Application of Symbolic Analysis to Power and Ground Interconnect Optimization

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OUTLINE

- **Motivation**
- **Specific Power and Ground Problems and Solutions**
- **Symbolic Analysis Role**
- **Modeling of Power and Ground Buses**
- **Admittance and Current Estimation Techniques**
- **Moment Computations**
- **Time Domain Transformation**
- **Preliminary Experimental Results**
- **Conclusions**
- **Future Work**

MOTIVATION

- **High-speed circuits ->Gigahertz clock rates -> Increased switching activity**
- **Feature sizes to deep submicron levels**
- **Vdd levels going down to conserve power**
- **Power and Ground buses cannot be assumed to be perfect conductors**
- **P&G buses must be designed carefully to ensure that supply voltage levels are maintained at appropriate levels.**
- **Better CAD techniques needed for estimation of voltage drops in P&G buses for functional and physical reliability in integrated circuits.**

SPECIFIC P&G PROBLEMS

- **Functional and Physical reliability**
  - **Increased resistivity in P&G distribution network due to smaller feature sizes**
  - **Reduces the current carrying capabilities of P&G networks**
  - **High current transients in P&G buses**
  - **Voltage drops across the P&G networks**
  - **Can lead to incorrect logic operation**
  - **and/or reduction in switching speeds**
  - **Lifetime reductions and complete failure due to electromigration**
  - **Also contributes to an increase in dynamic power dissipation**

PROBLEM SOLUTIONS

- **Accurate estimates of voltage drops and current densities in P&G buses**
  - **Use circuit level analog simulators (like SPICE) for current and voltage estimations**
  - **The main problems are the Time and Computational cost due to size of P&G networks**
- **Faster simulation times achieved by using**
  - **custom tools targeted towards solving this specific problem**
  - **simplified device models**
  - **make use of tree or mesh structures of P&G networks**
  - **find reduced order approximation of the transfer functions**
  - **problems include loss of accuracy and algorithm stability**
- **All solutions proposed so far are numeric in nature**

SOME EXISTING ALGORITHMS

- **J. E. Hall, D. E. Hocevar, P. Yang, and M. J. McGraw, "SPIDER-- a CAD system for modeling VLSI metallization patterns," IEEE Transactions on CAD, Nov. 1987.**
  - **Estimates Median-Time-to-Failure for each section of a metal bus**
  - **Makes use of SPICE runs**
  - **User must provide estimates of current sources**
  -
- **S. Choudhury and M. A. Breuer, "Optimum design of IC power/ground nets subject to reliability constraints," IEEE Transactions on CAD, July 1988.**
  - **More general technique**
  - **Takes into consideration electromigration and voltage drops from leaf to node**
  - **Transform the problem into an unconstrained optimization problem**

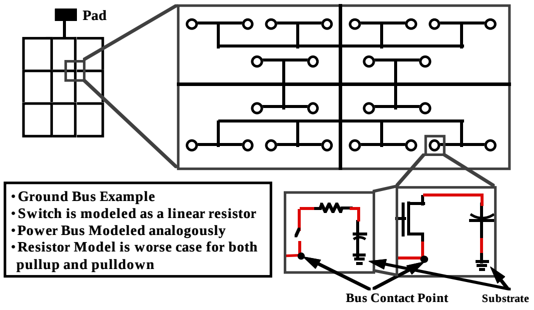
SOME EXISTING ALGORITHMS (cont ...)

- R. Dutta and M. Marek-Sadowska, “Automatic sizing of power/ground P/G networks in VLSI,” Proceedings DAC, pp. 783-786, 1989.
  - Modeled as a nonlinear programming problem
  - Cost function is the sum of the segments areas
  - Constraints are wire width
  - Convergance is not necessary, meaningful results are achieved at any time the iterations are terminated
- T. Mitsuhashi and E. S. Kuh, “Power and ground network topology optimization for cell-based VLSIs,” in Proceedings of the ACM/IEEE Design Automation Conference, pp. 524-529, 1992.

