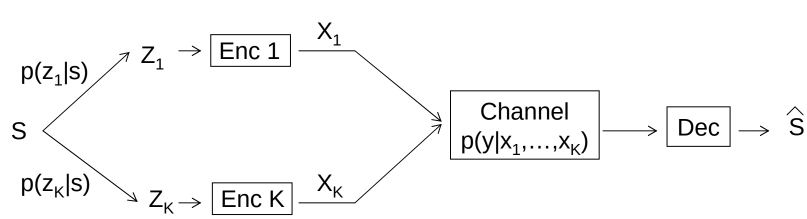
EE 8510: Sensor Network Notes

- Sensor Network Model
- Single Sensor Capacity
- Multi-terminal Source Coding
- CEO Problem

Sensor Network Model

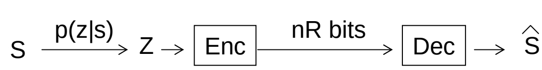


- S: source (random variable)
- $Z_1,\ldots,Z_K$: sensor observations
- $X_1,\ldots,X_k$: sensor transmissions
- Y: channel output
- $\hat{S}$: estimate of source

Simplified Models

1. Single sensor
   - Separation theorem holds
2. $S=(S_1,\ldots,S_k)$, $Z_i = S_i$
   - Multi-terminal source coding and MAC
3. No channel (sensors send bits to decoder)
   - CEO problem

Single Sensor, No Channel

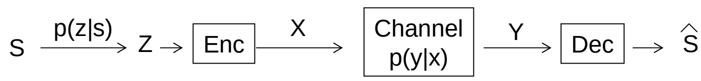


- Remove channel for simplicity
- Decoder transmits R bits per symbol
- Rate distortion with noisy observation [1]:

$$R(D) = \min_{p(\hat{s}\mid z)} I(Z;\hat{S})$$
$$\text{s.t.} \quad E[d(s,\hat{s})] \le D$$

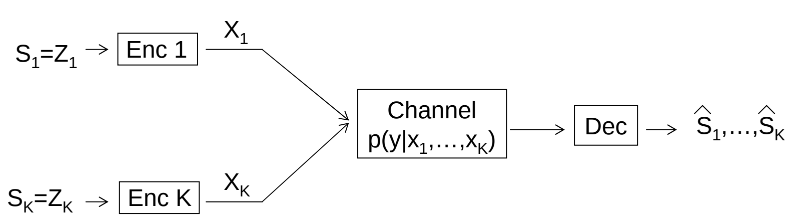
- Only difference is that must map from observation z to estimate of s, instead of from s to estimate of s (since s not known at encoder)

Single Sensor (# 1)



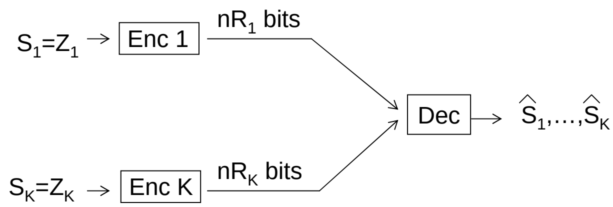
- Separation theorem holds: Can get estimate of S satisfying distortion D if and only if R(D) < C [1]
- Implication:
  - First perform quantization of Z (to achieve R(D))
  - Next perform channel coding treating quantization index as the message
  - Receiver decodes message (i.e. quantization index), and estimates using reconstruction codebook

Arbitrary Source (# 2)



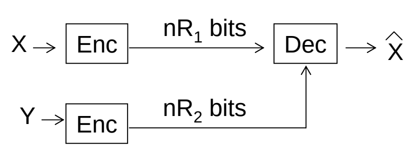
- Arbitrary source $S=(S_1,\ldots,S_k)$
- $Z_i = S_i$ (no measurement noise)
- Question: What sources can be reconstructed at decoder?
- If require lossless coding of S, is MAC with correlated sources [2]
- Same conclusion for lossy coding
  - Capacity unknown, separation does not hold

