Energy-Limited vs. Interference-Limited Ad Hoc Network Capacity

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***Abstract*— In a multi-user system in which interference is treated as noise, increasing the power of all transmissions eventually makes thermal noise negligible and causes the network to be interference-limited. This paper attempts to determine the power level at which a random-access ad hoc network becomes interference limited. Furthermore, when the network is not interference-limited (i.e., when signal power does not completely overwhelm noise), the relationship between power and area spectral efficiency is quantified. It is shown that the key quantity is the energy per information bit, commonly referred to as $\frac{E_b}{N_0}$. Roughly speaking, a network becomes interference limited for $\frac{E_b}{N_0}$ values above approximately 15 dB; increasing $\frac{E_b}{N_0}$ leads to a negligible capacity increase, but decreasing $\frac{E_b}{N_0}$ below this value does lead to a non-negligible capacity decrease. Furthermore, as $\frac{E_b}{N_0}$ approaches the Shannon limit of -1.59 dB, network capacity is seen to be extremely sensitive to the value of $\frac{E_b}{N_0}$.**

I. Introduction

In a multi-user communication network with simultaneous and interfering transmissions, the two fundamental impediments to reliable communication are thermal noise and multi-user interference. If the power of each simultaneous transmission is increased, signal and interference power increase proportionally while thermal noise power remains constant. Thus, at some point thermal noise becomes approximately negligible, i.e., the network becomes *interference-limited*, and any further increase in transmission power provides essentially no benefit. On the other hand, thermal noise is not negligible when the transmission power is not so large relative to the noise power. The objective of this paper is to (a) determine the power level at which an ad hoc network becomes interference-limited, and (b) quantify the relationship between transmission power and network capacity.

To allow for analytical tractability, we analyze a random-access based ad-hoc network consisting of transmitter-receiver pairs distributed on the two-dimensional plane. More specifically, the network we consider has the following key characteristics:

- Transmitter locations are a realization of a homogeneous spatial Poisson process.
- Each transmitter communicates with a single receiver that is a distance d meters away.
- All transmissions occur at power ρ and rate R bits/sec, and the noise spectral density is N_0.
- Each receiver treats multi-user interference as noise.

By considering such a network, the *transmission capacity* framework, which was first developed in [1] and quantifies the probability of successful transmission in terms of the transmission density and SINR threshold, can be utilized.

At a basic level, the objective of the work is to determine the relationship between ρ, transmission power, and the capacity (i.e., density of transmissions) of such a network. Rather than fixing all operating parameters and then varying ρ, we find it more meaningful to optimize the network for each value of ρ. In the random access setting considered here, the relevant design variable is the fraction of the total bandwidth that each rate R bits/sec communication occupies. We consider the case where the system bandwidth of W Hz is divided into N equal sub-bands of $\frac{W}{N}$ Hz, with each transmission occurring on a randomly chosen sub-band. The number of sub-bands N is optimized (separately) for each value of ρ, and the optimized capacity is then studied as a function of transmission power.

By posing the problem of sub-band optimization in terms of the spectral efficiency of each communication (equal to $\frac{R}{W/N}$), we are able to derive simple expressions for the optimal operating point that depend only on the path-loss exponent of the network and the energy per information bit of each transmission [2] (i.e., $\frac{E_b}{N_0} = \frac{P}{N_0 R}$, where P is the received power, N_0 is the noise spectral density, and R is the rate). Furthermore, this perspective shows that the value of $\frac{E_b}{N_0}$ determines whether a network is interference-limited or not.

When thermal noise is negligible relative to the received signal power (i.e., $\frac{E_b}{N_0} \to \infty$), the network becomes purely interference-limited and the optimal spectral efficiency is a function of the path loss exponent (α) alone. Furthermore, for reasonable path loss exponents the optimal spectral efficiency lies between the low-SNR and high-SNR regimes. For example, the optimal is 1.3 bps/Hz and 2.3 bps/Hz for $\alpha = 3$ and $\alpha = 4$, respectively.