THIS ALGORITHM & SYMBOLIC ANALYSIS

- Motivation: Use of SPICE is computationally expensive and other current methods use simplified current estimation techniques but are limited to performing DC analysis
- Sapatnekar and Shah presented a method (1996) that uses frequency domain techniques:
  - not limited to a resistive or any one class of networks
  - can perform transient analysis (with simplified models)
  - near linear algorithm (in the size of the network)
  - results within 1% of SPICE
- The algorithm is very suitable for symbolic analysis because of its linear models, frequency domain techniques and repetitive evaluations

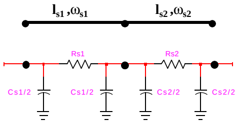
P&G BUS MODEL



- Ground Bus Example
- Switch is modeled as a linear resistor
- Power Bus Modeled analogously
- Resistor Model is worse case for both pullup and pulldown

SEGMENTED RC MODEL of P&G BUS

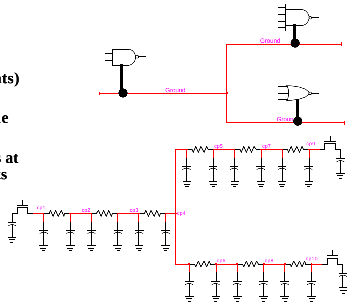
- Modeled as a set of segments connected in the form of a tree
- Each segment is modeled using a lumped RC
- π model
- $R_i = \rho l_i / \omega_i$
- $C_i = \beta l_i \omega_i$



- where l_i is the length of the ith segment, ω_i is the width of the ith segment, ρ is the sheet resistance and β is the wire capacitance per unit area

EXAMPLE MODEL FOR GROUND NET

- The P&G network is modeled as a tree T having m+1 nodes
- m edges (wire segments)
- one or more switches connected to each node
- Assume switches undergo state changes at predefined time points



SYMBOLIC ANALYSIS APPLICATION

- The algorithm computes the time domain voltage waveform at each node for every time instance
- Two steps, which are repeated for every time point at which one or more switches changed states.
  1) Solve a set of equations of the form

  $(G + sC)\ V(s) = J(s)$

  using an efficient path tracing algorithm, to obtain V(s).

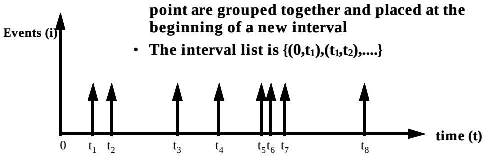
  2) Given V(s) at every node, compute the time domain response by approximating V(s) with a rational Padé approximation.

Both steps are perfectly suitable for symbolic analysis:

Linear, fixed topology network that requires many computational iterations

ALGORITHM BASIS

- Event-driven algorithm where the events specified are the time points at which one or more switches change their state.
- The events are typically generated by an event-driven simulator
- All the switches that switch at the same time point are grouped together and placed at the beginning of a new interval
- The interval list is $\{(0,t_1),(t_1,t_2),....\}$



ALGORITHM OUTLINE

- Construct interval
- Select every element in the interval list in that order
- Set the corresponding switch states
- Process the interval (compute new states)
- Repeat with the the new switch states and computed initial conditions for next interval.
- Continue until all the intervals are processed.

COMPUTATION WITHIN AN INTERVAL

- The initial conditions on the capacitors are taken into consideration
- Compute the moments of each node voltage in the P&G networks
- Generate a stable Padé approximation
- Transform to a time domain voltage waveform
- The response is used to compute the initial conditions at all the nodes at the time of the next switching event
- Repeat the process

COMPUTATION WITHIN AN INTERVAL (continued...)

- **S = Set of switches connected to the P&G bus**
- **$E = \{e_1,....,e_m\}$ is the set of all the segments in T**
- **$I = \{i_k \mid k = 1.....I_{max}\}$ is the interval list**
- **i_k is the k^{th} interval**
- **Associated with each interval i^k is a subset of switches s_k and their states p_k**
- **t_k is the time span of interval i_k**

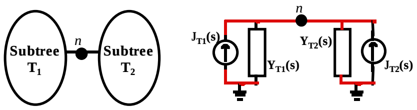
```
BEGIN ALGORITHM Vdrop()
 I = IntervalList();
 foreach $i_k$ in I
     foreach j in $s_k$ and state(j) in $p_k$
         SetSwitch(T,j,state(j))
         PropagateActivity(T,j,state(j));
     foreach n in Nodes(T)
         $f_n(t)$ = TimeDomainResponse(moments(n));
         SetInitialCondition(n,$f_n$(tk))
END ALGORITHM Vdrop()
```

MOMENT COMPUTATION BY PATH TRACING

- **Computation of moments of the voltage at any given node**
- **Recursively reduce the subtrees rooted at the given node n to an admittance $Y_{Ti}(n)$ and a current source $J_{Ti}(n)$**

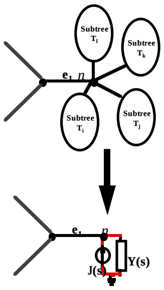
where

- **$Y_{Ti}(n)$ is the equivalent admittance of the subtree T_i, as seen from node n to the ground**
- **$J_{Ti}(n)$ is the combined effect of all the switch elements in T_i which have contributed to the current in the ground net.**



MOMENT COMPUTATION (continued...)

