EE8510 Project

Using Noncoherent Modulation for Training

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Noncoherent Channel Model

$$\boldsymbol{X} = \sqrt{\frac{\rho T}{M}}\boldsymbol{\Phi}\boldsymbol{H} + \boldsymbol{W}$$

- Rayleigh flat block-fading, T: channel coherence interval
  Marzetta & Hochwald [IT'99]
- $\boldsymbol{\Phi} \in \mathcal{C}^{T\times M}$: Transmitted signal matrix
- $\boldsymbol{X} \in \mathcal{C}^{T\times N}$: Received signal matrix
- $\boldsymbol{H} \in \mathcal{C}^{M\times N}$: Unknown channel matrix, *i.i.d.* $\mathcal{CN}(0,1)$
- $\boldsymbol{W} \in \mathcal{C}^{T\times N}$: Additive noise matrix, *i.i.d.* $\mathcal{CN}(0,1)$
- Power Constraint: $\mathbb{E}\{\mathrm{Tr}\ (\boldsymbol{\Phi}^{\dagger}\boldsymbol{\Phi})\} = M$
- ρ: Average SNR per receive antenna

Known Results on Coherent Capacity

- Coherent Capacity ($\boldsymbol{H}$ known to Rx but not Tx)

  Foschini [*Bell Labs. Tech. J*'96], Telatar [*ETT*'99]

$$C_{coherent}(\rho) = \mathbb{E}\{\log_2 \det(\boldsymbol{I}_M + \frac{\rho}{M}\boldsymbol{H}\boldsymbol{H}^\dagger)\}$$
$$= \mathbb{E}\{\log_2 \det(\boldsymbol{I}_N + \frac{\rho}{M}\boldsymbol{H}^\dagger\boldsymbol{H})\}$$

- Asymptotically ($M = N$):

$$\lim_{M\to\infty}\lim_{\rho\to\infty}\left[\frac{C_{coherent}(\rho)}{M} - \log_2(\frac{\rho}{e})\right] = 0$$

Known Results on Noncoherent Capacity

- Noncoherent Capacity ($\boldsymbol{H}$ unknown to either Tx or Rx)
  Zheng & Tse [IT'02]
  high SNR $\rho \gg 1, T \geq 2M = 2N$

$$C_{M,M}(\rho) = (1 - \frac{M}{T})C_{coherent}(\rho) + c(T,M) + o(1)$$

- Asymptotically (fix the ratio $\alpha = M/T$):

$$\frac{C_{M,M}(\rho)}{M} \rightarrow (1-\alpha)\log_2\left[(\frac{\rho}{e}) \cdot 2^{\frac{k(\alpha)}{(1-\alpha)\ln 2}}\right]$$

where

$$k(\alpha) = \frac{(1-\alpha)^2}{2\alpha}\ln(1-\alpha) + \frac{\alpha}{2}\ln\alpha + \frac{1-\alpha}{2} < 0$$

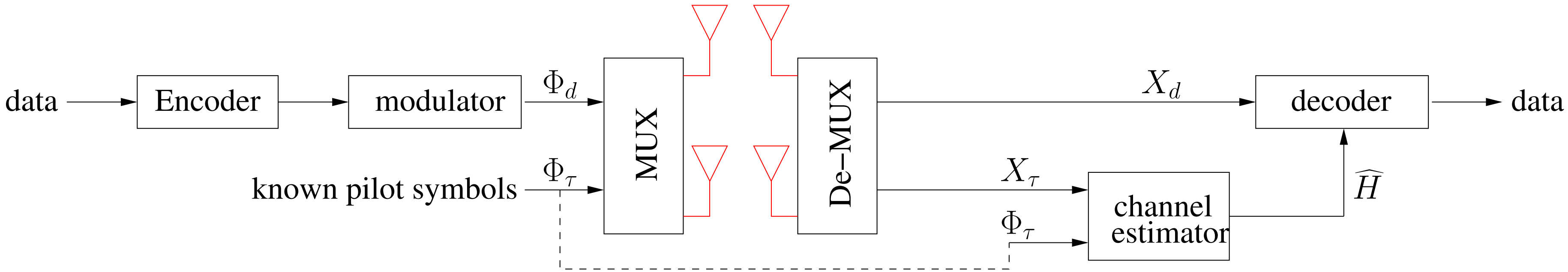
Unitary Space Time Modulation (USTM)

