

# Capacity of Ad-Hoc Networks with Node Cooperation

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*Abstract* — This paper examines communication between a cluster of closely-packed nodes with another cluster of closely-packed nodes. The nodes within each cluster are separated by small distances, relative to the distance between the two clusters. The effect on capacity of cooperation between nodes in the transmitting cluster and cooperation between nodes in the receiving cluster is investigated.

## I. SYSTEM MODEL

Consider a system with two transmitters and two receivers, as shown in Fig. 1. TX 1 wishes to communicate to RX 1, and TX 2 wishes to communicate to RX 2. It is assumed that the distance between each of the four transmitter-receiver pairs is the same, and each channel gain is normalized to have amplitude one. Thus, the channels between each transmitter-receiver pair are identical, except for a random uniformly distributed phase. The channel can be written as:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} e^{j\theta_1} & e^{j\theta_1} \\ e^{j\theta_1} & e^{j\theta_1} \end{bmatrix}$$

where  $n_1$  and  $n_2$  are independent  $N(0,1)$  noises. Since the amplitudes of all gains are equal, phase differences are required for  $\mathbf{H}$  to be full rank. The channel gains and phases are assumed to be fixed and known at all nodes. Furthermore, perfect synchronization between nodes is assumed. In addition to the direct communication channel, there is also an AWGN channel (with gain  $\sqrt{G}$ ) between the two transmitters and between the two receivers that allows for cooperation. We consider the scenario where there are three equal frequency bands, two for cooperation and one for direct communication, and we also consider the scenario where there is a single frequency band that must be divided into cooperation bands and a direct communication band. Finally, a power constraint of  $P$  is imposed on the total transmit energy, *i.e.* the power used over all three channels must be no larger than  $P$ .

## II. COOPERATIVE COMMUNICATION

Without node cooperation, the channel in question is a two-user strong interference channel, for which the capacity region is known [1]. This capacity region is compared to rates achievable using node cooperation. Transmitter cooperation is also considered in [2] and the references therein. In this work, the following schemes are analyzed:

**TX Cooperation:** If the transmitters were allowed to jointly encode their messages, the channel would be a multiple-antenna broadcast channel, for which dirty paper coding is capacity-achieving. Motivated by this, a strategy where the two transmitters first exchange their intended messages using an optimal fraction of the total power  $P$ , and then *jointly* encode both messages using dirty paper coding with the remaining power, is considered.

**RX Cooperation:** Since the channels of each of the signals are statistically very similar, the information decodable

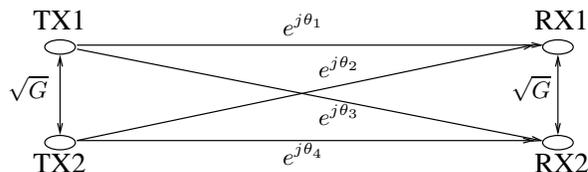


Figure 1: System Model

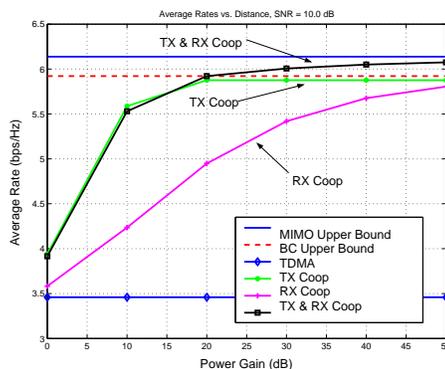


Figure 2: Rate vs.  $G$  for non-cooperative SNR = 10 dB

at each receiver is roughly the same. Thus relay strategies that involve first decoding a message seem fruitless. An alternative cooperation scheme is for each receiver to amplify-and-forward its received signal to the other receiver. Each receiver uses an optimal fraction of power to amplify-and-forward its received signal, effectively resulting in a noisy (due to noise amplification) second antenna for each receiver.

The combination of these strategies is also considered, *i.e.* the transmitters exchange messages and cooperatively transmit, and the receivers perform amplify-and-forward. The capacity of the implicit broadcast channel, multiple-access channel, and MIMO channel bound the achievable rates in TX, RX, and joint cooperation modes, respectively.

Achievable rates with these different cooperation modes are investigated for different values of  $G$  and different SNR's. In Fig. 2 achievable rates (averaged over random channels) are plotted as a function of  $G$ , the cooperative channel strength, for a channel in which frequency bands are set aside for cooperation. These results indicate that TX cooperation performs significantly better than RX cooperation, and increases larger than 50% are feasible. Furthermore, cooperation at both sides is only beneficial for extremely large values of  $G$ .

## REFERENCES

- [1] H. Sato, "The capacity of the Gaussian interference channel under strong interference", *IEEE Trans. Inform. Theory*, Nov. 1981, pp. 786-788.
- [2] A. Host-Madsen, "A new achievable rate for cooperative diversity based on generalized writing on dirty paper", Proc. of Int. Symp. Inform. Theory, June 2003, pp. 317.