- **The equivalent current sources and admittances are computed for every node in linear time using a path tracing algorithm.**
- **The algorithm proceeds from the leaf-nodes, which are the contact points at the terminating segments of the ground net.**
- **A leaf-node could have multiple switches but exactly one segment connected to it.**



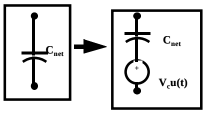
SYMBOLIC MODEL COMPUTATION

- J(n) and Y(n) can be computed symbolically
- It is the same as a symbolic implementation of Norton's theorem
- SCAPP used to generate a sequence of expressions
- A simpler symbolic implementation possible
- The equivalent current source of a single switch i is given by:

$$J^i = \frac{V_c}{s}\frac{sC_{net}}{1+R_d s C_{net}}$$

SYMBOLIC MODEL COMPUTATION

- V_c is the initial voltage on the capacitor C_{net} at the start of the given interval.
- The pole at the origin in indicates that C_{net} is modeled as



$$J^i = \frac{V_c}{s}\frac{sC_{net}}{1+R_d s C_{net}}$$

SYMBOLIC MODEL COMPUTATION (continued....)

- Using Maclaurin series polynomial approximations we have

$$J^i(n) = V_c(sC_{net} - R_d s C_{net}^2 + R_d s^2 C_{net}^3 + ...)$$

- Total admittance and equivalent current is:

$$Y^i = \frac{sC_{net}}{1+R_d s C_{net}}$$

- The expressions includes the effect of all the switches with binary operators controlling the closing and opening of the switches during the evaluation phase

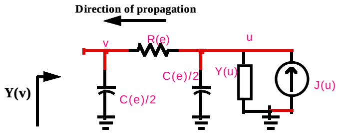
$$Y^i = sC_{net} - R_d s^2 C_{net}^2 + R_d s^3 C_{net}^3 + ...$$

$$Y(n) = \sum Y^i \forall i \in \text{ Switches at n}$$

$$J(n) = \sum J^i \forall i \in \text{ Switches at n}$$

EFFECT OF THE CURRENT SOURCES

- **The effect of the equivalent current sources is propagated to every node in the network**
- **Terminate segment e in the ground network, connecting a single contact point u to the rest of the network, rooted at v**



EQUIVALENT ADMITTANCE

- **The admittance and the equivalent current source, as seen into edge e from node v is computed as**

$$Y(v) = \frac{1}{R(e) + \{Y_0(u) + (Y_1(u) + 0.5C(e))s + \ldots + Y_{2n-1}(u)s^{2n-1}\}^{-1}} + 0.5C(e)$$

$$= \frac{Y_0(u) + (Y_1(u) + 0.5C(e))s + \ldots + Y_{2n-1}(u)s^{2n-1}}{(1 + R(e)Y_0(u)) + R(e)(Y_1(u) + 0.5C(e))s + \ldots + (R(e)Y_{2n-1}(u)s^{2n-1}} + 0.5C(e)$$

which yields

$$Y(v) = Y_0(v) + Y_1(v)s + Y_2(v)s^2 + \ldots + Y_k(v)s^k + \ldots$$

where $$Y_k(v) = \frac{Y_k(u) - \sum_{i=1}^{k} R(e)Y_{k-i}(v)Y_i(u)}{1 + R(e)Y_0(u)}$$

EQUIVALENT CURRENT SOURCE

Using Norton's theorem (shorting node v to ground) we get

$$J_k(v) = J_0(v) + J_1(v)s + J_2(v)s^2 + \ldots + J_k(v)s^k + \ldots$$

where

$$J_k(v) = \frac{J_k(u) - \sum_{i=1}^{k} R(e)J_{k-i}(v)Y_i(u)}{1 + R(e)Y_0(u)}$$

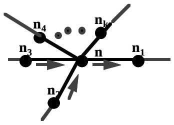
Thus, we say that the equivalent admittance and current source have been propagated from node u to v

GENERALIZING THE ALGORITHM

- **The above case only considers the switches connected to leaf-nodes**
- **In general, switches can be connected to any node**
- **In cases where a node has more then one segment and multiple switches connected, the switches are processed in a similar manner**
- **The admittance and current source can be propagated if and only if a maximum of one segment is left unprocessed at the given node.**

GENERAL NODE CASE

- **Since each node n in a P&G network is a root of a subtree T_i, the equivalent devices seen through edge e_i can be computed by**



$$J^{e_i}(n) = \sum_{j=1, j\neq i}^{k} J_{T_j}(n) \qquad Y^{e_i}(n) = \sum_{j=1, j\neq i}^{k} Y_{T_j}(n)$$

GENERAL NODE CASE (continued...)

