A Low Complexity Linear Multiuser MIMO Beamforming System with Limited Feedback

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***Abstract*—In this paper, we propose a low complexity linear multiuser beamforming system for the multiple-input multiple-output (MIMO) broadcast channel. We consider the specific case of transmission of a single information stream to two users with two or more receive antennas. Unlike past work in which an iterative algorithm is required to design the beamformers, we first provide a low complexity non-iterative solution via the generalized eigenvector decomposition to jointly optimize transmit beamforming and receive combining vectors. The proposed beamforming technique works for two or more transmit and receive antennas where perfect channel state information is available at the transmitter. To enable practical implementation, a new non-uniform limited feedback algorithm is also proposed that exploits the structure of the algorithm to avoid full channel quantization especially for two transmit antenna systems. The feedback overhead is independent of the number of receive antennas. Simulation results show that the proposed method performs close to the sum capacity of the MIMO broadcast channel even with limited feedback.**

***Index Terms*—Multiuser MIMO, broadcast channels, limited feedback**

I. Introduction

It is well known that dirty paper coding (DPC) is the sum capacity achieving optimal transmit strategy for the MIMO broadcast channel. As DPC is difficult to implement in practice [1]–[6], there has been considerable interest in more practical linear beamforming techniques.

Channel inversion (zero-forcing beamforming) is linear but has some drawbacks [7]. It is only known to work for one receive antenna per user, $i.e.$, the number of transmit antennas must be equal to the total number of receive antennas in the network. Further, it suffers from a power penalty. Coordinated beamforming algorithms work similarly to zero-forcing beamforming but allow fewer streams than the number of receive antennas but require iterative computations [8]–[11]. The authors in [12] proposed a coordinated interference-aware beamforming technique for the MIMO broadcast channel. This technique also has the disadvantage that each user is required to know the channel information and noise variance of other users to estimate the received symbols.

Previous work on linear beamforming [8]–[14] has assumed perfect channel state information at the transmitter. It is, however, not practical in frequency division duplex (FDD) systems. The impact of limited feedback on the performance of multiuser MIMO channels has been analyzed in [15]–[17]. In [15], the performance degradation due to quantized channel information for zero-forcing beamforming when a single antenna is employed was analyzed. More recently, [16], [17] proposed antenna combining techniques using multiple receive antennas at each user. The number of receive antennas at the user terminal, however, must still be smaller than the number of transmit antennas at the transmitter. To solve this dimensionality constraint, in [18] we derived a closed-form expression for the iterative coordinated beamforming algorithm when perfect channel state information is not available at the transmitter, but our solution was valid only for two transmit antennas. [1]

In this paper, we assume that two users are served through two or more transmit antennas. The presence of control channel overhead in practical systems makes it reasonable to consider two users for simultaneous transmission. Here we propose a low complexity non-iterative solution via the generalized eigenvector decomposition for jointly optimizing the transmit beamforming and receive combining vectors. The proposed beamforming technique works for two or more transmit and receive antennas.

To enable practical implementation, we also propose a novel limited feedback solution that requires only quantized channel state information from each user to be sent to the transmitter. The proposed non-uniform quantization is valid for two transmit antenna systems with two or more receive antennas.

[1]The author in [19] proposed a generalized zero-forcing optimized simple beamforming solution but assumed perfect channel state information at the transmitter and the solution was also only valid for two transmit antenna systems.

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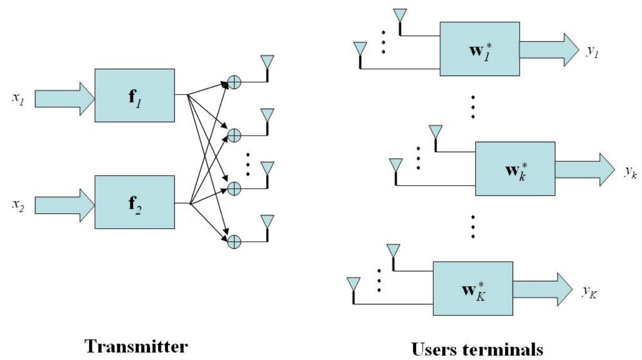

Fig. 1. System Model. The transmitter and the receivers are equipped with N_t and N_r antennas, respectively, while the transmitter supports only two users.