When thermal noise is not negligible (i.e., $\frac{E_b}{N_0}$ is moderate), the optimal spectral efficiency is shown to be a fraction of the maximum spectral efficiency achievable in the absence of interference (i.e., the AWGN capacity at that $\frac{E_b}{N_0}$). For example, if $\frac{E_b}{N_0} = 0$ dB and $\alpha = 4$, the maximum spectral efficiency is 1 bps/Hz [2] but the optimal operating point is 0.5 bps/Hz; for $\alpha = 3$ the optimal is $\frac{1}{3}$ bps/Hz.

Furthermore, a network is seen to become interference-limited when $\frac{E_b}{N_0}$ is in the range of $15-20$ dB; increasing $\frac{E_b}{N_0}$

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beyond this value provides only a negligible capacity increase, while decreasing $\frac{E_b}{N_0}$ below this range does lead to a non-negligible capacity reduction. Furthermore, as $\frac{E_b}{N_0}$ approaches the interference-free limit of -1.59 dB, capacity is seen to be extremely sensitive to the value of $\frac{E_b}{N_0}$.

The remainder of this paper is organized as follows. In Section II the network model and relevant transmission capacity results are described. In Section III the sub-band optimization is performed for all values of $\frac{E_b}{N_0}$, while Sections IV and V contains detailed analyses of the interference-limited (large $\frac{E_b}{N_0}$) and energy-limited ($\frac{E_b}{N_0} \to -1.59$ dB) regimes. For the sake of brevity results are stated without proofs, although full proofs are available in [3].

II. Preliminaries

A. Network Model

We consider a set of transmitting nodes at an arbitrary snapshot in time with locations specified by a homogeneous Poisson point process (PPP) of intensity λ on the infinite 2-D plane. We assume that all nodes simultaneously transmit with the same power ρ, and that the noise spectral density is N_0. Moreover, nodes decide to transmit independently and irrespective of their channel conditions, which corresponds roughly to slotted ALOHA (i.e., no scheduling is performed). The stationarity of the PPP allows us to analyze the behavior of a single reference receiver (RX 0). The reference transmitter (TX 0) is placed a fixed distance d away, and X_i denotes the distance of the i-th transmitting node to the reference receiver. By the properties of the PPP, the locations of the other transmitting nodes (i.e., the interfering nodes) form a homogeneous PPP. Received power is modeled by path loss with exponent $\alpha > 2$.

If the transmit signal of the i-th transmitter is denoted as U_i, the reference received signal is:

$$Y_0 = U_0 d^{-\alpha/2} + \sum_{i \in \Pi(\lambda)} U_i X_i^{-\alpha/2} + Z_i$$

where Z_i is additive Gaussian noise with power η. The resulting SINR therefore is:

$$\mathrm{SINR}_0 = \frac{\rho d^{-\alpha}}{\eta + \sum_{i \in \Pi(\lambda)} \rho X_i^{-\alpha}},$$

where $\Pi(\lambda)$ indicates the point process describing the (random) interferer locations. If Gaussian signaling is used by all nodes, the mutual information between input U_0 and channel output Y_0 conditioned on the transmitter locations is $I(U_0; Y_0 | \Pi(\lambda)) = \log_2(1 + \mathrm{SINR}_0)$. As discussed in more detail below, we consider an outage setting where a communication is successful if and only if the mutual information is larger than the transmitted information.

A few comments in justification of the above model are in order. Although the model contains many simplifications to allow for tractability, it contains many of the critical elements of a real decentralized network. First, the spatial Poisson distribution means that nodes are randomly and independently located; this is reasonable particularly in a network with indiscriminate node placement or substantial mobility [4], [5]. The fixed transmission distance of d is clearly not a reasonable assumption; however our prior work [1], [6] has shown rigorously that variable transmit distances do not result in fundamentally different capacity results, so a fixed distance is chosen because it is much simpler analytically and allows for crisper insights. Our model does not include fading, but our past work has shown that the effect of fading in the interference-limited regime is only a multiplicative constant; the effect of fading in the energy-limited regime is of interest but it beyond the scope of this work. Finally, scheduling procedures (e.g., using carrier sensing to intelligently select a sub-band) may significantly affect the results and are definitely of interest, but this opens many more questions and so is left to future work.

