EE8510 Course Project

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Maximizing the Worst-User's Capacity for a Multi-User OFDM Uplink Channel

Juyul Lee

Department of Electrical & Computer Engineering

University of Minnesota

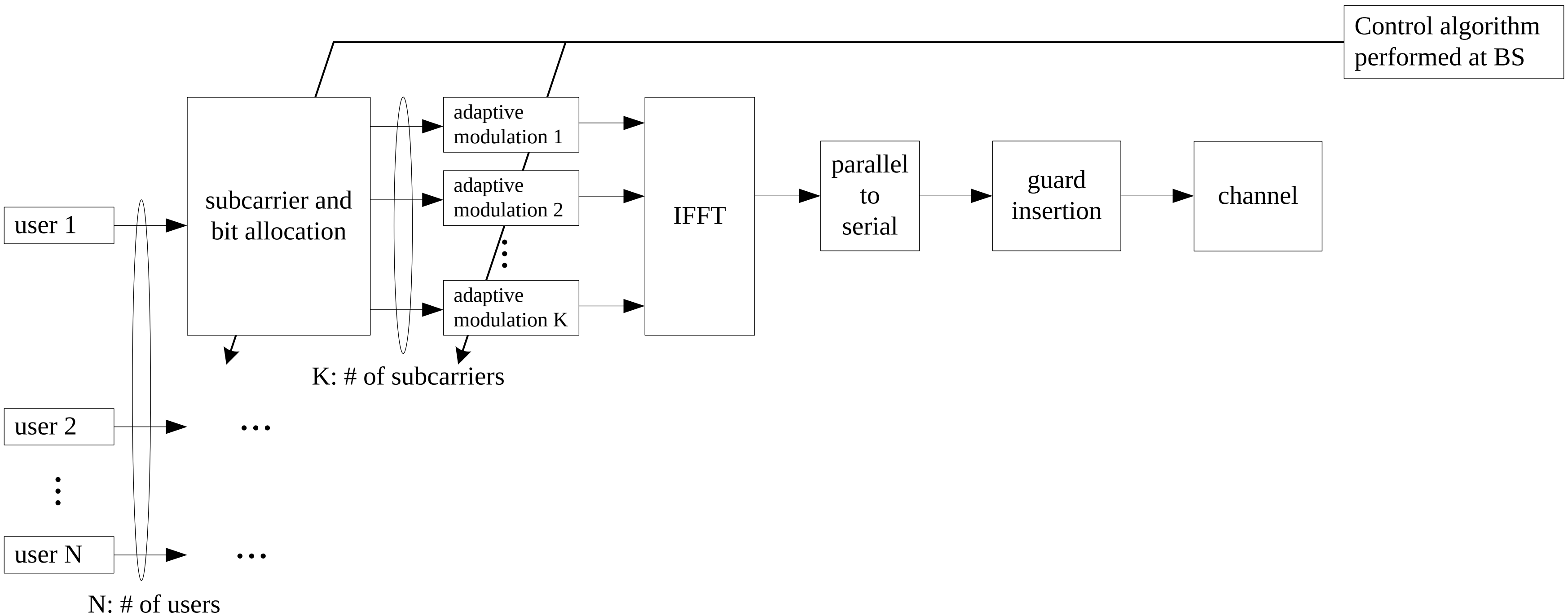
Outline

- OFDM channel
- Problem description
- Literature
- Multi-user scalar channel
- Single-user vector channel
- Multi-user vector channel
- Conclusion

