

# SIMULATION OF ELECTRIC MACHINE AND DRIVE SYSTEMS USING MATLAB AND SIMULINK

## Introduction

This package presents computer models of electric machines leading to the assessment of the dynamic performance of open- and closed-loop ac and dc drives. The *Simulink/Matlab* implementation is adopted because of its inherent integration of vectorized system representations in block diagram form, of numerical analysis methods, of graphical portrayal of time evolutions of signals combined with the simple implementation of the functionality of controllers and power electronic excitations. The development of **Simulink** models of drive assemblies is a relatively simple task consisting of combining input-output block representation of the various components making up the system. This approach provides a powerful design tool because of the ease of observing the effects of parameter modifications and of changes in system configurations and control strategies.

Under the rubric *Animations*, a series of movie clips portrays the motion of electric machines, magnetic fields, and space vectors.

**TABLE of CONTENTS**

SIGNALS & CIRCUITS  
sigckt.m

DC MOTOR DRIVES  
dcm.m

INDUCTION MACHINES  
im.m

SYNCHRONOUS MACHINES  
sm.m

STEADY-STATE CHARACTERISTICS  
script.m

ANIMATIONS  
animations.m

TIPS

INFO

REFS

WEB

**LIBRARY**

SIMULATION OF ELECTRIC MACHINE & DRIVE SYSTEMS

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machine3D

machine2D

## The approach

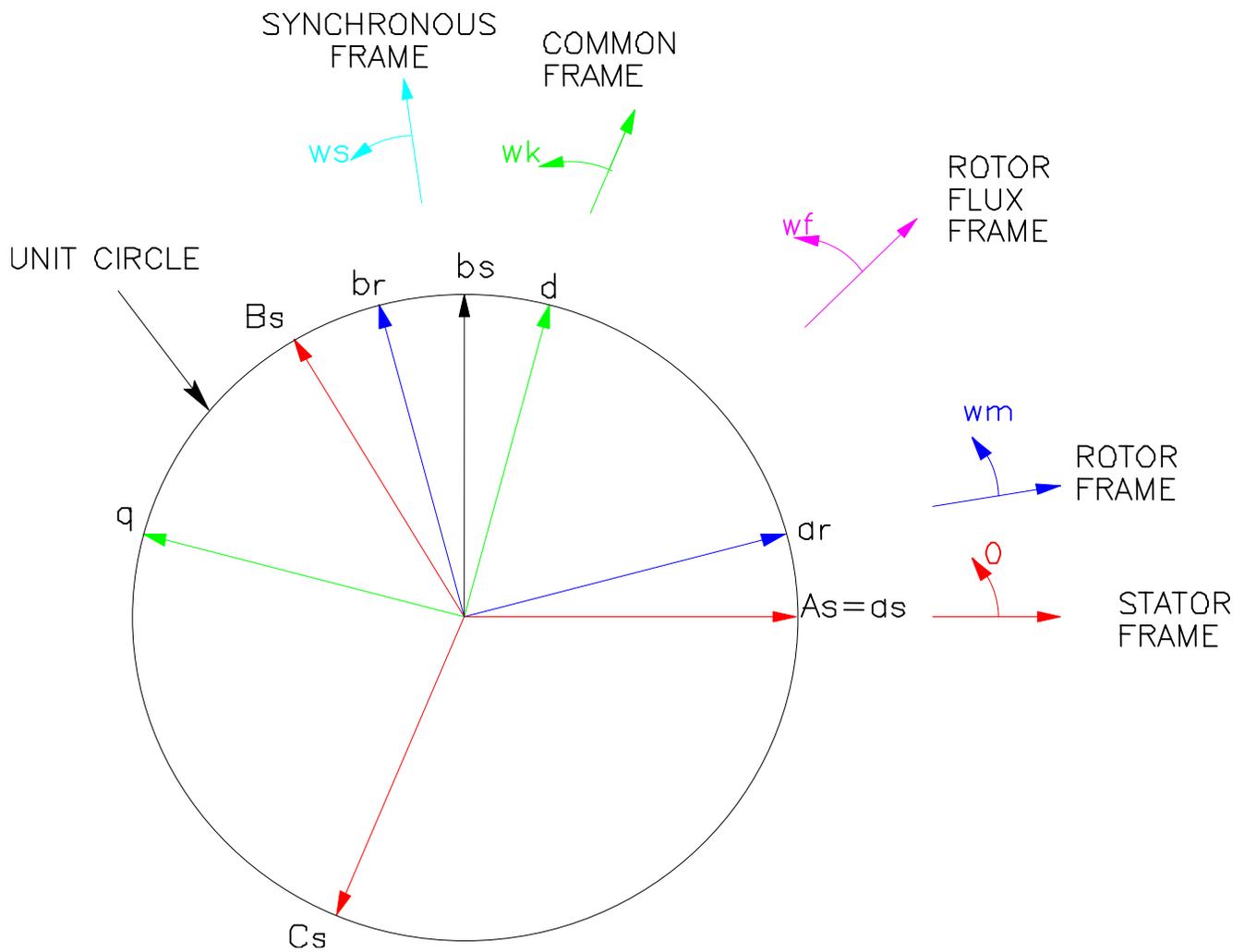
### Electric machines

The starting step in the mathematical modeling of ac machines is to describe them as coupled stator and rotor polyphase circuits in terms of so-called phase variables, namely stator currents  $i_{as}$ ,  $i_{bs}$ ,  $i_{cs}$ ; rotor currents  $i_{ar}$ ,  $i_{br}$ ,  $i_{cr}$  for an induction machine or  $i_f$ ,  $i_{kd}$ ,  $i_{kq}$  for a synchronous machine; the rotor speed  $\omega_m$ ; and the angular displacement  $\theta$  between stator and rotor windings. The magnetic coupling is expressed in terms of an inductance matrix which is a function of position  $\theta$ . The matrix expression of the machine equations are readily formulated in Matlab or Simulink language. A detailed example of this approach is given in a later section.

The next step is to transform the original stator and rotor **abc** frames of reference into a common **k** or **dq** frame in which the new variables for voltages, currents, and fluxes can be viewed as 2-D space vectors. In this common frame the inductances become constant independent of position. Figure 1 illustrates various reference frames (coordinate systems): the triplet [ $A_s$   $B_s$   $C_s$ ] denotes a three-phase system attached to the stator while the pair [ $a_s$   $b_s$ ] corresponds to an equivalent two-phase system (zero-sequence components can be ignored in Y-connected ac machines in which the neutral is normally isolated). Among possible choices of **dq** frames are the following:

- a) Stator frame where  $\omega_k = 0$
- b) Rotor frame where  $\omega_k = \omega_m$
- c) Synchronous frame associated with the frequency  $\omega_s$  (possibly time varying) of the stator excitation.
- d) Rotor flux frame in which the d-axis lines up with the direction of the rotor flux vector.

The choice of the common **dq** frame is usually dictated by the symmetry constraints imposed by the construction and excitation of the machine. With the complete symmetry encountered in a three-phase induction machine with balanced sinusoidal excitation, any one of the five frames can be used, although the synchronous frame is more convenient in as much as all signals appear as constant dc in steady state. However, certain control strategies may require the adoption of a specific frame, as is the case of vector control where the reference frame is attached to the rotor flux vector. In the presence of asymmetry, the common frame is attached to the asymmetrical member: an induction motor with unbalanced excitation or asymmetrical stator windings (the case of a capacitor motor) will be modeled in the stator frame where as a synchronous machine is represented in the rotor frame. In the common **dq** frame, the machine dynamic equations appear as differential equations with constant coefficients (independent of rotor position) and nonlinearities confined to products of variables associated with speed voltages and torque components.



**Figure 1. Reference frames in ac machine analysis**

$\omega$  denotes the rotational speed or angular frequency of a frame (in electrical rad/s) with respect to the stationary stator. The angular position is obtained by integrating speed over time, that is  $\theta = \int \omega dt$ .