B. Transmission Capacity Model

In the outage-based transmission capacity framework, an outage occurs whenever the SINR falls below a prescribed threshold β, or equivalently whenever the instantaneous mutual information falls below $\log_2(1+\beta)$. Therefore, the system-wide outage probability is:

$$\mathbb{P}\left(\frac{\rho d^{-\alpha}}{\eta + \sum_{i \in \Pi(\lambda)} \rho X_i^{-\alpha}} \le \beta\right).$$

This quantity is computed over the distribution of transmitter positions and clearly is an increasing function of the intensity λ. The SINR threshold β and the noise power η are treated as external constants here, but are related to R, W, and N in the following section.

If λ_ϵ is the maximum intensity of *attempted* transmissions such that the outage probability (for a fixed β) is no larger than ϵ, the transmission capacity is then defined as $c_\epsilon = \lambda_\epsilon(1-\epsilon)b$, which is the maximum density of *successful* transmissions times the spectral efficiency b of each transmission. In other words, transmission capacity is like area spectral efficiency (ASE) subject to an outage constraint. Using tools from stochastic geometry, in [1] it is shown that the maximum spatial intensity λ_ϵ is upper bounded by the quantity $\overline{\lambda}_\epsilon$, which is defined as:

$$\overline{\lambda}_\epsilon = \frac{-\ln(1-\epsilon)}{\pi d^2}\left(\frac{1}{\beta} - \frac{\eta}{\rho d^{-\alpha}}\right)^{\frac{2}{\alpha}} \tag{1}$$

Furthermore, this bound is asymptotically tight for small values of ϵ:

$$\lambda_\epsilon = \overline{\lambda}_\epsilon + O(\epsilon^2), \tag{2}$$

and in [1][6] $\overline{\lambda}_\epsilon$ is seen to be quite accurate for values of ϵ below approximately 20% for path loss exponents not too close to 2.

III. Sub-Band Optimization

We now address the problem of determining the number of sub-bands that maximize the density of transmissions such that the outage probability is no larger than ϵ. If the system bandwidth is not split ($N = 1$), each node utilizes the entire bandwidth of W Hz. The SINR required (β) to achieve a rate of R bps is determined by inverting the AWGN capacity expression:

$$R = W \log_2(1+\beta),$$

which gives $\beta = 2^{\frac{R}{W}} - 1$. The maximum intensity of transmissions can be determined by plugging in this value of β along with $\eta = N_0 W$ into (1).

If the system bandwidth is split into $N > 1$ orthogonal sub-bands each of width $\frac{W}{N}$, and each transmitter-receiver pair uses one *randomly selected* sub-band, the required SINR $\beta(N)$ is determined by inverting the rate expression:

$$R \ = \ \frac{W}{N}\log_2(1+\beta(N)) \quad \rightarrow \quad \beta(N) = 2^{\frac{NR}{W}} - 1. \tag{3}$$

Because each transmitter randomly chooses a sub-band, the users on each sub-band are still a PPP and are independent across bands. As a result, the maximum intensity of transmissions *per sub-band* is λ_ϵ as defined in Section II-B with SINR threshold $\beta(N)$ and noise power $\eta = N_0 \frac{W}{N}$. Since the N sub-bands are statistically identical, the maximum total intensity of transmissions, denoted by λ_ϵ^T, is the per sub-band intensity λ_ϵ multiplied by N. Therefore, from (1) we have $\lambda_\epsilon^T(N) \leq \overline{\lambda_\epsilon^T}(N)$ with

$$\overline{\lambda_\epsilon^T}(N) = N\left(\frac{-\ln(1-\epsilon)}{\pi d^2}\right)\left(\frac{1}{\beta(N)} - \frac{N_0\left(\frac{W}{N}\right)}{\rho d^{-\alpha}}\right)^{\frac{2}{\alpha}}. \tag{4}$$

The only design parameter is N, the number of sub-bands, and thus transmission density is maximized by choosing the N that maximizes $\lambda_\epsilon^T(N)$. To allow for analytical tractability we maximize the upper bound, i.e., we solve the following one-dimensional optimization:

$$N^* = \arg\max_N \overline{\lambda_\epsilon^T}(N) \tag{5}$$

Rather than maximizing with respect to N, it is more convenient to maximize with respect to the *operating spectral efficiency*, which is equal to the transmission rate divided by the bandwidth of each sub-band (not the total bandwidth):

$$b \triangleq \frac{R}{W/N} \text{ bps/Hz}. \tag{6}$$

It is important to note that the operating spectral efficiency b is a design parameter even though the per-transmission rate R and system bandwidth W are fixed[1].