More general non-uniform quantization will be considered in our future work. In this paper, we mostly focus on two transmit antenna systems. Note that, however, the proposed beamforming technique can be used for any number of antenna scenarios where perfect channel state information is available at the transmitter.

By employing the structure of the receive combining operation, our limited feedback solution requires quantizing the entries of symmetric Hermitian matrices derived from the channel. In our solution, the feedback overhead is independent of the number of receive antennas. Note that the proposed method quantizes normalized channel magnitude as well as direction while the algorithm in [17], [20] quantizes only channel directions. Unlike our work in [18], in this paper we propose a non-uniform channel quantization. Through Monte Carlo simulations, we show that the proposed method performs close to the sum capacity of the MIMO broadcast channel even with limited feedback.

This paper is organized as follows. In Section II, we introduce the system model for the linear multiuser MIMO systems, specialized to the case of one data stream for each user. In Section III we present the proposed non-iterative beamforming algorithms, followed by a limited feedback method in Section IV. Performance evaluation and conclusion are given in Sections V and VI, respectively.

II. System Model

Consider a multiuser MIMO system with N_t multiple antennas at the transmitter and N_r multiple receive antennas for each of K users as shown in Fig. 1. Note that only two users will be selected based on some scheduling algorithm. We assume that the channel is flat fading, which can be obtained in practice using multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM). The channel between the transmitter and the first user is represented by a $N_r \times N_t$ matrix $\mathbf{H}_1$. Let x_1 denote the transmit symbol for the first user, and $\mathbf{n}_1$ be the additive white Gaussian noise vector observed at the receiver. Let $\mathbf{f}_1$ and $\mathbf{w}_1$ denote the unit-norm transmit beamforming and receive combining vectors, respectively. For the second user, we use the same notation. Then the signal at the first and second users after receiver combining are [2]

$$\begin{aligned} y_1 =& \mathbf{w}_1^*\mathbf{H}_1\mathbf{f}_1 x_1 + \mathbf{w}_1^*\mathbf{H}_1\mathbf{f}_2 x_2 + \mathbf{w}_1^*\mathbf{n}_1 \\ y_2 =& \mathbf{w}_2^*\mathbf{H}_2\mathbf{f}_2 x_2 + \mathbf{w}_2^*\mathbf{H}_2\mathbf{f}_1 x_1 + \mathbf{w}_2^*\mathbf{n}_2. \end{aligned} \tag{1}$$

Using the coordinated transmission strategies [8]–[10], the transmitter chooses the transmit beamforming and receive combining vectors such that zero multi-user interference is experienced at each receiver. This implies that transmit beamforming vector is chosen in the null space of $\mathbf{w}_l^*\mathbf{H}_l$ $(\forall l \neq k)$, that is,

$$\mathbf{w}_k^*\mathbf{H}_k\mathbf{f}_l = 0 \tag{2}$$

where $k = 1$ or 2. If chosen in this way, $\mathbf{f}_k$ will then cause zero interference to user l by completely removing the interference term in (1).

Although it can be assumed, as in [8], that any number of data streams could be sent to any user, we restrict ourselves to one stream per user. Therefore we cannot achieve every rate vector in the capacity region. This limitation, however, is acceptable in real systems, where spatial division multiplexing access (SDMA) is used in conjunction with single user MIMO transmissions using adaptive switching [21]. Therefore, SDMA targets high cell throughput, which is optimal with infinite number of users in the network [22], while spatial multiplexing to a single user targets high peak rates.

It can be shown that even with dirty-paper coding, transmitting to no more than N_t users at a time incurs only a marginal penalty. In that case, transmitting only one stream per user is asymptotically optimal at high SNR and practically acceptable in low to medium SNR regions [23].