## Excitations and controllers

The simulation of the inputs to the machines involves the mathematical representation of programmed time sequence of events such as the sudden application or removal of mechanical loads, the ramping of the magnitude and frequency of the applied voltages, or even the changes in parameter values (for instance, rotor resistance). Similarly the functionality of power electronics type of excitations can generally be realized in the form of simple mathematical expressions: as an example, a PWM signal can be described by the function

$\text{signum}[m - \text{tri}(t)]$  where  $m$  is the modulation factor ( $-1 \leq m \leq 1$ ) and

$\text{tri}(t) = \frac{2}{\pi} \sin^{-1}[\sin(\omega t - \gamma)]$  is a triangular waveform of unit amplitude,

frequency  $\omega$ , and phase  $\gamma$

This function is readily translated into Simulink block form.

The simulation of speed or position controllers in drive systems is achieved by using a relay block in a hysteresis type of controller, and a simple combination of gains, summers and integrator (incorporating limiters to include antiwindup features) in a PI type of controller.

## Initial conditions

Initial conditions are established by specifying a steady-state operating condition. The simplest case is encountered in the simulation of the starting of a motor for which all initial conditions are zero. In some other case, the initial conditions can simply be calculated before running the simulation: for example, under steady-state sinusoidal excitation, an induction motor running at a specified speed can be quickly analyzed in terms of a standard phasor equivalent circuit; by using phasor techniques, one can compute the corresponding load torque and initial conditions. However, in most instances, a specified operating condition can only be obtained after running the simulation for a time that will depend on the starting setting of these initial conditions. This situation will occur if, in the above case of the induction motor, torque instead of speed is specified or if the mechanical load is a nonlinear function of speed. This is the normal situation encountered with power electronic input signals: a steady-state condition is reached when an output signal waveform is repeated every switching cycle so that the values at the beginning and end of a cycle are equal. When a steady-state condition is attained, one can then save this so-called final state and use it later as the initial state in a renewed simulation which now includes the specified time sequenced input events.

## Space vector model of the induction machine

(SI units)

Electrical system equations:

$$\bar{v}_s = R_s \bar{i}_s + \frac{d\bar{\lambda}_s}{dt} + \omega_k M \bar{\lambda}_s$$

$$\bar{v}_r = R_r \bar{i}_r + \frac{d\bar{\lambda}_r}{dt} + (\omega_k - \omega_m) M \bar{\lambda}_r$$

where the space vector  $\bar{f} = \begin{bmatrix} f_d \\ f_q \end{bmatrix}$  and the  $\frac{\pi}{2}$  rotational operator  $M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Flux linkage-current relations:

$$\bar{\lambda}_s = L_s \bar{i}_s + L_m \bar{i}_r \quad \text{or} \quad \bar{i}_s = \Gamma_s \bar{\lambda}_s - \Gamma_m \bar{\lambda}_r$$

$$\bar{\lambda}_r = L_m \bar{i}_s + L_r \bar{i}_r \quad \bar{i}_r = -\Gamma_m \bar{\lambda}_s + \Gamma_r \bar{\lambda}_r$$

where  $L_s = L_m + L_{sl}$  and  $\Gamma_s = \frac{L_r}{\Delta}$   $\Gamma_r = \frac{L_s}{\Delta}$   $\Gamma_m = \frac{L_m}{\Delta}$   
 $L_r = L_m + L_{rl}$   $\Delta = L_m L_{sl} + L_m L_{rl} + L_{sl} L_{rl}$

Mechanical system equations:

$$T_e = J \frac{d\omega_{mec}}{dt} + B_m \omega_{mec} + T_L$$

where  $T_e = k(\bar{\lambda}_s \otimes \bar{i}_s) = k(M \bar{\lambda}_s \bullet \bar{i}_s) = (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})$

$$= k(\bar{i}_r \otimes \bar{\lambda}_r) = k L_m (\bar{i}_r \otimes \bar{i}_s) = k \frac{L_m}{L_r} (\bar{\lambda}_r \otimes \bar{i}_s) = k \Gamma_m (\bar{\lambda}_r \otimes \bar{\lambda}_s)$$

and  $\omega_{mec} = \frac{2}{p} \omega_m$   $k = \frac{3p}{2}$

Nomenclature:

$\bar{v}$  = voltage space vector [V]

$\bar{i}$  = current space vector [A]

$\bar{\lambda}$  = flux linkage space vector [Wb]

R = Resistance [ $\Omega$ ]

L = Inductance [H]

$\Gamma$  = Inverse inductance [ $H^{-1}$ ]

$f_0$  = Base frequency [Hz]

operators:  $\otimes \Rightarrow$  cross product

subscripts: s  $\Rightarrow$  stator r  $\Rightarrow$  rotor

$\omega_0 = 2\pi f_0$  = Base frequency [rad/s]

$\omega_k$  = speed of dq frame [rad/s]

$\omega_m$  = rotor speed [rad/s]

$T_e$  = electromagnetic torque [N.m]

$T_L$  = load torque [N.m]

J = moment of inertia [ $kg \cdot m^2$ ]

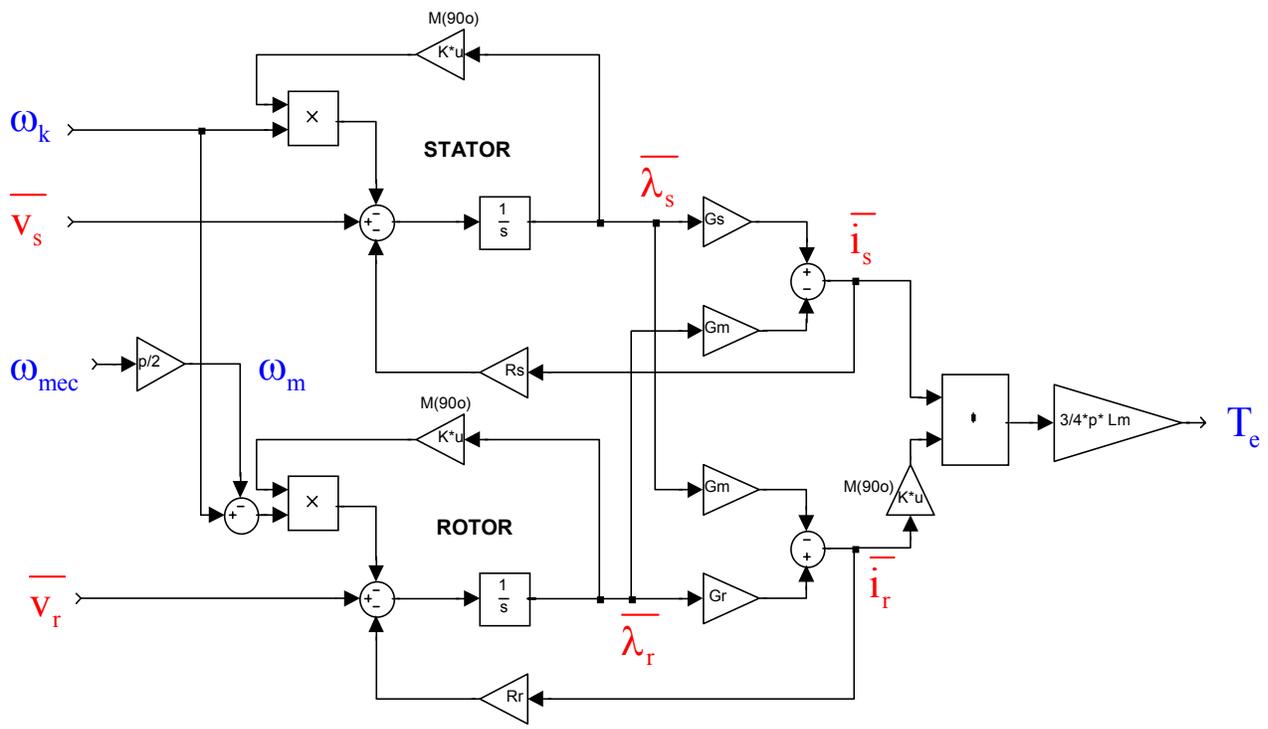
p = number of poles

$\bullet \Rightarrow$  dot product M  $\Rightarrow$  rotation

d  $\Rightarrow$  direct axis q  $\Rightarrow$  quadrature axis

## Simulink block diagram model

These mathematical equations can be represented as shown below in a block diagram form that preserves the one-to-one correspondence between the 2D space vectors of the equations and the vectorized signals (of width 2) appearing in the Simulink representation. It is significant to point out that flux linkages are selected as state variables in the simulation. Note that, apart from the speed signals (scalars) indicated in blue, all the variables are 2-element vectors shown in red.