With this substitution we can write the transmission density as a function of b:

$$\overline{\lambda_\epsilon^T}(b) = \left(\frac{W}{R}\right)\left(\frac{-\ln(1-\epsilon)}{\pi d^2}\right) b \left(\frac{1}{2^b-1} - \frac{1}{b}\frac{N_0 R}{\rho d^{-\alpha}}\right)^{\frac{2}{\alpha}} \tag{7}$$

[1]If we consider only bandwidth optimization, b should be limited to integer multiples of $\frac{R}{W}$. However, if we consider a more general scenario where we are designing the sub-band structure as well as the length of transmission (e.g., in a packetized system), then these two parameters allow us to operate at any desired b. We therefore consider arbitrary $b > 0$ for the remainder of the paper, although it is not difficult to see the effect of constraining N to be an integer.

Noting that the constant $\frac{\rho d^{-\alpha}}{N_0 R} \triangleq \frac{E_b}{N_0}$ is the received energy per information bit [2] and defining the constant $k \triangleq \left(\frac{W}{R}\right)\left(\frac{-\ln(1-\epsilon)}{\pi d^2}\right)$, this can be further simplified this as:

$$\overline{\lambda_\epsilon^T}(b) = kb\left(\frac{1}{2^b-1} - \frac{1}{b\frac{E_b}{N_0}}\right)^{\frac{2}{\alpha}}. \tag{8}$$

The optimal spectral efficiency b^* is therefore the solution to the following optimization:

$$b^* = \arg\max_{b>0} \ b\left(\frac{1}{2^b-1} - \frac{1}{b\frac{E_b}{N_0}}\right)^{\frac{2}{\alpha}}. \tag{9}$$

Note that the optimal b^* depends only on the path loss exponent α and $\frac{E_b}{N_0}$, and thus any dependence on power and rate is completely captured by $\frac{E_b}{N_0}$. By posing the problem in terms of spectral efficiency, we are also able to remove any direct dependence on W. Furthermore, the problem is independent of the outage constraint ϵ, assuming the bound in (1) is accurate.

The problem in (9) is only feasible for b satisfying $\frac{1}{2^b-1} - \frac{1}{b\frac{E_b}{N_0}} \geq 0$, which corresponds to the SINR threshold $\beta = 2^b - 1$ being no larger than the interference-free SNR $\frac{N\rho d^{-\alpha}}{N_0 W}$. Some simple manipulation shows that this condition is equivalent to $b \leq \mathsf{C}\left(\frac{E_b}{N_0}\right)$, where $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ is the maximum spectral efficiency of an AWGN channel and thus is the solution to [2, Equation 23]:

$$\frac{2^{\mathsf{C}\left(\frac{E_b}{N_0}\right)} - 1}{\mathsf{C}\left(\frac{E_b}{N_0}\right)} = \frac{E_b}{N_0}. \tag{10}$$

The domain of the maximization is thus $0 \leq b \leq \mathsf{C}\left(\frac{E_b}{N_0}\right)$. If $\frac{E_b}{N_0} \leq \ln 2 = -1.59$ dB the problem is infeasible for any b because this corresponds to operating beyond interference-free capacity.

If $\alpha = 2$ we have $\overline{\lambda_\epsilon^T}(b) = k\left(\frac{b}{2^b-1} - \frac{1}{\frac{E_b}{N_0}}\right)$ and it is easy to verify that this is a *decreasing* function of b for any value of $\frac{E_b}{N_0} \geq -1.59$ dB. As a result, transmission density is maximized by choosing b as small as possible, which corresponds to always selecting $N = 1$ (universal frequency reuse). However, for $\alpha > 2$, the optimum value of b is always strictly positive. Thus, the remainder of the paper deals with the non-trivial case of $\alpha > 2$.