III. Beamformers Design

In our prior work [18], we derived a closed-form expression for the transmit beamformer using the power iteration for the two transmit antenna system. In this paper, we propose a low complexity non-iterative solution for two or more antenna systems with two users. With the maximal ratio combining strategy at the receiver ($i.e.$, $\mathbf{w} = \mathbf{Hf}/\|\mathbf{Hf}\|$), which is a reasonable design choice (but not necessarily the only one) since we come very close to capacity under the zero interference constraint, we have the optimization problem as follows:

$$\mathbf{f}_{1,opt}, \mathbf{f}_{2,opt} = \arg \max_{\mathbf{f}_1:\|\mathbf{f}_1\|=1, \mathbf{f}_2:\|\mathbf{f}_2\|=1} \left\{ \log_2\left(1 + |\mathbf{f}_1^*\mathbf{R}_1\mathbf{f}_1|^2\right) + \log_2\left(1 + |\mathbf{f}_2^*\mathbf{R}_2\mathbf{f}_2|^2\right) \right\} \tag{3}$$

$$s.t. |\mathbf{f}_1^*\mathbf{R}_1\mathbf{f}_2| = 0, |\mathbf{f}_2^*\mathbf{R}_2\mathbf{f}_1| = 0. \tag{4}$$

[2] Upper case and lower case boldface are used to denote matrices $\mathbf{A}$ and vectors $\mathbf{a}$, respectively. If $\mathbf{A}$ denotes a complex matrix, and $\mathbf{A}^*$ and $\mathbf{A}^{-1}$ denote the conjugate transpose and inverse of $\mathbf{A}$, respectively. $[\mathbf{A}]_k$ and $\|\mathbf{A}\|_F$ denote the k-th column and the Frobenius norm of matrix $\mathbf{A}$.

where $\mathbf{R}_1 = \mathbf{H}_1^*\mathbf{H}_1/\|\mathbf{H}_1\|_F^2$, $\mathbf{R}_2 = \mathbf{H}_2^*\mathbf{H}_2/\|\mathbf{H}_2\|_F^2$ are the $N_t \times N_t$ normalized matched channel matrices and $\mathbf{f}_1$, $\mathbf{f}_2$ are the transmit beamformers of size $N_t \times 1$.

There may be several transmit beamformer vectors that satisfy the zero interference constraints for more than two transmit antenna systems. Also, to the best of the authors' knowledge, there is no non-iterative solution yet for the transmit beamformers for two or more transmit and receive antennas. Here we propose a low complexity algorithm for finding the beamformers.

Theorem 1: If $\mathbf{t}_m$, $\mathbf{t}_n$ are generalized eigenvectors of ($\mathbf{R}_1$, $\mathbf{R}_2$) and they correspond to distinct eigenvalues, then any $\mathbf{t}_m$, $\mathbf{t}_n$ satisfy the zero inter-user interference constraint (4), where $m, n = 1, 2, \cdots$, the number of generalized eigenvectors, $m \neq n$.

Proof: The condition is that:

$$\mathbf{R}_1\mathbf{t}_m = \lambda_m \mathbf{R}_2\mathbf{t}_m \tag{5}$$

$$\mathbf{R}_2\mathbf{t}_n = \lambda_n \mathbf{R}_2\mathbf{t}_n \tag{6}$$

for some scalars $\lambda_m \neq \lambda_n$. Therefore,

$$\mathbf{t}_n^*\mathbf{R}_1\mathbf{t}_m = \lambda_m \mathbf{t}_n^*\mathbf{R}_2\mathbf{t}_m \tag{7}$$

where we used (5) to get the first equality. Using (6) we get:

$$\mathbf{t}_n^*\mathbf{R}_2\mathbf{t}_m = \mathbf{t}_m^*\mathbf{R}_2\mathbf{t}_n \tag{8}$$

$$= (1/\lambda_n)\mathbf{t}_m^*\mathbf{R}_1\mathbf{t}_n. \tag{9}$$

This means that

$$\lambda_n \mathbf{t}_n^*\mathbf{R}_2\mathbf{t}_m = \mathbf{t}_m^*\mathbf{R}_1\mathbf{t}_n \tag{10}$$

$$= \mathbf{t}_n^*\mathbf{R}_1\mathbf{t}_m. \tag{11}$$

Therefore:

$$\lambda_m \mathbf{t_n}^*\mathbf{R}_2\mathbf{t}_m = \lambda_n \mathbf{t}_n^*\mathbf{R}_2\mathbf{t}_m \tag{12}$$

which implies $\mathbf{t}_n^*\mathbf{R}_2\mathbf{t}_m = 0$ because $\lambda_m \neq \lambda_n$. The same argument shows that $\mathbf{t}_n^*\mathbf{R}_1\mathbf{t}_m = 0$.