- Constellations of $T \times M$ space-time signals
  Hochwald & Marzetta [IT'00]

$$\{\Phi_l, l = 1, \ldots, L\} : \Phi_l^\dagger \Phi_l = \boldsymbol{I}_M$$

- Capacity achieving when $T \gg M$ or $\rho \gg 1$ with $M \leq \min\{N, T/2\}$
- Designed by numerical optimizations, no particular algebraic structure
- Exponential demodulation complexity
  Constellation size L: 2^{RT} for a given rate of R bits per symbol

Training-Based Schemes



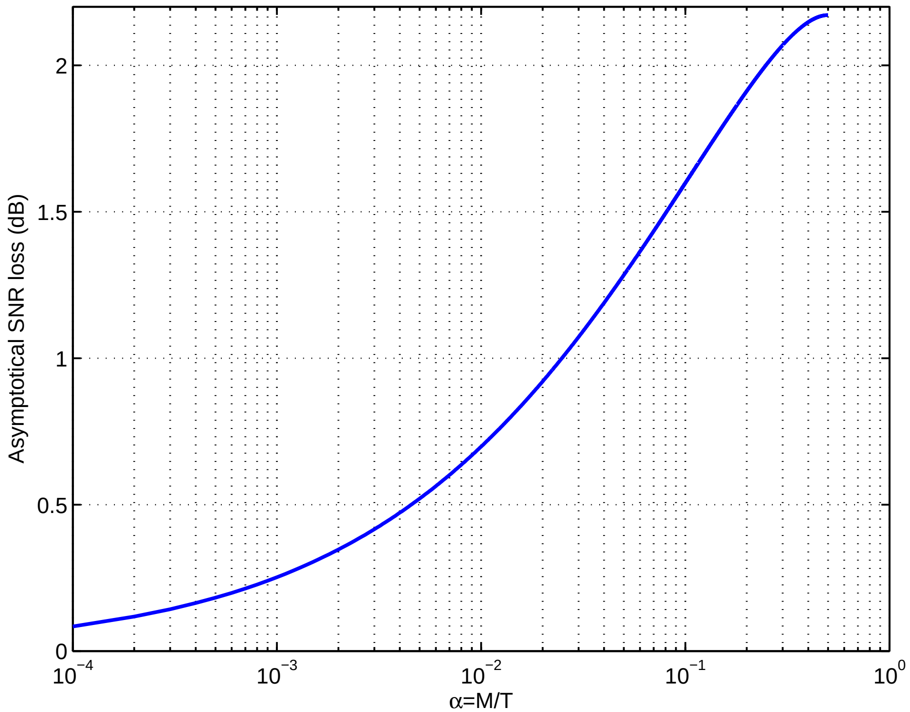
- Multiplexing known pilot symbols with data symbols
- A tight capacity lower bound ($\boldsymbol{\Phi}_\tau^\dagger \boldsymbol{\Phi}_\tau = \boldsymbol{I}_M$)
  Zheng & Tse [IT'02], Hassibi & Hochwald [IT'03]
  $C^L_{known}(\rho) = (1 - \frac{M}{T}) C_{coherent}(\rho_{eff})$
- Suffers SNR loss (due to estimation error) at high SNR: $\rho_{eff} < \rho$
- Optimal for noncoherent channel when T is large

  Q: How large T is enough?