By taking the derivative of $\overline{\lambda_\epsilon^T}(b)$ and setting it equal to zero, we are able to characterize b^* in terms of a fixed point equation parameterized by only α and $\frac{E_b}{N_0}$:

Theorem 1: The optimum operating spectral efficiency b^* is the *unique* positive solution of the following equation:

$$\frac{E_b}{N_0}\frac{2}{\alpha}b^2 2^b \ln 2 - \frac{E_b}{N_0}b\left(2^b-1\right) + \left(1-\frac{2}{\alpha}\right)\left(2^b-1\right)^2 = 0. \tag{11}$$

Proof: See [3]. ■

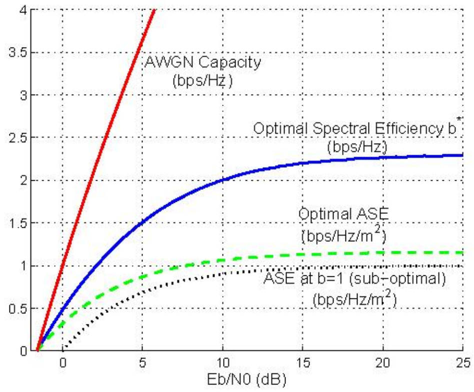

Fig. 1. Optimal Spectral Efficiency/Density vs. $\frac{E_b}{N_0}$ for $\alpha = 4$.

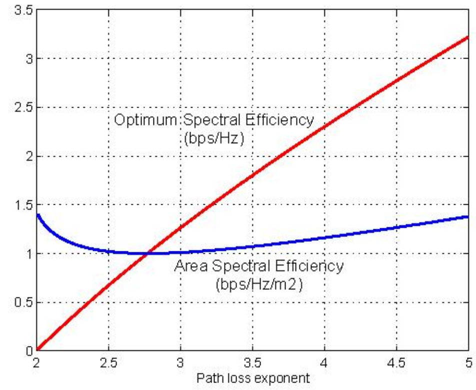

Fig. 2. Optimal Spectral Efficiency/Density for Interference-Limited Networks.

Theorem 1 characterizes the optimal number of sub-bands ($N^* = \frac{b^*}{R/W}$), i.e., the optimal operating point as a function of $\frac{E_b}{N_0}$, and the most relevant capacity characterization is the area spectral efficiency (ASE). Since each communication occurs at rate R bps and a total of W Hz are used, the upper bound to the system-wide ASE (assuming operation at b^*) is a function only of $\frac{E_b}{N_0}$ and is given by:

$$\begin{aligned} ASE\left(\frac{E_b}{N_0}\right) &\triangleq \overline{\lambda_\epsilon^T}(b^*)\left(\frac{R}{W}\right) \quad (12) \\ &= cb^*\left(\frac{1}{2^{b^*}-1} - \frac{1}{b^*\frac{E_b}{N_0}}\right)^{\frac{2}{\alpha}} \end{aligned}$$

where b^* is characterized in Theorem 1 and $c = \frac{-\ln(1-\epsilon)}{\pi d^2}$. Because c is only a multiplicative constant, for simplicity we assume $c = 1$ for the remainder of the paper.

Although we are not able to find a general closed-form expression for (11) or (12), these expressions are easily solved numerically and we can find analytical solutions in the asymptotic regimes ($\frac{E_b}{N_0} \to \infty$ and $\frac{E_b}{N_0} \to -1.59$ dB). In Fig. 1 the numerically computed optimum spectral efficiency b^* and $ASE\left(\frac{E_b}{N_0}\right)$ are plotted versus $\frac{E_b}{N_0}$ for $\alpha = 4$, along with the capacity of an interference-free AWGN channel $\mathsf{C}\left(\frac{E_b}{N_0}\right)$. Also plotted is the ASE achieved if spectral efficiency $b = 1$ (which corresponds to $N = \frac{W}{R}$) is used at all values of $\frac{E_b}{N_0}$ rather than b^*. For large values of $\frac{E_b}{N_0}$ this quantity is relatively close to the optimal ASE, but notice that it is much smaller than the optimal for smaller values of $\frac{E_b}{N_0}$. From this figure, we can identify two different asymptotic regimes of interest:

- **Interference-Limited Networks:** When $\frac{E_b}{N_0}$ is sufficiently large, thermal noise becomes negligible and performance depends only on multi-user interference. As a result, the optimal b^* and $ASE\left(\frac{E_b}{N_0}\right)$ both converge to constants as $\frac{E_b}{N_0} \to \infty$.
- **Energy-Limited Networks:** When $\frac{E_b}{N_0}$ is close to its minimum value of -1.59 dB, b^* scales linearly with $\frac{E_b}{N_0}$ (dB) and shows characteristics very similar to the AWGN wideband capacity [2]. In this regime performance is determined by interference and thermal noise, and b^* and $ASE\left(\frac{E_b}{N_0}\right)$ go to zero as $\frac{E_b}{N_0}$ decreases towards -1.59 dB.