■

From Theorem 1, it is clear that the generalized eigenvectors of $\mathbf{R}_1$ and $\mathbf{R}_2$ satisfy the zero inter-user interference constraints (4). The authors recognize that this solution is not essentially optimal for arbitrary antenna configurations. The idea here is to use the proposed transmit beamformers to obtain zero inter-user interference. Note that this solution can be directly used where the transmitter has perfect channel state information.

IV. Codebook for Channel Quantization

In this section, we focus on $N_t = 2$ for channel quantization. The general non-uniform quantization will be considered in our future work. Here we propose to quantize the sufficient channel state information $\mathbf{R}_k$ at the receiver k and send it to the transmitter through a limited feedback link so that the transmitter can compute the transmit beamformers. As the transmitter needs the complete channel matrix $\mathbf{R}_k$, and not just its subspace information as with Grassmannian beamforming [20], we use direct quantization exploiting the symmetry of $\mathbf{R}_k$. In this section, we omit the user index k for simplicity, though we still analyze two user systems.

For analysis, we model the elements of each users channel matrix $\mathbf{H}$ as independent complex Gaussian random variables with zero mean and unit variance $\mathcal{CN}(0, 1)$. The sufficient channel information that the transmitter needs to know to compute the beamformers is $\mathbf{R}_1$, $\mathbf{R}_2$, where the matrix $\mathbf{R}$ without user index is given by

$$\mathbf{R} = \frac{\mathbf{H}^*\mathbf{H}}{||\mathbf{H}||_F^2} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}. \tag{13}$$

We quantize the elements in the matched channel matrix $\mathbf{R}$ in (13) using scalar parameters α, β, and γ. Since $\mathbf{R}$ is a Hermitian matrix with unit Frobenius norm,

$$R_{11} = \alpha \text{ and } R_{22} = 1 - \alpha, \tag{14}$$

$$R_{12} = \beta + j\gamma \text{ and } R_{21} = R_{12}^* \tag{15}$$

where $0 \leq \alpha \leq 1$ and $-0.5 \leq \beta, \gamma \leq 0.5$.

Theorem 2: R_{11} has a beta distribution with parameters (N_r, N_r).

Proof: See [24].

■

Therefore, the probability density function (pdf) and cumulative density function (cdf) of R_{11} are given by

$$f_{R_{11}}(x) = \frac{(2N_r - 1)!}{((N_r - 1)!)^2}\Big((1 - x)x\Big)^{N_r - 1}, \tag{16}$$

$$F_{R_{11}}(x) = \frac{B(x; N_r, N_r)}{B(N_r, N_r)}, \tag{17}$$

where $B(x; a, b)$ is the incomplete beta function, *i.e.*, $B(x; a, b) = \int_0^x t^{a-1}(1 - t)^{b-1}dt$.

Theorem 3: The real and imaginary parts of R_{12} have the same distribution as R_{11} with a shift by $1/2$.

Proof: See [24].

■

Since all the elements in $\mathbf{R}$ have the same distribution with a different mean, we need to generate only one codebook. Suppose that Q-bit scalar codewords denoted by $C = \{c_1, c_2, \cdots, c_{2^Q}\}$ are used for the channel quantization. For α, we find the codebook that satisfies the following condition:

$$\int_{c_{i-1}}^{c_i} f_{R_{11}}(x)dx = \frac{1}{2^Q + 1}, \tag{18}$$

where $i = 1, 2, \cdots, 2^Q$. Since the CDF $F_{R_{11}}$ in (17) is a regularized beta function $I(x; N_r, N_t)$, we can rewrite (18) as follows:

$$I(c_i, N_r, N_t) - I(c_{i-1}, N_r, N_t) = \frac{1}{2^Q + 1}, \ i = 1, \cdots, 2^Q \tag{19}$$

