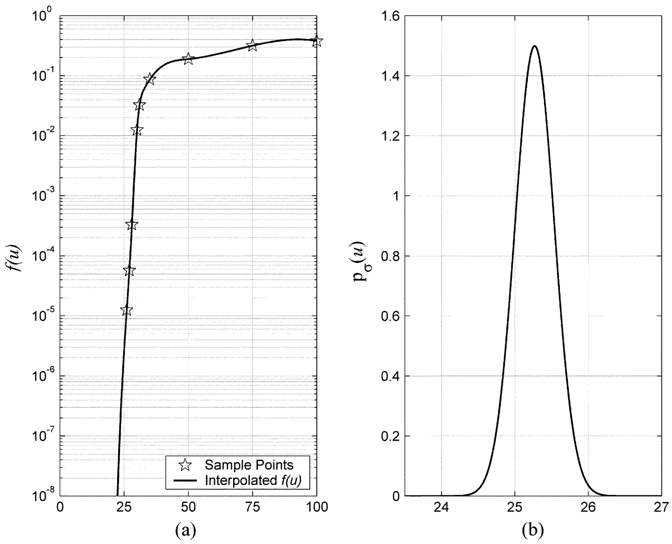

Fig. 2. Decomposition of the BER(σ) function into (a) cubic spline interpolation of the $f(u)$ curve for the PR4 channel and (b) chi density function $p_\sigma(u)$ for $\sigma = 0.38$ ($\mathrm{SNR} = 9$ dB) and $n = 4509$.

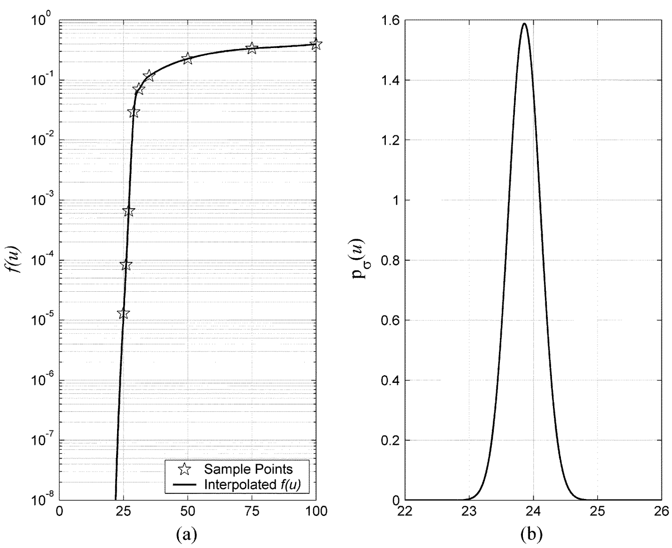

Fig. 3. Decomposition of the BER(σ) function into (a) cubic spline interpolation of the $f(u)$ curve for the EPR4 channel and (b) chi density function $p_\sigma(u)$ for $\sigma = 0.35$ ($\mathrm{SNR} = 9.5$ dB) and $n = 4511$.

Function $f(u)$ captures the properties of the code and represents the fraction of erroneously decoded message bits when $\|\mathbf{v}\| = u$. Hence, through this decomposition, e.g., shown in Figs. 2 and 3, we attempt to separate the characteristics of the noise from the structure of the code represented by $f(u)$ in the BER expression. If $f(u)$ is (approximately) known, we can numerically evaluate the integral in (2) (using Riemann sums).

For a given encoder–decoder pair, $f(u)$ depends on the parameters of the code. In this paper, we approximate $f(u)$ by interpolating between a small number of its sample points. These sample points can be determined by simulating the code (for a few message frames) in noise $\boldsymbol{v}$ that is uniformly distributed on

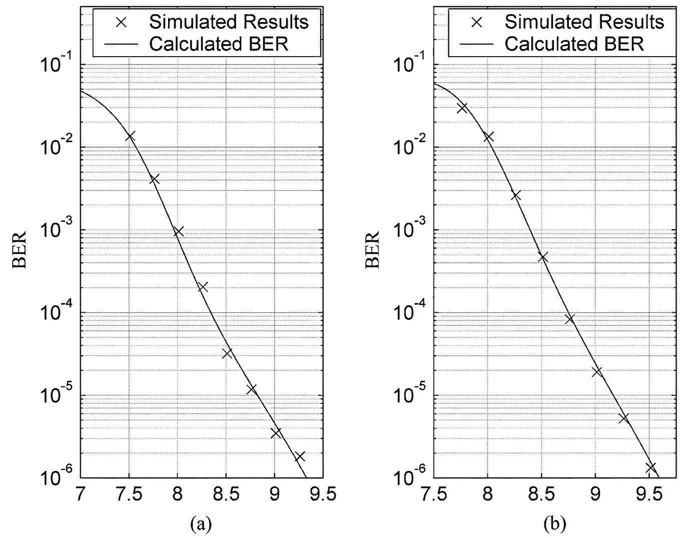

Fig. 4. Comparison of the simulated and calculated BER performance versus SNR for a rate 8/9 convolutional code after ten iterations on (a) PR4 channel and (b) EPR4 channel.

an n–dimensional sphere of radius u, which is centered at the coordinate origin. Such noise can be generated as

$$\boldsymbol{v} = \sqrt{u}\frac{\boldsymbol{\zeta}}{\|\boldsymbol{\zeta}\|} \tag{4}$$

where $\boldsymbol{\zeta}$ is a vector whose elements are i.i.d. zero mean Gaussian random variables with unit variance [11].