In Section IV the interference-limited regime is explored and a closed form expression for the optimal value of b^* is derived. Once a system is in this regime, ASE is virtually unaffected by further increasing transmission power. In Section V the energy-limited regime is explored and simple expressions for b^* and ASE, in terms of α and $\frac{E_b}{N_0}$, are given that are accurate as $\frac{E_b}{N_0} \to -1.59$ dB. Although intuition might suggest that noise becomes dominant and thus interference becomes negligible, this is not the case as evidenced by the fact that the optimum spectral efficiency b^* is considerably smaller than the interference-free spectral efficiency $\mathsf{C}\left(\frac{E_b}{N_0}\right)$. Furthermore, increasing transmission power does significantly increase ASE in this regime. Between these two regimes (approximately from 2-3 dB to 15-20 dB), b^* increases sub-linearly with $\frac{E_b}{N_0}$ (dB). Although we do not have a closed-form expression for b^* for this range of $\frac{E_b}{N_0}$ values, the intuition is a combination of the insights derived for the interference- and energy-limited regimes.

It should be noted that Fig. 1 provides numerical answers to the two basic questions posed earlier: the optimized ASE essentially hits its asymptotic value for $\frac{E_b}{N_0}$ values between 15 and 20 dB, and thus this marks the beginning of the interference-limited regime, and the the slope of the $ASE\left(\frac{E_b}{N_0}\right)$ curve quantifies the marginal benefit of increasing/decrease the value of $\frac{E_b}{N_0}$. Although not shown here, the shape of the $ASE\left(\frac{E_b}{N_0}\right)$ curve is very similar for all path loss exponents between 2.5 and 5, although the magnitude is larger for large exponents.

IV. Interference-Limited Networks

In systems with sufficiently powered devices (i.e., large $\frac{E_b}{N_0}$), thermal noise is essentially negligible. In the limiting case where $N_0 = 0$ (i.e., $\frac{E_b}{N_0} \to \infty$) the density is given by:

$$\overline{\lambda_\epsilon^T}(b) = kb\left(2^b - 1\right)^{-\frac{2}{\alpha}}. \quad (13)$$

In this limiting regime, a closed-form solution for b^* can be reached.

Theorem 2: The optimum operating spectral efficiency b^* in the absence of thermal noise ($N_0 = 0 \leftrightarrow \frac{E_b}{N_0} = \infty$) is the unique solution to:

$$b^* = (\log_2 e)\frac{\alpha}{2}(1 - 2^{-b^*}), \quad (14)$$

which can be written in closed form as:

$$b^* = \log_2 e\left[\frac{\alpha}{2} + \mathcal{W}\left(-\frac{\alpha}{2}e^{-\frac{\alpha}{2}}\right)\right] \quad (15)$$

where $\mathcal{W}(z)$ is the principle branch of the Lambert $\mathcal{W}$ function and thus solves $\mathcal{W}(z)e^{\mathcal{W}(z)} = z$.

Proof: See [3]. ■

The optimum depends only on the path loss exponent α, and it is not difficult to show that b^* is an increasing function of α, b^* is upper bounded by $\frac{\alpha}{2}\log_2 e$, and that $b^*/(\frac{\alpha}{2}\log_2 e)$ converges to 1 as α grows large. In Fig. 2 the optimal spectral efficiency b^* is plotted (in units of bps/Hz) as a function of the path-loss exponent α, along with the optimized ASE.