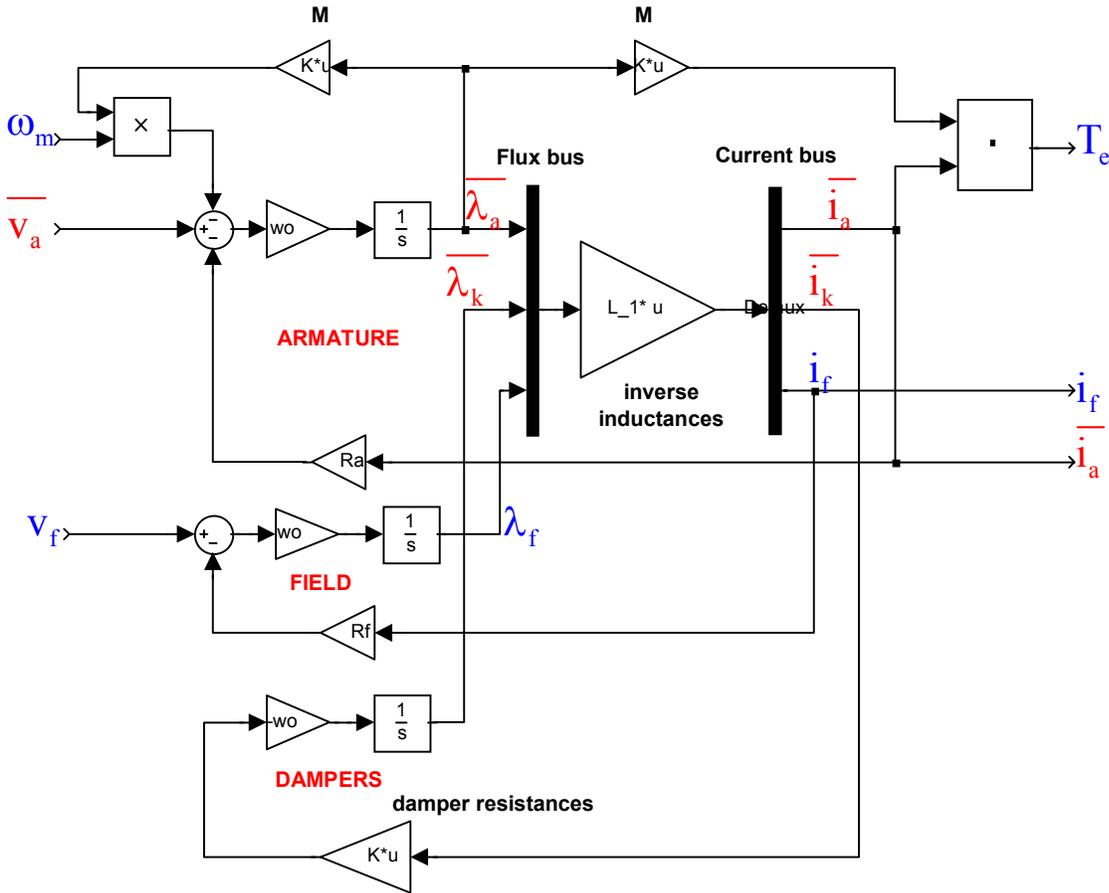
Space vector model of an induction machine

## Remark

It is important to point out at this stage the difference between the space vectors introduced here and those mostly used in the current literature (see references cited). First, a matter of notation: here a 2D space vector is represented by a column vector having 2 real elements such as  $\vec{f} = \begin{bmatrix} f_d \\ f_q \end{bmatrix}$ ; the usual representation in the complex plane is  $\underline{f} = f_d + jf_q = |\underline{f}|e^{j\theta}$  (often referred to as a complex space vector, somewhat of a misnomer since it appears as a complex scalar). While the manipulation of expressions and equations using complex quantities is perhaps easier to perform, the process masks the underlying physical and geometric understanding achievable with real vectors in a real 2D plane. A geometric vector has a meaning irrespective of the coordinate system in which it may be expressed; it can be scaled, rotated, added to or projected on another vector, or multiplied with another vector as a cross product. In mechanics, the work done by a force  $\vec{f}$  moving a body along a path  $\vec{s}$  is expressed as the dot (scalar) product  $\vec{f} \cdot \vec{s}$  and the torque exerted by this force relative to an origin specified by a radius vector  $\vec{r}$  is the cross (vector) product  $\vec{f} \otimes \vec{r}$ . In the present context, electromagnetic torque is the cross product of 2 space vectors such as  $\vec{\lambda}_s \otimes \vec{i}_s$ , where as in the complex notation it will appear as  $\text{Im}(\underline{\lambda}_s^* \underline{i}_s)$  or in terms of the dq components as  $(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})$ . Similarly, power is neatly the dot product  $\vec{v} \cdot \vec{i}$  and energy is simply  $\vec{\lambda} \cdot \vec{i}$ . Finally, for computer simulation, it becomes necessary to recast the differential equations from their complex expressions into real forms by assembling real and imaginary parts. In the case of an induction machine, the process results in four equations (2 for the stator in d and q, 2 for the rotor in d and q). These equations can then be represented within Simulink in a cumbersome scalar form; or preferably the dq components can be recombined so as to recover a vectorized model (a long way around!). These observations should not detract from the fact that direct access to the components of some of the vectors (via Demux blocks) may often be required for control purposes as is certainly the case of the vector control of an induction machine.

### Block diagram model of the synchronous machine

The armature and damper circuits are shown in vectorized form with the variables indicated in red. The scalar signals are indicated in blue.



Space vector model of the synchronous machine  
(per-unit formulation)

### **Conventions & formulations:**

Because the emphasis in this work is focused on drives, the so-called motor conventions are adopted through out. This means that electric power in and mechanical power out are considered positive for motoring action. Furthermore, the currents are assumed to be positive when directed into the input stator and rotor ports of the electric machine. Positive rotation is counter clockwise from d axis to q axis.

The models are formulated in both the SI and per-unit systems. The per-unit system uses the rating of the machine as a basis, normally horsepower output for induction machines and KVA input for synchronous machines; the base voltage is rated peak line-to-neutral voltage [Volt]; the base speed is synchronous speed at base frequency. Although both systems utilize practically the same notations, they can readily be identified by context. In particular, the time differential  $dt$  (with  $t$  in seconds) becomes  $\omega_0 dt$  in a dimensionless expression. Furthermore, such factors as  $3/2$  or  $p/2$  do not appear in a per-unit formulation.