Asymptotic SNR Loss of Training with Known Symbols

$$T \geq 2M = 2N \to \infty, \rho \to \infty, \text{ but } \alpha = M/T \text{ fixed}$$

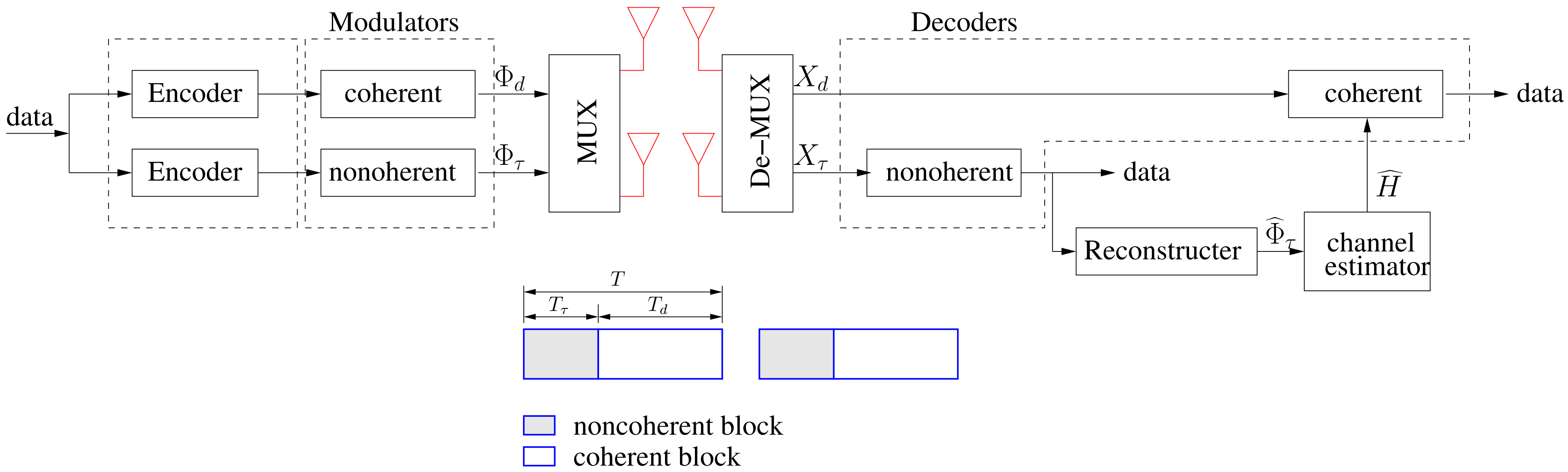
$$\rho_{loss}(\alpha) = \left[1 + 2\sqrt{\alpha(1-\alpha)}\right] \cdot 2^{\frac{k(\alpha)}{(1-\alpha)\ln 2}}$$



$\rho_{loss}(0.5) = 2.17\text{dB}, \rho_{loss}(10^{-1}) = 1.598\text{dB},$

$\rho_{loss}(10^{-2}) = 0.698\text{dB}, \ \rho_{loss}(10^{-3}) = 0.252\text{dB}$

Using Noncoherent Modulation for Training



- Pilot symbols are unknown to the receiver, and can carry data information
- T_τ is only a fraction of T, leading to less complexity
- The tradeoff between complexity and SNR loss can be obtained by selecting a suitable T_τ

Using USTM as Training Symbols

- Choose $\mathbf{\Phi}_\tau$ as USTM: $\mathbf{\Phi}_\tau^\dagger \mathbf{\Phi}_\tau = \boldsymbol{I}_M$

$$C^L_{unknown}(\rho) = \alpha_1 I_{USTM(\rho)} + (1-\alpha_1) C_{coherent}(\rho_{eff}),$$

$\alpha_1 = T_\tau / T$: Time-Sharing factor!