Arbitrary Source, No Channel



- Multi-terminal source coding: What rates required to reconstruct S according to certain fidelity?
- Lossless
  - Capacity region known (Slepian-Wolf) [3]
  - No sum rate loss relative to cooperative coding: $H(S_1,\ldots S_K)$
- Lossy
  - Capacity region unknown
  - Inner/Outer bounds (Berger-Tung) [4] (similar to Wyner-Ziv)
  - Generally rate loss relative to cooperative coding

Source Coding with Side Info



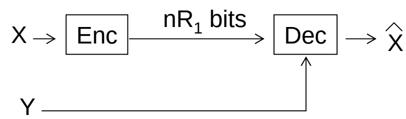
- Want to decode X losslessly using Y only as side information
- (R_1,R_2) achievable iff (Wyner) [5]:

$$R_1 \geq H(X\,|\,U)$$
$$R_2 \geq I(Y;U)$$

for some $p(u|y)$

- Achievability: Cover Y with reconstruction r.v. U, use random binning on X and Markov lemma to find typical (X^n, U^n) pair in bin
- Markov Lemma: If X->Y->Z, (x^n,y^n) typical, and z^n chosen iid $p(z^n|y^n)$, then (x^n,y^n,Z^n) typical with nearly probability one
  - By definition X->Y->U. Since (x^n, y^n) typcial with high probability, also have (x^n,y^n,u^n) typical, which implies (x^n,u^n) typical

Rate-Distortion with Side Info



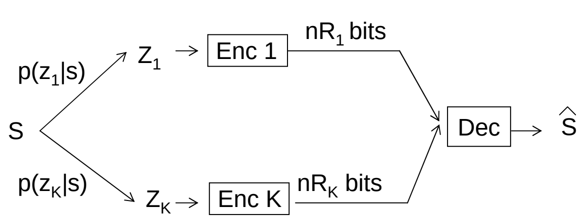
- Want to lossy decoding of X using Y as side information (Wyner-Ziv) [6]

$$R_Y(D) = \min_{\substack{p(u|x),\, f(y,u) \\ E[d(x, f(y,u))] \le D}} I(X;U) - I(Y;U)$$

- Generally is <u>strict</u> rate loss relative to cooperative encoder (i.e. encoder who knows X and Y)

- Achievability: Generate $2^{nI(X;U)}$ codewords iid p(u) and randomly place in 2^{nR} bins. Choose codeword typical with x^n and send bin index. Decoder finds (u^n, y^n) typical (Markov lemma) in correct bin. Given correct u^n, computes f(u,y) to estimate X.

CEO Problem (# 3)



- Sensors (agents) observe source S, transmit quantized measurements to decoder (CEO) [7]
- Decoder estimates S according to distortion criteria (lossless or lossy reconstruction)
- Question: What rates $R_1,\ldots,R_K$ are required to achieve certain distortion?
  - Region not known in general

Lossless Coding of S

- Discrete source S must be reconstructed losslessly (P_e -> 0) at decoder
- Conditionally (on S) iid observations
- Fix sum of rates to be R
- If K goes to infinity and sensors could cooperate, could pool observations to recover S perfectly and if R > H(S) could losslessly transmit S
- Non-cooperative sensors <u>cannot</u> achieve perfect reconstruction at decoder for any finite R [7]

Quadratic CEO Problem

- $Z_i = S + N_i$
- $S \sim N(0, P_S)$, $N_i \sim N(0,P_N)$ (independent)
- $D(S,\hat{S}) = (S-\hat{S})^2$
- Berger-Tung achievable region achieves capacity [8,9,10]
- Achievability:
  - Sensors perform single-user quantization on observation Z_i
  - Decoder recovers each quantized observation, which add additional Gaussian noise to Z_i
  - Decoder estimates S (via MMSE) based on quantized observations, which are jointly Gaussian with S

References

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