OFDM History

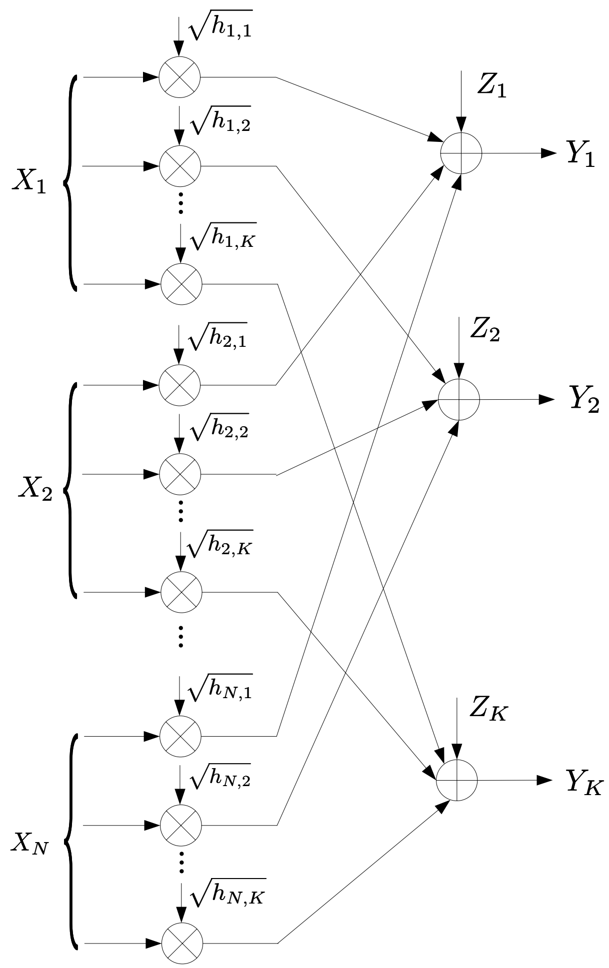
- 1950s
  - Concept of multicarrier modulation with non-overlapping subchannels
- 1960s
  - Orthogonal subchannels
- 1970s
  - A US patent, applications in military
- 1980s
  - OFDM employing QAM with DFT technique
- 1990s
  - Various standards for wireline/wireless systems based on OFDM
- 2000 –
  - Application to cellular environments
    - FLASH-OFDM(Flarion), OFDM-HSDPA(Nortel), VSF-Spread OFDM(NTT DoCoMo), HPi(ETRI+Samsumg), HMm(ETRI),...

Uplink OFDM Model



No ICI, No non-linear effect… ➔ Everything is perfect!!!

Multi-User Vector Channel Model



$$\mathbf{Y} = \sum_{n=1}^{N} \mathbf{H}_n \mathbf{X}_n + \mathbf{Z}$$

N : Number of users

K : Number of subchannels

$\mathbf{Y}, \mathbf{X}_n, \mathbf{Z}$: $K \times 1$ vector

$\mathbf{H}_n$: diagonal matrix

Problem Description

- Assumptions
  - Noises are Gaussian.
  - Channel gains are known.
  - Each user has its own power constraint.
  - All variables are real (for simplicity).

- Problem: Maximizing the worst-user's rate (vector MAC)
  - worst-user: who has the lowest maximum rate (capacity) due to small channel gain and small power constraint

- To answer
  - What is the optimal rate for each user?
  - How to achieve the rate in an information-theoretic point of view? (That is, how to allocate power)

Related Works

$$\begin{array}{ll} \max & \mu\mathbf{R} = \mu_1 R_1 + \mu_2 R_2 \\ \text{subj} & P \end{array}$$

$$\min P - \mu\mathbf{R} \quad \mu : Lagrangian$$

$$\begin{array}{ll} \min & P \\ \text{subj} & \mathbf{R} \end{array}$$

- Downlink (Broadcast channel)
  - Maximization a given rate vector [Tse 97]
    - Show the duality with minimizing power to support a given rate vector
  - Considering proportional fairness [Shen, Andrews, Evans 03]
  - Maximizing the worst-user's rate [Rhee, Cioffi 00]
- Uplink (Multiple access channel)
  - Vector channel capacity region structure [Tse, Hanly 98]
  - Maximizing sum-rate capacity [Yu, Rhee, Boyd, Cioffi 04]
    - Iterative solution

- Multi-user scalar channel
  - How the optimal rate is determined.
- Single-user vector channel
- Multi-user vector channel

Multi-User Scalar Channel (1)

- No allocation strategy is possible. Instead, examine which point in the capacity region is the solution of the problem.
- Two-user multiple access channel model

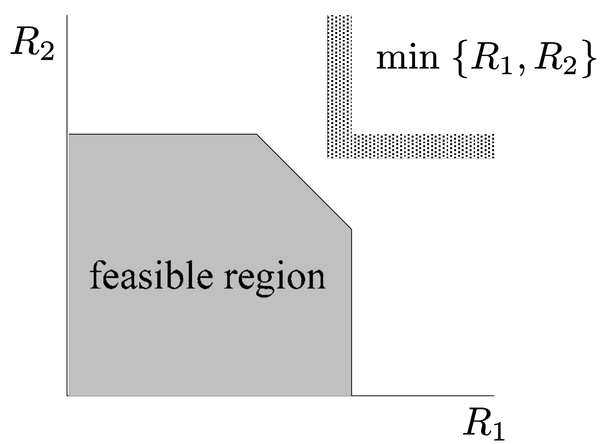
$$Y = \sqrt{h_1}X_1 + \sqrt{h_2}X_2 + Z \qquad Z \sim \mathcal{N}(0,1)$$