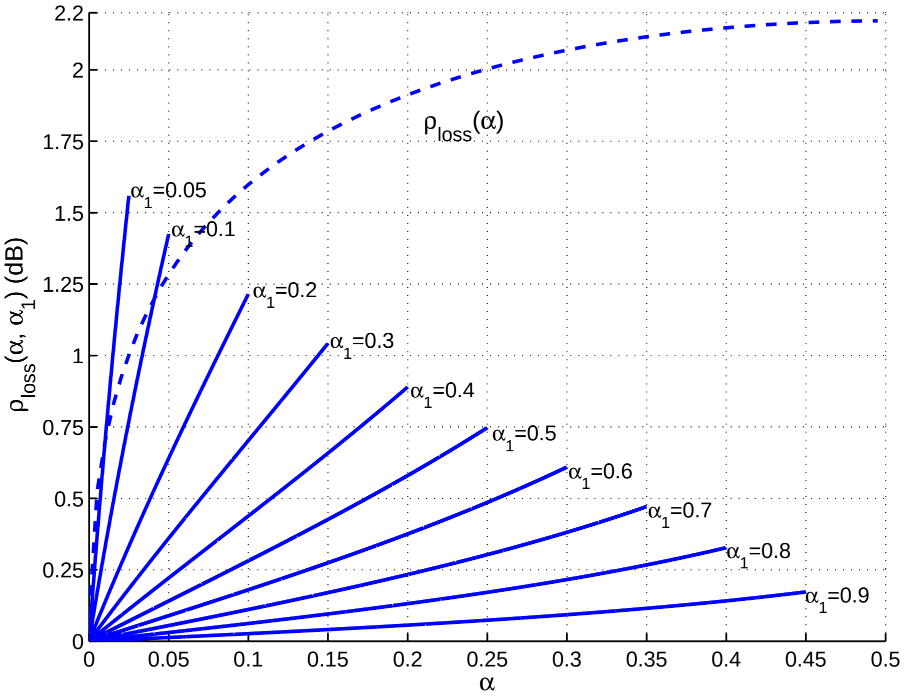
- Asymptotically,
$T \geq T_\tau \geq 2M = 2N \to \infty, \rho \to \infty$, but $\alpha = M/T, \alpha_1 = T_\tau/T$ fixed

$$\frac{C^L_{unknown}(\rho)}{M} \to (1-\alpha)\log_2\left[\left(\frac{\rho}{e}\right) \cdot \left(1 + \frac{\alpha}{\alpha_1}\right)^{\left(\frac{-1+\alpha_1}{1-\alpha}\right)} \cdot 2^{\frac{\alpha_1 k\left(\frac{\alpha}{\alpha_1}\right)}{(1-\alpha)\ln 2}}\right]$$

$$\text{noncoherent capacity: } \frac{C_{M,M}(\rho)}{M} \to (1-\alpha)\log_2\left[\left(\frac{\rho}{e}\right) \cdot 2^{\frac{k(\alpha)}{(1-\alpha)\ln 2}}\right]$$

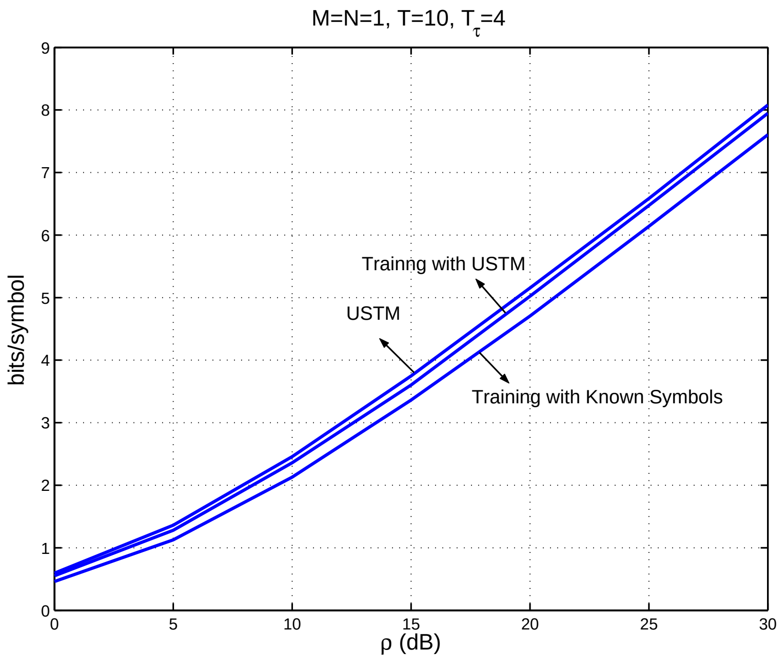
Asymptotic SNR Loss of Training with USTM Symbols