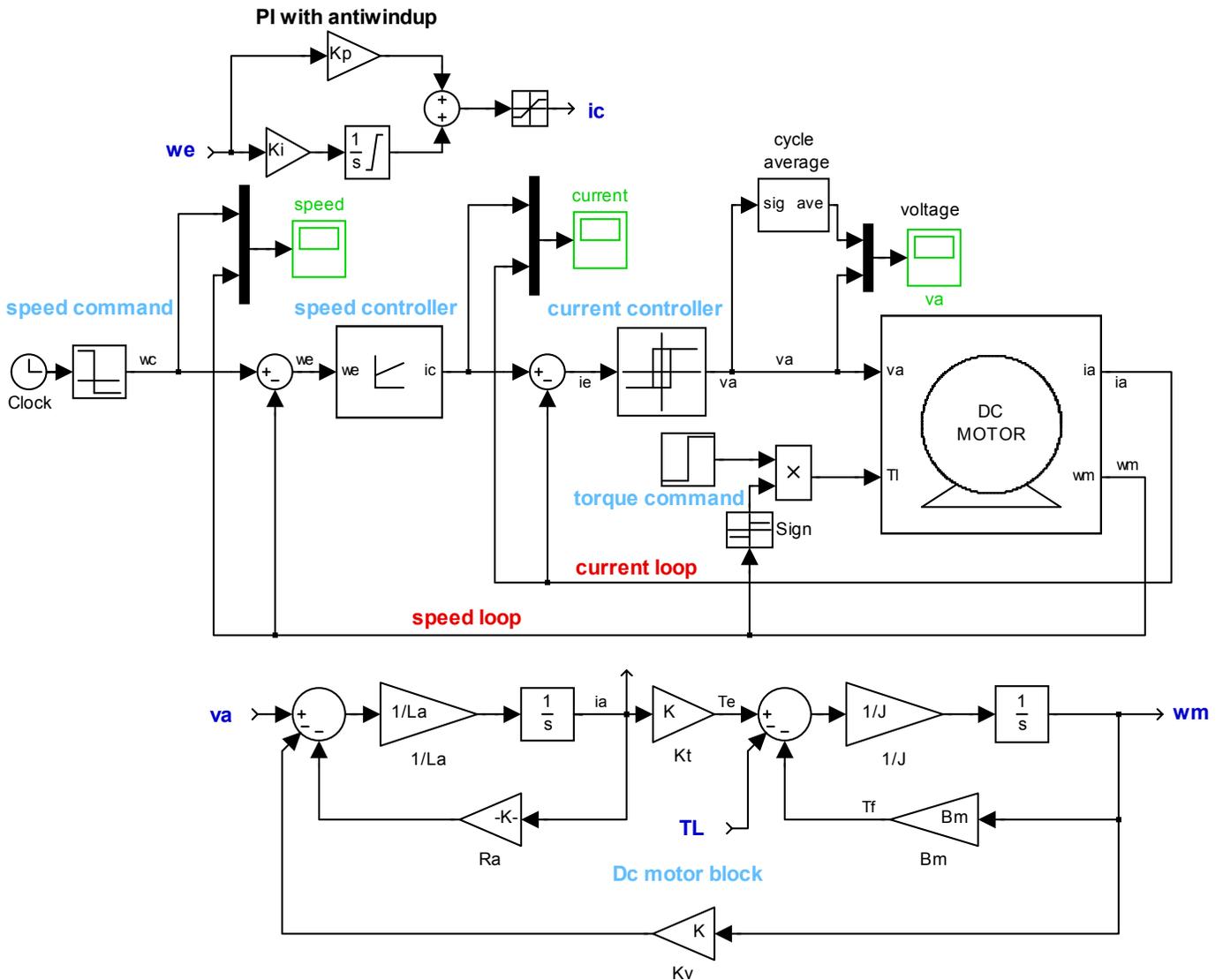
## A dc motor speed drive

The mathematical model of dc motor (permanent magnet type) can be expressed by these equations

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \quad \text{where } e_a = K\omega_m$$

$$T_e = J \frac{d\omega_m}{dt} + T_L \text{sign}(\omega_m) + B_m \omega_m \quad \text{where } T_e = K i_a$$

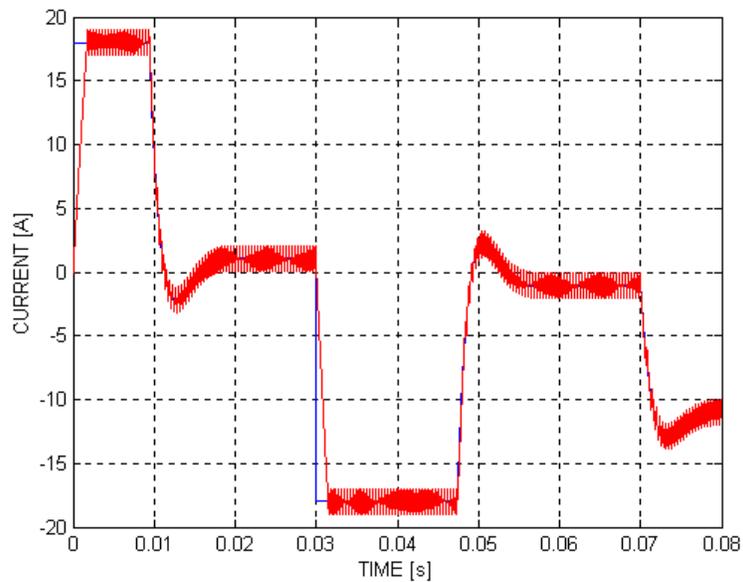
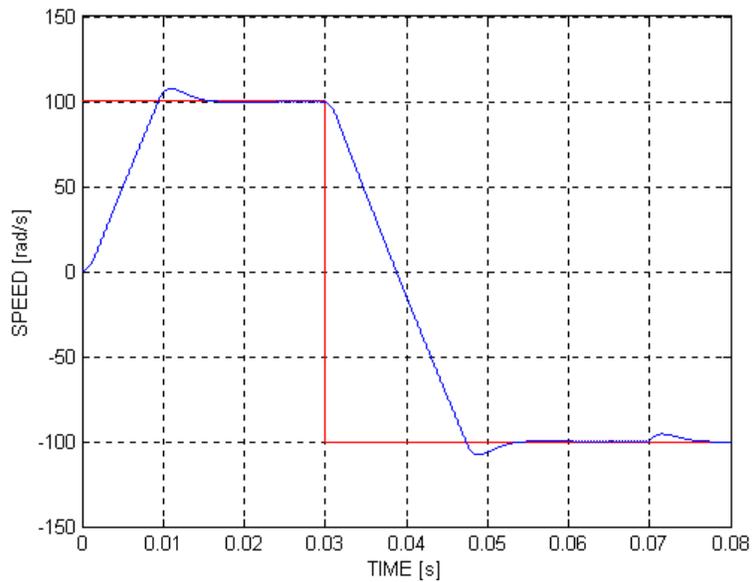
The block diagram of a cascade closed-loop speed control of the dc motor is shown below.



**CASCADE SPEED CONTROL OF A DC MOTOR DRIVE**

Typical dynamic responses are also shown. The motor is initially at standstill and at no load when a step command in speed is applied; when steady-state conditions are reached, a reversal of speed is commanded followed by a step load application.

The system is highly nonlinear due to the introduction of saturation needed to limit both the current delivered and the voltage applied to the motor. The system is in the saturation mode when the errors are large; as a consequence, the controller functions as a constant current source, that is torque, resulting in the ramping of the speed since the load in this example is a pure inertia. The inclusion of saturation limits on the PI integrator is therefore necessary to provide antiwindup action. The presence of the signum function in the torque expression is required in order to insure that the load is passive whether the speed is positive or negative (as is the case here).



# MATLAB MATRIX FORMULATION OF INDUCTION MACHINE STATE EQUATIONS

## Phase variable model

### INDUCTION MOTOR RUNNING UNDER PULSED LOADS

The purpose of this Matlab pseudo-script file is to show how the state equations of a three-phase induction machine are assembled in the context of a simple mode of operation: the motor is running in steady state at a specified loading when a pulsating load is applied. The dynamic equations are expressed in canonical first-order matrix form using the abc phase variables, specifically the state variables are the stator currents  $i_{as}$ ,  $i_{bs}$ ,  $i_{cs}$ ; the rotor currents  $i_{ar}$ ,  $i_{br}$ ,  $i_{cr}$ ; the rotor speed  $\omega_m$ ; and the angular displacement  $\theta$  between stator  $a_s$  axis and rotor  $a_r$  axis. The equations are solved using ODE45.

To perform the present simulation, simply run the m\_file named **IMabcrun0**.

A Simulink rendition is portrayed below and is given in the mdl\_file named **IMabcsim0**.