- **Voltage moments at n:**

$$V(n) = \frac{J(n)}{Y(n)} \quad \text{where} \quad J(n) = \sum_{i=1}^{k} J_{T_i}(n), \quad Y(n) = \sum_{i=1}^{k} Y_{T_i}(n)$$

Moments of V(n):

$$V(n) = V_0(n) + V_1(n)s + V_2(n)s^2 + \ldots + V_k(n)s^k + \ldots$$

where $$V_k(n) = \frac{J_k(n) - \sum_{i=1}^{k} V_{k-i}(n) Y_i(u)}{Y_0(n)}$$

GENERAL NODE CASE (continued...)

- **The voltage moments can be computed at all nodes if equivalent Ys and Js have been propagated twice along each segment: once in each direction. Hence a linear time path tracing algorithm which traces each segment once in either direction can be used for the computation of voltage moments at all nodes.**

COMPUTING TIME DOMAIN RESPONSE

- **Use moment matching techniques**
  - **We have V(s) for each node in the network**
  - **2N moments for the node voltages have been computed**
  - **Largest order of approximation can be N, let M=N-1**
  - **Approximate V(s) with an M-zero N-pole expression**
  - **Padé Approximation**

$$\tilde{V}(s) = \frac{a'_0 + a'_1 s + a'_2 s^2 + \ldots + a'_M s^M}{1 + b'_1 s + b'_2 s^2 + \ldots + b'_N s^N}$$

TIME DOMAIN (continued ..)

- **The denominator coefficients are computed from the Hankel matrix**

$$\begin{bmatrix} V_0 & V_1 & \cdots\cdots & V_N \\ V_1 & V_2 & \cdots\cdots & V_{N+1} \\ \vdots & \vdots & & \vdots \\ \cdot & \cdot & \cdots\cdots & \cdot \\ V_N & V_{N+1} & \cdots\cdots & V_{N+M-1} \end{bmatrix} \begin{bmatrix} b'_N \\ b'_{N-1} \\ \vdots \\ \cdot \\ b'_1 \end{bmatrix} = -\begin{bmatrix} V_{N+1} \\ V_{N+2} \\ \vdots \\ \cdot \\ V_{N+M} \end{bmatrix}$$

The numerator coefficients are computed by equating powers of s

$$a'_0 = V_0$$
$$a'_1 = V_1 + b'_1 V_0$$
$$\cdot$$
$$a'_M = V_L + \sum_{i=1}^{M} b'_i V_{M-i}$$

TIME DOMAIN (cont ...)

- **Given the above symbolic coefficients, the time domain response can be found numerically by writing**

$$\tilde{V}(s) = \hat{c} + \sum_{i=1}^{N} \frac{\hat{k}_i}{s - \hat{p}_i}$$

- **which is transformed to the time domain as**

$$\tilde{v}(t) = \hat{c}\delta(t) + \sum_{i=1}^{N} \hat{k}_i e^{\hat{p}_i t}$$

PRELIMINARY EXPERIMENTAL RESULTS

- **Two randomly generated ground nets**
  - **gnet1: 400 contact points**
    total wire length=2cm
    wire width=3µ
  - **gnet2: 4000 contact points**
    total wire length=10cm
    wire width=3µ
- **Accuracy was compared with HSPICE at four selected nodes A, B, C and D**
- **Comparing peak voltage and root mean square error**
- **Sample time points were 0.01ns**
- **Maximum order of the Padé Approximations was set to 3**

PRELIMINARY EXPERIMENTAL RESULTS (continued ....)

- **gnet1: Peak and RMSE Voltages**

Node	Peak (Volts) Vdrop	Peak (Volts) HSPICE	V_{RMSE} (Volts)
A	0.350	0.342	0.00130
B	0.432	0.426	0.00150
C	0.688	0.689	0.00087
D	0.461	0.463	0.00054

- **gnet2: Peak and RMSE Voltages**

Node	Peak (Volts)	Peak (Volts)	V_{RMSE}
	Vdrop	HSPICE	(Volts)
A	0.216	0.229	0.00017
B	0.225	0.243	0.00013
C	0.419	0.419	0.00028
D	0.129	0.125	0.00039

CONCLUSIONS

- **Expand the applications of symbolic analysis to a new field**
- **Application of symbolic analysis to Power and Ground interconnect analysis**
- **Provides mechanism for reducing cost of highly iterative process**
- **More testing of the algorithm in general and the symbolic role in particular is required**
- **Expand algorithm to perform P&G bus optimization, ie. produce an optimal P&G network based on the results**