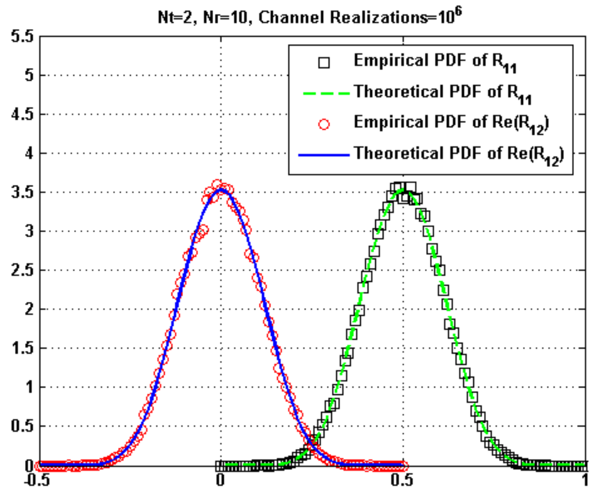

Fig. 2. Channel quantization of R_{11} and R_{12} where $N_t = 2$ and $N_r = 10$. Note that R_{11} is real and R_{12} are complex.

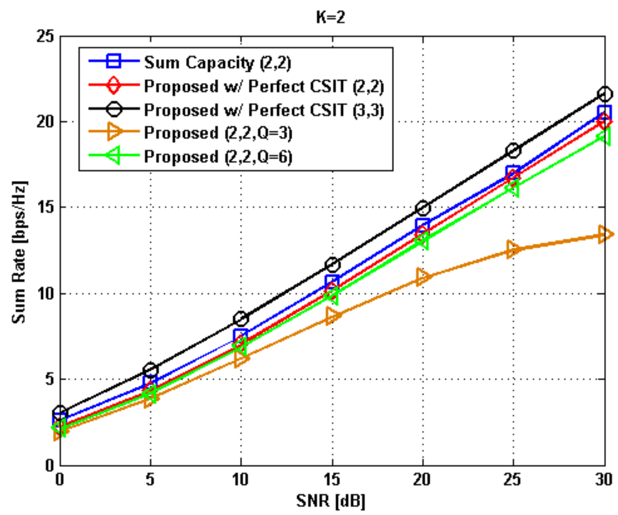

Fig. 3. Sum rates vs. SNR for (2,2) and (3,3) scenarios with two users where (a, b) means a transmit antennas and b receive antennas. We used the same codebook size Q per parameter for simplicity thus total $3Q$ bits are required per user.

where

$$I(x; N_r, N_r) = \sum_{j=N_r}^{2N_r-1} \frac{(2N_r-1)!}{j!(2N_r-1-j)!} x^j (1-x)^{2N_r-1-j}. \tag{20}$$

and c_0=0. For β and γ, we use the same codebook after shifting the mean by $\frac{1}{2}$. Upon receiving the codebook indices of R_{11}, $Re(R_{12})$, and $Im(R_{12})$ from the receiver over a control channel, the transmitter can estimate $\mathbf{R}$ and compute the transmit beamformers before transmitting the data. In this paper, we use the same quantization level Q for all elements, for simplicity. Vector quantization could be used to optimize the feedback overhead but we leave this issue for future research.

In [17], it was proposed that each user quantizes its effective channel after multiplication by the pre-combining vector that produces the lowest quantization error, using a random codebook. This approach, however, has some drawbacks. A search over the codebooks for the different number of receive antennas is required to find the best quantization. Computational complexity increases as the number of receive antennas increases. In our solution, the feedback overhead remains the same regardless of the number of receive antennas, since $\mathbf{R}$ is always a matrix of size 2×2. Note that the proposed method quantizes channel magnitude as well as direction while the algorithm in [17] quantizes only channel directions.

From the results above, we can now compute the transmit beamformers and the receive combining vectors where the transmitter has only limited feedback from the receivers. The procedure to compute the transmit beamformers and the combining vectors is as follows. Based on the limited feedback information, first, the transmitter computes the matched channel matrices $\tilde{\mathbf{R}}_1$ and $\tilde{\mathbf{R}}_2$ where $\tilde{\mathbf{R}}$ is the estimated information of $\mathbf{R}$ and finds all generalized eigenvectors of $\tilde{\mathbf{R}}_1$ and $\tilde{\mathbf{R}}_2$. Then let $\mathbf{T}$ be the set of eigenvectors of $\tilde{\mathbf{R}}_1$ and $\tilde{\mathbf{R}}_2$. The eigenvector pair that maximizes the sum rate is selected as follows:

$$\begin{aligned} \mathbf{f}_1, \mathbf{f}_2 = \arg & \max_{\mathbf{t}_n, \mathbf{t}_m \in \{\mathbf{T}\}, \mathbf{t}_n \neq \mathbf{t}_m} \\ & \left\{ \log_2 \left(1 + \frac{P}{N_t \sigma^2} |\mathbf{t}_n^* \tilde{\mathbf{R}}_1 \mathbf{t}_n|^2 \right) \right. \\ & \left. + \log_2 \left(1 + \frac{P}{N_t \sigma^2} |\mathbf{t}_m^* \tilde{\mathbf{R}}_2 \mathbf{t}_m|^2 \right) \right\} \end{aligned} \tag{21}$$

where P is the total power at the transmitter. Note that there are two eigenvectors when $N_t = 2$ so we need to compute (21) using only two beamformer pairs $\{\mathbf{f}_1, \mathbf{f}_2\} = [\{\mathbf{t}_1, \mathbf{t}_2\}$ or $\{\mathbf{t}_2, \mathbf{t}_1\}]$. Thus the computational complexity for finding the beamformers is marginal.

V. Performance Evaluation

With perfect channel state information at the transmitter, there is no inter-user interference thanks to the zero-interference constraint. This is, however, not possible in the limited feedback system since the transmitter computes the transmit beamformers based on the quantized channel information through a low rate limited feedback channel. Therefore, we use the achievable sum rate given by

$$\begin{aligned} \mathcal{R} = & \log_2 \left(1 + \frac{\frac{P_1}{\sigma^2} |\mathbf{w}_1^* \mathbf{H}_1 \mathbf{f}_1|^2}{\frac{P_1}{\sigma^2} |\mathbf{w}_1^* \mathbf{H}_1 \mathbf{f}_2|^2 + 1} \right) \\ & + \log_2 \left(1 + \frac{\frac{P_2}{\sigma^2} |\mathbf{w}_2^* \mathbf{H}_2 \mathbf{f}_2|^2}{\frac{P_2}{\sigma^2} |\mathbf{w}_2^* \mathbf{H}_2 \mathbf{f}_1|^2 + 1} \right) \text{ [bps/Hz]} \end{aligned}$$

where $\mathbf{w}_1$ and $\mathbf{w}_2$ are the receive combining vectors ($\mathbf{w}_i = \mathbf{H}_i\mathbf{f}_i\|\mathbf{H}_i\mathbf{f}_i\|$ where $i = 1$ or 2), respectively. In this paper, we assume that each user can estimate the effective channel vectors $\mathbf{H}_i\mathbf{f}_i$ using dedicated pilot channels. Note that the transmit beamformer $\mathbf{f}_i$ is obtained through the estimated channel matrices $\tilde{\mathbf{R}}_i$ and $\mathbf{H}$ is the perfect channel matrix.

In Fig. 2, we compare the derived theoretical PDFs of R_{11} and the real part of R_{12} in (13) with the empirical PDFs generated by many channel realizations where $N_t = 2$ and $N_r = 10$ as an example. In the simulation, we generated 10^6 random channels to validate the derived PDFs with the empirical PDFs. Fig. 3 illustrates the sum rates of the proposed non-iterative coordinated beamforming with limited feedback (using the proposed non-uniform quantization), coordinated beamforming with perfect channel state information, and the sum capacity where two users are in the network. In this case, no scheduling algorithm is needed. This situation illustrates that the proposed method yields good performance even without the help of multiuser diversity, in which case opportunistic beamforming methods [25] and unitary codebook-based precoding methods [26], [27] fail. To illustrate the effect of limited feedback, we use three different feedback sizes $Q = 3$, and $Q = 6$ Note that the total feedback overhead is $3Q$ per user in this scenario. As explained in Section III, we can see that the proposed beamforming algorithm still works for more than two transmit antennas (in this case, with perfect channel state information).

VI. Conclusion

This paper presented a downlink multiuser MIMO algorithm tailored for practical implementation. In particular, for the downlink channel where the transmitter is equipped with two or more transmit antennas, it supports transmission of one stream to each of two users simultaneously. For this scenario, we proposed a low complexity solution for the transmit beamformers using the generalized eigenvalue decomposition, avoiding the need for iterative computation. We also proposed a new limited feedback algorithm for two transmit antennas that has the same feedback overhead regardless of the number of receive antennas. As a part of future work, we will investigate more general cases like the more than two user scenario.

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