### MACHINE PARAMETERS [SI]

$V_{ll}=220$  Line-to-line rms voltage rating [V]  
 $R_s=0.531$  Stator resistance [Ohm]  
 $R_r=0.408$  Rotor resistance [Ohm]  
 $L_{sl}=2.5e-3$  Stator leakage inductance [H]  
 $L_{rl}=2.5e-3$  Rotor leakage inductance [H]  
 $L_m=84.7e-3$  Magnetizing inductance [H]  
 $f_{req0}=60$  Base frequency [Hz]  
 $\omega_0=2*\pi*f_{req0}$  Base frequency [rad/s]  
 $p=4$  Number of poles  
 $J=0.02$  Moment of inertia [kg.m<sup>2</sup>]  
 $T_{Li}=10$  Initial load torque [N.m]  
 $T_{Lf}=2$  Final load torque [N.m]  
 $B_m=.01$  Frictional coefficient  
Rotor-stator turns ratio = unity  
 $V_s=V_{ll}*\sqrt{2/3}$  Peak per-phase voltage [V]

### ODE

$x_0=[0\ 0\ 0\ 0\ 0\ 0\ \omega_0\ 0]'$  Approximate initial conditions  
 $t_f=8.0$  Final time  
 $tspan=[0\ t_f]$   
 $options=odeset('RelTol',1e-3,'maxstep',1e-3)$

$[t,x]=ode45(@IMabcf0,tspan,x_0,options,R_s,R_r,L_m,L_{sl},L_{rl},p,V_s,\omega_0,J,B_m,T_{Li},T_{Lf})$

### OUTPUT VARIABLES

$spd=x(:,7)*60/(p*\pi)$  Speed [rpm]  
 $i_{as}=x(:,1)$  ;  $i_{bs}=x(:,2)$  ;  $i_{cs}=x(:,3)$  Stator currents [A]  
 $I_s=\sqrt{i_{as}^2+(i_{bs}-i_{cs})^2/3}$  Stator current magnitude [A]  
 $i_{ar}=x(:,4)$  ;  $i_{br}=x(:,5)$  ;  $i_{cr}=x(:,6)$  Rotor currents [A]  
 $gam=2*\pi/3$   
 $v_{as}=V_s*\cos(\omega_0*t)$   
 $v_{bs}=V_s*\cos(\omega_0*t-gam)$   
 $v_{cs}=V_s*\cos(\omega_0*t+gam)$  Three-phase sinusoidal stator voltages (Balanced) [V]

## STATE EQUATIONS

State variable vector:  $x=[i_{as} \ i_{bs} \ i_{cs} \ i_{ar} \ i_{br} \ i_{cr} \ \omega_m \ \theta]$

function dxdt=IMabcf0(t,x,Rs,Rr,Lm, Lsl,Lrl, p,Vs,wo,J,Bm,TLi,TLf)

$L_{m1}=2/3*L_m$       **Maximum mutual inductance between any two phases xy**  
 $M_{xy} = L_{m1}*\cos(\text{angle between the 2 phases xy})$   
 $L_s=L_{sl}+L_{m1}$ ;  $L_r=L_{rl}+L_{m1}$ ;  $\text{gam}=2*\pi/3$

### Inputs:

**Load time profile:**

if  $t < 1.5$ ,  $TL=TL_i$ ; elseif  $t \geq 1.5$  &  $t < 5$ ,  $TL=TL_f$ ; elseif  $t > 5$ ,  $TL=TL_i$ ; end

**Stator voltage time profile:**

$v_{as}=V_s*\cos(\omega_0*t)$ ;  $v_{bs}=V_s*\cos(\omega_0*t-\text{gam})$ ;  $v_{cs}=V_s*\cos(\omega_0*t+\text{gam})$

### Variables:

$V=[v_{as} \ v_{bs} \ v_{cs} \ 0 \ 0 \ 0]'$       **Voltage vector [V]**  
 $I=x(1:6)$       **Current vector [A]**  
 $\omega_m=x(7)$       **Speed [electrical rad/s]**  
 $\theta=x(8)$       **Angular displacement between stator  $a_s$  axis and rotor  $a_r$  axis [rad]**

### Parameters:

**Resistance matrix :**      **[ $\Omega$ ]**  
 $R=\text{diag}([R_s \ R_s \ R_s \ R_r \ R_r \ R_r])$

**Inductance submatrices :**      **[H]**  
 $L_{ss}=[L_s \ -L_{m1}/2 \ -L_{m1}/2$   
 $\quad -L_{m1}/2 \ L_s \ -L_{m1}/2$   
 $\quad -L_{m1}/2 \ -L_{m1}/2 \ L_s]$

$L_{rr}=[L_r \ -L_{m1}/2 \ -L_{m1}/2$   
 $\quad -L_{m1}/2 \ L_r \ -L_{m1}/2$   
 $\quad -L_{m1}/2 \ -L_{m1}/2 \ L_r]$

$L_{sr}=L_{m1}*\begin{bmatrix} \cos(\theta) & \cos(\theta+\text{gam}) & \cos(\theta-\text{gam}) \\ \cos(\theta-\text{gam}) & \cos(\theta) & \cos(\theta+\text{gam}) \\ \cos(\theta+\text{gam}) & \cos(\theta-\text{gam}) & \cos(\theta) \end{bmatrix}$

$dL_{sr}=L_{m1}*\begin{bmatrix} -\sin(\theta) & -\sin(\theta+\text{gam}) & -\sin(\theta-\text{gam}) \\ -\sin(\theta-\text{gam}) & -\sin(\theta) & -\sin(\theta+\text{gam}) \\ -\sin(\theta+\text{gam}) & -\sin(\theta-\text{gam}) & -\sin(\theta) \end{bmatrix}$

**Inductance matrix L :**

$L=[L_{ss} \ L_{sr}$   
 $\quad L_{sr}' \ L_{rr}]$

**dL/d $\theta$  :**

$dL_{d\theta}=[\text{zeros}(3) \ dL_{sr}$   
 $\quad dL_{sr}' \ \text{zeros}(3)]$

# Matrix formulation of induction machine equations in phase variables:

## Constitutive flux-current relations:

$$\lambda = L \cdot I \quad \text{or} \quad I = L \backslash \lambda$$

## Torque:

$$T_e = \frac{\partial(W_m)}{\partial\theta_m} = \frac{p}{2} \cdot \frac{\partial(0.5 \cdot I' \cdot L \cdot I)}{\partial\theta} \quad W_m = \text{magnetic co-energy}$$

$$= \frac{p}{4} \cdot I' \cdot \frac{dL}{d\theta} \cdot I$$

## Electrical system equations:

$$V = R \cdot I + d\lambda/dt = R \cdot I + d(L \cdot I)/dt$$

$$= (R + \omega_m \cdot dL/d\theta) \cdot I + L \cdot dI/dt$$

## Mechanical system equations:

$$T_e = J \cdot d\omega_m/dt + T_L + B_m \cdot \omega_m$$

$$\omega_m = d\theta/dt$$

$$T_e = \frac{p}{4} \cdot I' \cdot \frac{dL}{d\theta} \cdot I$$

Electromagnetic torque [N.m]

$$dI/dt = L \backslash (V - (R + \omega_m \cdot dL/d\theta) \cdot I)$$

Electrical system equations

$$d\omega_m/dt = \frac{p}{2} \cdot (T_e - T_L) / J - B_m \cdot \omega_m / J ; \quad d\theta/dt = \omega_m$$