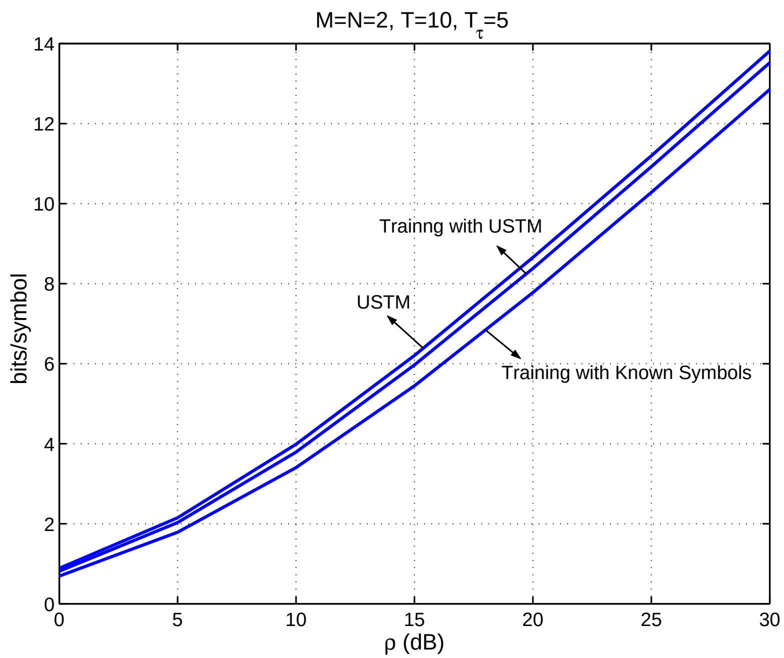
$$\rho'_{loss}(\alpha, \alpha_1) = \left(1 + \frac{\alpha}{\alpha_1}\right)^{\frac{1-\alpha_1}{1-\alpha}} \cdot 2^{\frac{k(\alpha) - \alpha_1 k\left(\frac{\alpha}{\alpha_1}\right)}{(1-\alpha)\ln 2}}$$



- For most interested (α, α_1) combinations $(\alpha > 0.05, \alpha > 0.1)$, $\rho'_{loss}(\alpha, \alpha_1) < \rho_{loss}(\alpha)$
- For sufficiently small α and α_1, $\rho'_{loss}(\alpha, \alpha_1) > \rho_{loss}(\alpha)$ benefit of power control > advantage of noncoherent training

Numerical Results





- $M = N = 1, T = 10, T_\tau = 4$– simulation results: 1.5dB, 0.4dB, asymptotic results: $\rho_{loss}(0.1) = 1.598\text{dB}$, $\rho'_{loss}(0.1, 0.4) = 0.438\text{dB}$
- $M = N = 2, T = 10, T_\tau = 5$– simulation results: 1.8dB, 0.55dB, asymptotic results: $\rho_{loss}(0.2) = 1.912\text{dB}$, $\rho'_{loss}(0.2, 0.5) = 0.580\text{dB}$

Conclusion

- Training with known pilot symbols converges to the optimal very slowly
- Training with unknown USTM symbols provides a tradeoff between complexity and performance

Thank You!