V. Energy-Limited Networks

The opposite extreme of interference-limited networks are energy-limited networks for which $\frac{E_b}{N_0}$ is close to its minimum value of -1.59 dB. We can obtain a simple solution for b^* that is accurate up to a quadratic term by solving the fixed point equation given in Theorem 1:

Theorem 3: The optimum operating spectral efficiency b^* in the energy-limited regime ($\frac{E_b}{N_0}$ slightly larger than -1.59 dB) is given by:

$$b^* = \left(1 - \frac{2}{\alpha}\right)\mathsf{C}\left(\frac{E_b}{N_0}\right) + O\left(b^2\right) \quad (16)$$

where $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ is the AWGN spectral efficiency at $\frac{E_b}{N_0}$ as defined in (10). Furthermore, the ASE is characterized as:

$$ASE\left(\frac{E_b}{N_0}\right) = \left((1-\delta)^{(1-\delta)}\delta^\delta 2^{-\delta}\right)\mathsf{C}\left(\frac{E_b}{N_0}\right) + O\left(b^2\right) \quad (17)$$

where $\delta \triangleq \frac{2}{\alpha}$ and $(1-\delta)^{(1-\delta)}\delta^\delta 2^{-\delta} < 1$ for all $\alpha > 2$.

Proof: See [3]. ■

Fig. 3 contains plots of the numerically computed optimal spectral efficiency b^*, denoted as optimal, the approximation $(1 - \frac{2}{\alpha})\mathsf{C}\left(\frac{E_b}{N_0}\right)$, and AWGN capacity $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ versus $\frac{E_b}{N_0}$ for $\alpha = 3$ and $\alpha = 4$. Fig. 4 contains plots of the numerically computed optimal $ASE\left(\frac{E_b}{N_0}\right)$, denoted as optimal, the approximation from (17), and AWGN capacity $\mathsf{C}\left(\frac{E_b}{N_0}\right)$ versus $\frac{E_b}{N_0}$ for $\alpha = 2.01$ and $\alpha = 3$ (the curve for $\alpha = 4$ is nearly indistinguishable from $\alpha = 3$, and thus is not shown). Note that the approximations to b^* and $\overline{\lambda_\epsilon^T}(b^*)$ are both accurate.

In addition to decreasing the operating spectral efficiency b^* to a fraction of $\mathsf{C}\left(\frac{E_b}{N_0}\right)$, multi-user interference also decreases

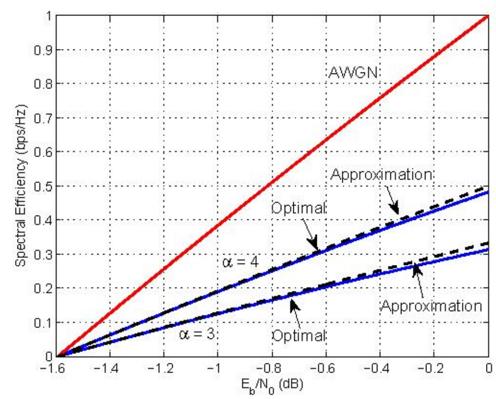

Fig. 3. Optimal Spectral Efficiency for Energy-Limited Networks.

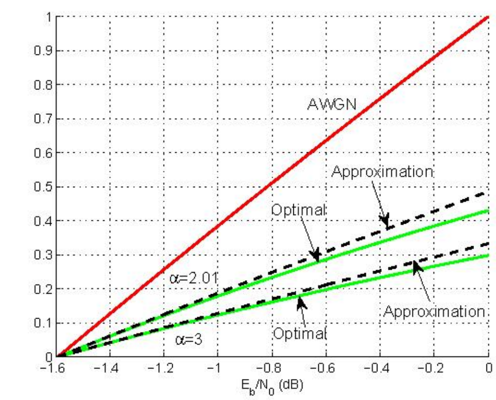

Fig. 4. Optimal ASE for Energy-Limited Networks.

the marginal benefit (in terms of ASE) of increased $\frac{E_b}{N_0}$ as compared to an interference-free channel. In an AWGN channel, spectral efficiency increases at a slope of 2 bps/Hz per 3 dB in the wideband regime ($\mathcal{S}_0 = 2$) [2], while (17) implies that ASE increases only at a rate of $2^{1-\delta}\left((1-\delta)^{(1-\delta)}\delta^\delta\right)$ (< 2) bps/Hz per 3 dB. For example, ASE increases at rates $\frac{2}{3}$ and $\frac{1}{\sqrt{2}}$ bps/Hz per 3 dB when $\alpha = 3,\ 4$, respectively.

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