Mechanical system equations

$$dx/dt = [dI/dt; d\omega_m/dt; d\theta/dt]$$

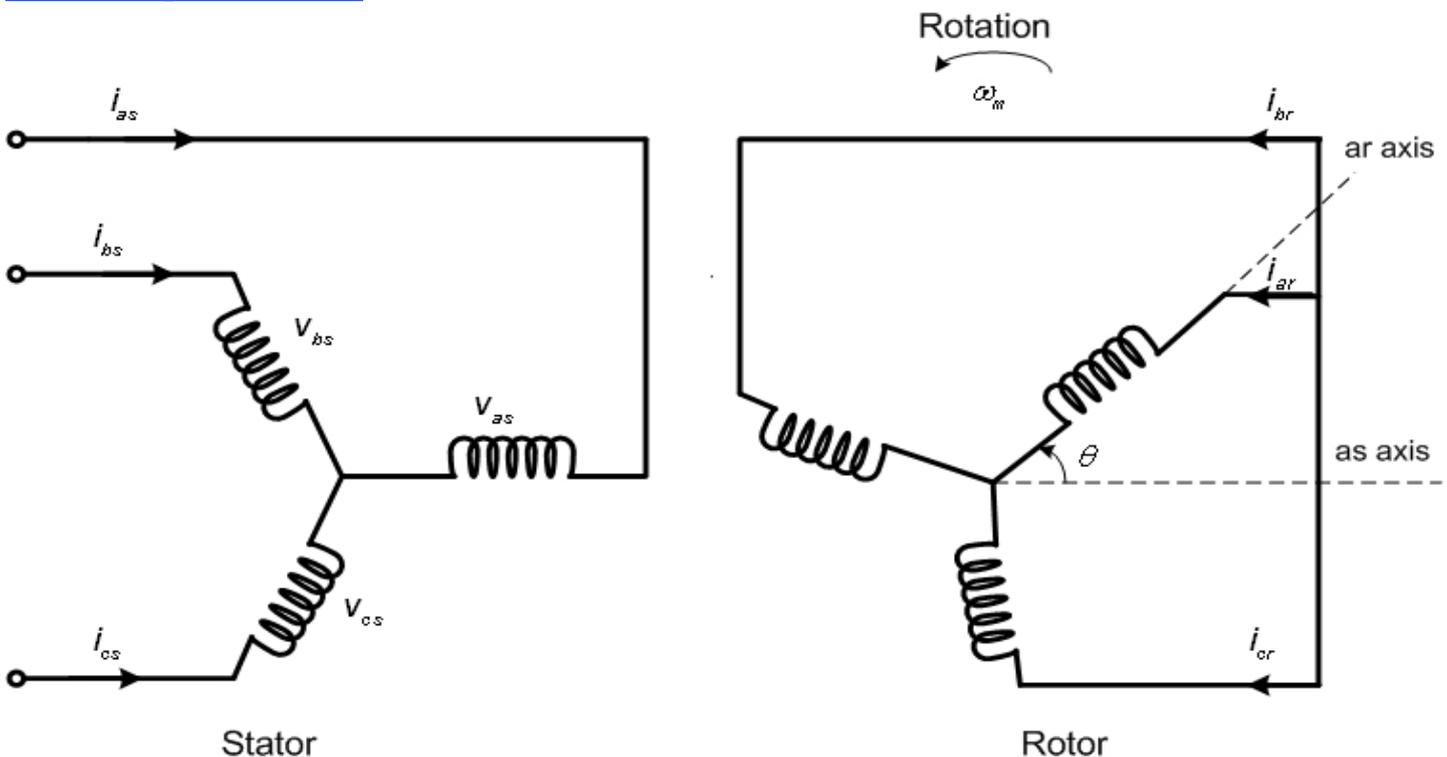
State equations arrived at by concatenation

## Note

In the phase model formulation of the machine equations, the variables  $V$ ,  $I$ ,  $\lambda$  appear as 6-element **column vectors** (in the matrix analysis connotation); so that, for instance, the current vector is  $I = [i_{as} \ i_{bs} \ i_{cs} \ i_{ar} \ i_{br} \ i_{cr}]'$ , representing stator and rotor currents expressed in their respective stator and rotor frames. The (mutual) inductance parameters are explicitly dependent upon rotor position  $\theta$ .

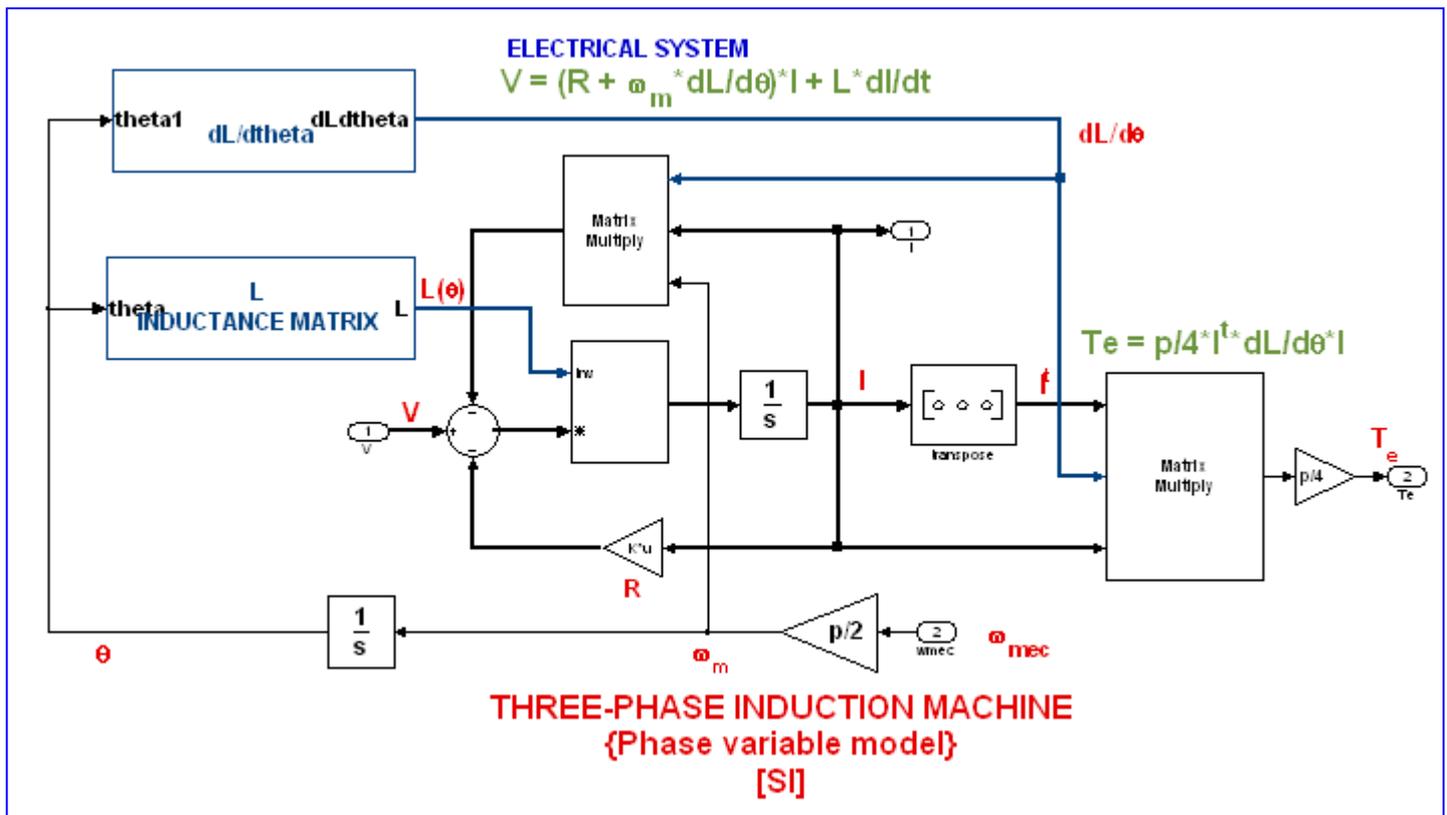
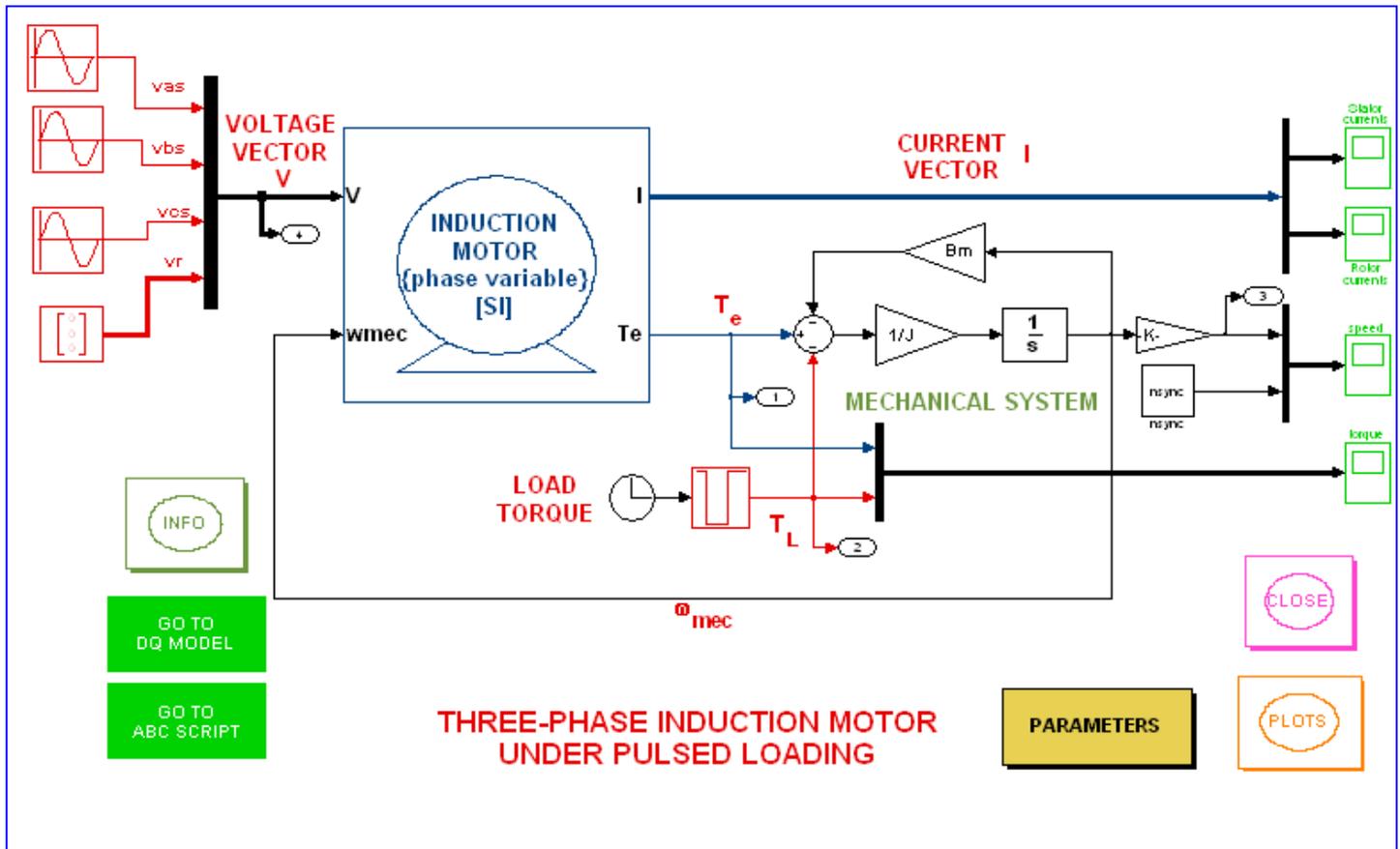
With the transformation of these original variables into a common DQ reference frame, they become **space vectors** (in a 2\_D geometric interpretation); now currents are defined as  $i_s = [i_{ds} \ i_{qs}]$  and  $i_r = [i_{dr} \ i_{qr}]$ . Furthermore, the inductance parameters become constant, independent of position.