III. Numerical and Simulation Results

We considered a message block size of 4000 bits and a rate 8/9 outer convolutional code based on puncturing a rate 1/2 recursive systematic convolutional code with feedback and feedforward polynomials given by $1 \oplus D \oplus D^4$ and $1 \oplus D \oplus D^3 \oplus D^4$, respectively. The parity bit streams are punctured so that in a block of eight parity bits, we only keep the first one. The partial response channel is either the $h_{\mathrm{PR4}}(D) = 1 - D^2$ or $h_{\mathrm{EPR4}}(D) = (1-D)(1+D)^2$, and the corresponding precoders are $1/(1 \oplus D)$ and $1/(1 \oplus D^2)$. The SNR is defined as $10\log_{10}(\|h_{\mathrm{(E)PR4}}\|^2/(R_c\sigma^2))$, where $R_c = 8/9$ is the overall code rate, $\|h_{\mathrm{PR4}}\|^2 = 2$ and $\|h_{\mathrm{EPR4}}\|^2 = 4$. We use a random interleaver and perform ten decoding iterations where the equalizer and the decoder use the Bahl–Cocke–Jelinek–Raviv algorithm [10].

We first consider the coded PR4 system. Fig. 2(a) shows the sample points of the function $f(u)$ corresponding to the overall coded system that was described in Section II. Each sample point of $f(u)$ was generated by simulating the coded system for a maximum of 300 message frames with noise that is uniformly distributed on an n–dimensional sphere as described in (4). We used $\sigma = 0.4$ for the likelihood calculations, and the time required to generate these points was about 3 h on a single PIV 2.4 GHz computer. Similarly, Fig. 3(a) shows the sample points of the function $f(u)$ for the coded EPR4 system.

We interpolated $f(u)$ using the cubic spline technique. Since iterative turbo decoders are known to perform close to the max-

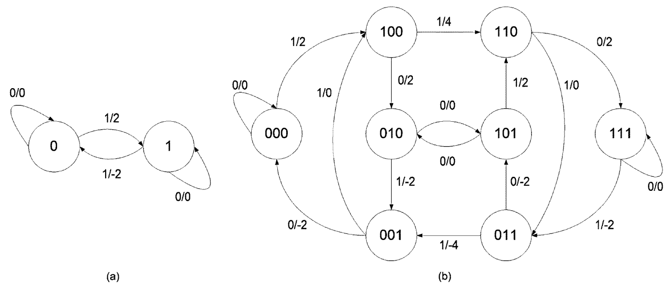

Fig. 5. State diagrams for (a) precoded PR4 channel and (b) precoded EPR4 channel.

imum likelihood decoder, we set $f(d_{\min}/2)$ to 10^{-300} for the interpolation purposes. We observe that the $f(u)$ curve consists of three different regions. Region 1 corresponds to large values of u and could be approximated by a straight line from 0.5 to 0.1. Region 2 corresponds to error values between 0.1 to 0.01, where the location of the bend in the curve is determined by the characteristics of the code (e.g., interleaver length in the overall coded system). Finally, region 3 corresponds to the rapid decay from 0.01 to 0 (which occurs at approximately $d_{\min}/2$). The last region is especially sensitive to interpolation and determines the slope of the BER curve at high SNR values.

Finally, the simulated BER performance for the PR4 and EPR4 channels, after ten iterations, is shown in Fig. 4(a) and (b). The proposed evaluation technique provides a very good match in the waterfall region of the BER curve as it is within about 0.1 dB of the simulated performance.

IV. Conclusion

We have proposed a novel performance evaluation technique for iteratively equalized magnetic recording systems. The calculated BERs in the waterfall region illustrate that the proposed method approximates the simulated BER results to within about 0.1 dB for the considered coded systems. Our future work extends the proposed technique to include evaluation of the error floor and investigation of higher order PR channels.

Appendix
Determining the Minimum Euclidean Distance for the Considered Coded Recording System

In order to accurately interpolate the $f(u)$ function, we need to determine the minimum Euclidean distance $d_{\min}$ of the overall concatenated code. A direct comparison to the all-zero sequence may not necessarily yield $d_{\min}$ as the inner encoder (PR4/EPR4 channel) is a nonlinear code. Hence, we describe a method to determine $d_{\min}$ for the considered coded systems.

Consider the state diagram of the precoded PR4 trellis shown in Fig. 5(a) and assume we start in state zero. Two ones in the codeword from the outer encoder cause the PR4 trellis to return to the zero state, resulting in a sequence that has the smallest Euclidean distance, $d_{\min} = 2\sqrt{2}$. Given the puncturing pattern of the convolutional code, we determined that a message sequence with ones in the 8th and 23rd positions (all other positions are zeros) produces a codeword with Hamming weight of 2.

The state diagram of the precoded EPR4 channel is shown in Fig. 5(b). The smallest Euclidean distance is $d_{\min} = 4$, which corresponds to an input sequence of [1, 0, 1, 0] with state transitions: 000 to 100 to 010 to 001 to 000. Furthermore, we observe that an odd number of zeros between the two ones does not change the minimum Euclidean distance. Hence, the codeword from the outer encoder must have a Hamming weight of 2. Using a random interleaver, there exists with high probability a message sequence of Hamming weight 2 corresponding to an interleaved codeword (from the outer encoder) of weight 2, in which the two ones are separated by an odd number of zeros.

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