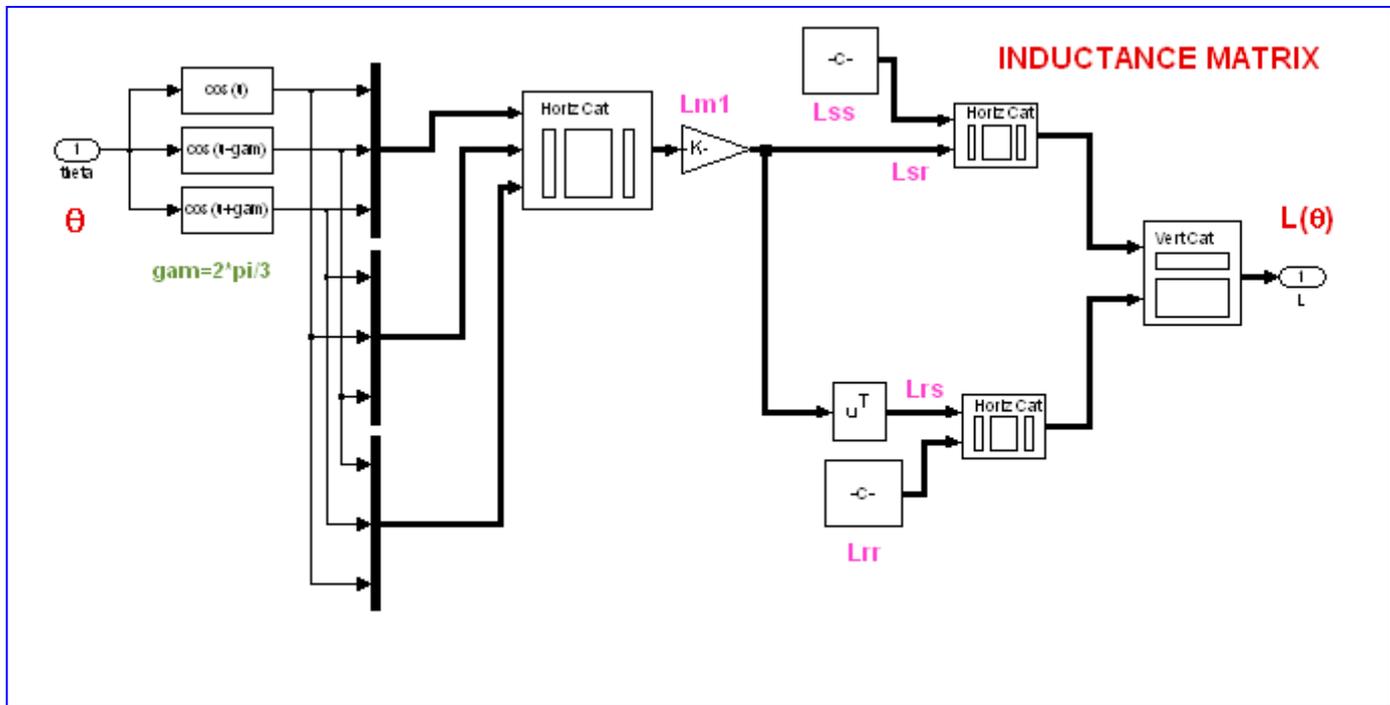
## Circuit representation



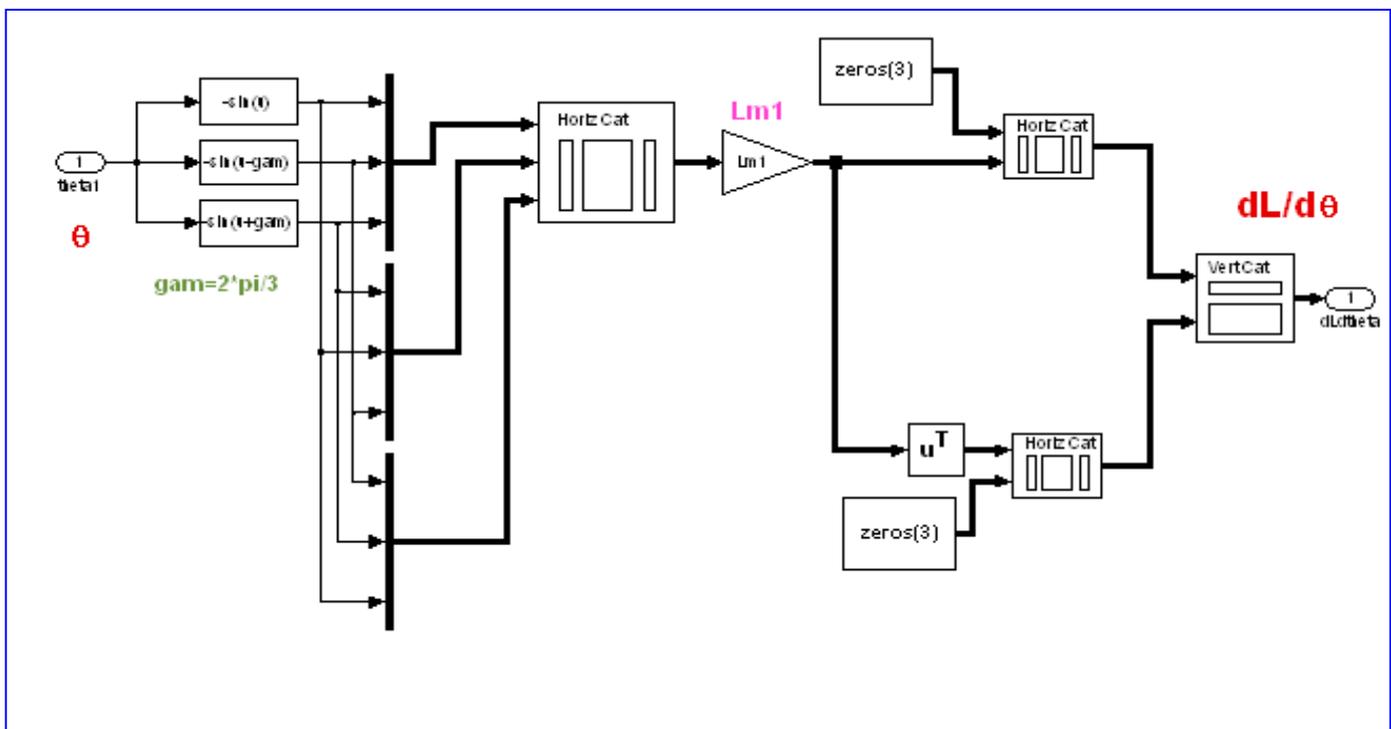
Three-phase coupled circuit representation of an induction machine

# Simulink models



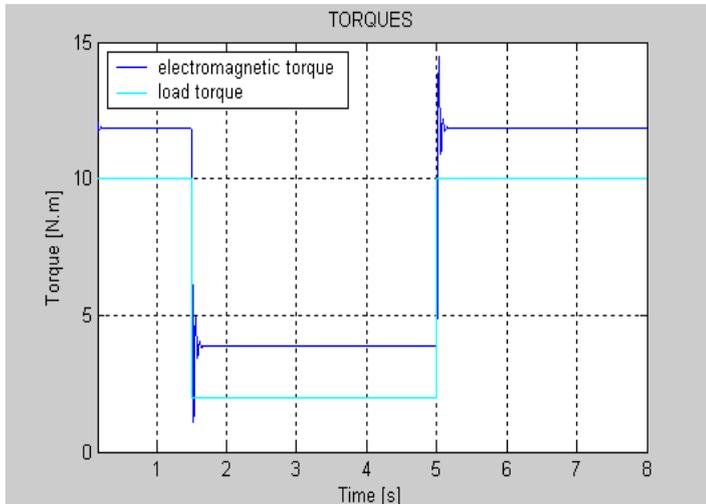


Concatenation used to assemble the inductance matrix  $L$  as a function of  $\theta$

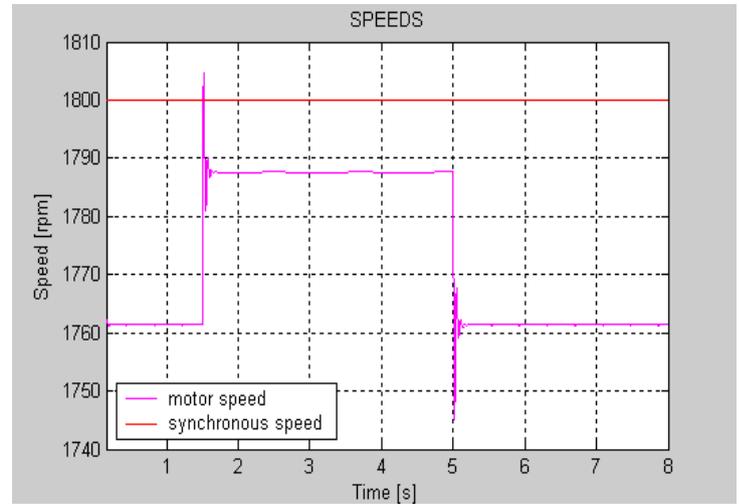


Concatenation used to assemble the matrix  $dL/d\theta$  as a function of  $\theta$

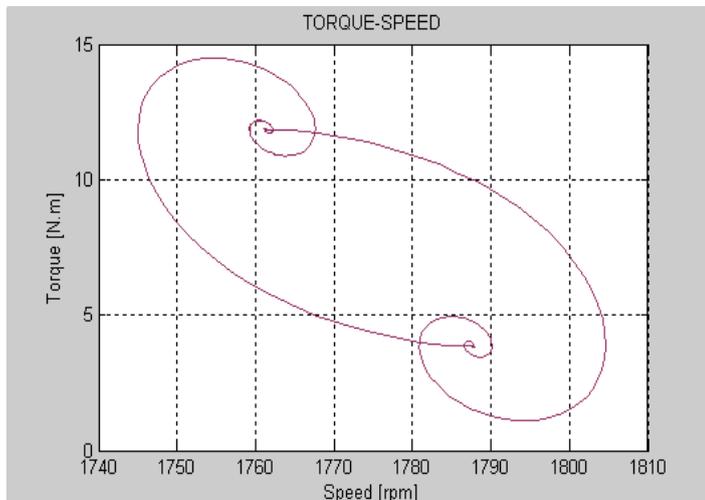
## Graphical displays



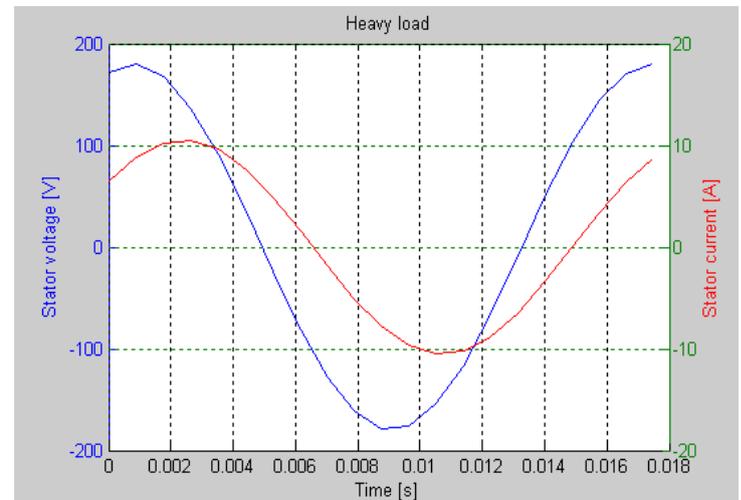
**Torques versus time**



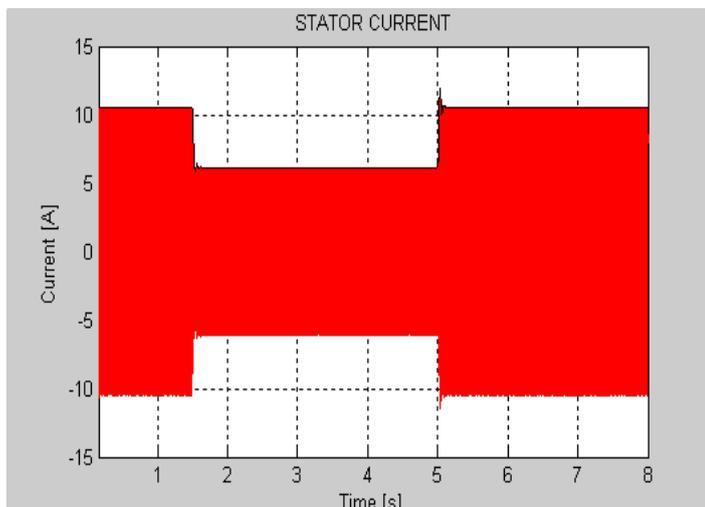
**Speeds versus time**



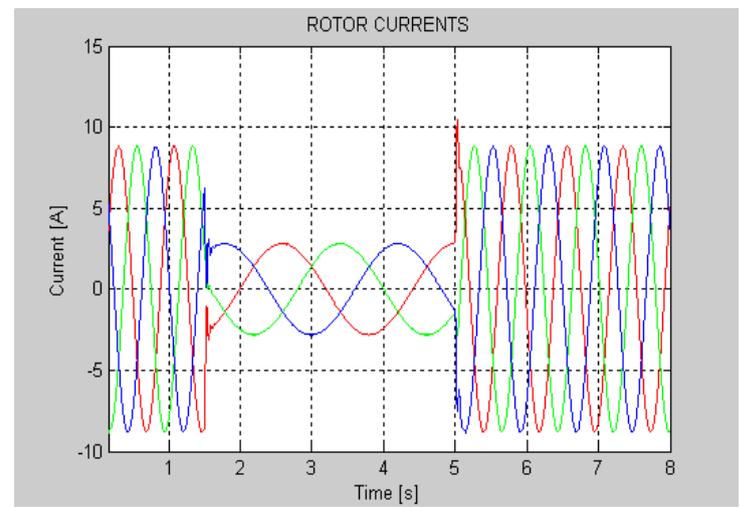
**Torque-speed curve**



**One cycle of stator voltage and current**



**Stator current  $i_{as}$**



**Three-phase rotor currents**