When there are power constraints ($E[|X_1|^2] \leq P_1$, $E[|X_2|^2] \leq P_2$), the capacity region is given by

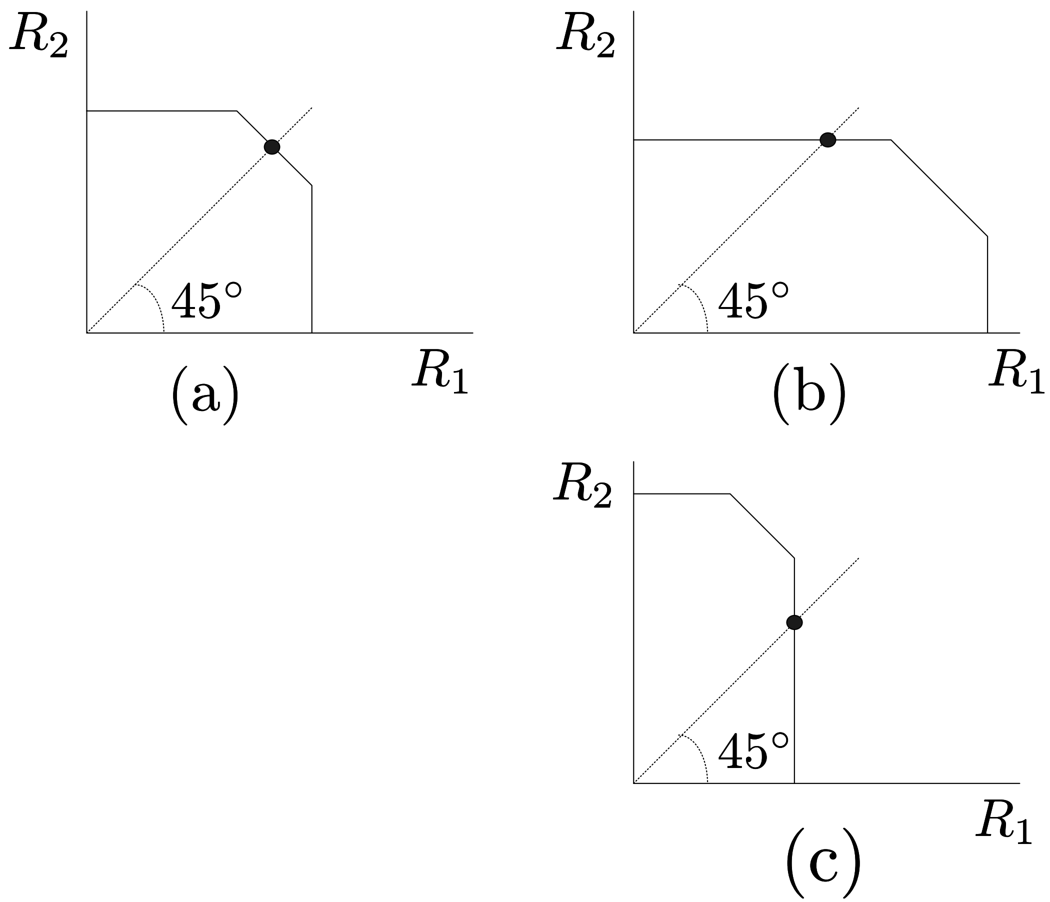
$$\begin{aligned} R_1 &\leq \frac{1}{2}\log_2\left(1 + h_1 P_1\right) \\ R_2 &\leq \frac{1}{2}\log_2\left(1 + h_2 P_2\right) \\ R_1 + R_2 &\leq \frac{1}{2}\log_2\left(1 + h_1 P_1 + h_2 P_2\right) \end{aligned}$$

Multi-User Scalar Channel (2)

$$\begin{aligned} \text{maximize} \quad & \min\{R_1, R_2\} \\ \text{subject to} \quad & R_1 \le \frac{1}{2}\log_2\left(1 + h_1 P_1\right) \\ & R_2 \le \frac{1}{2}\log_2\left(1 + h_2 P_2\right) \\ & R_1 + R_2 \le \frac{1}{2}\log_2\left(1 + h_1 P_1 + h_2 P_2\right) \end{aligned}$$

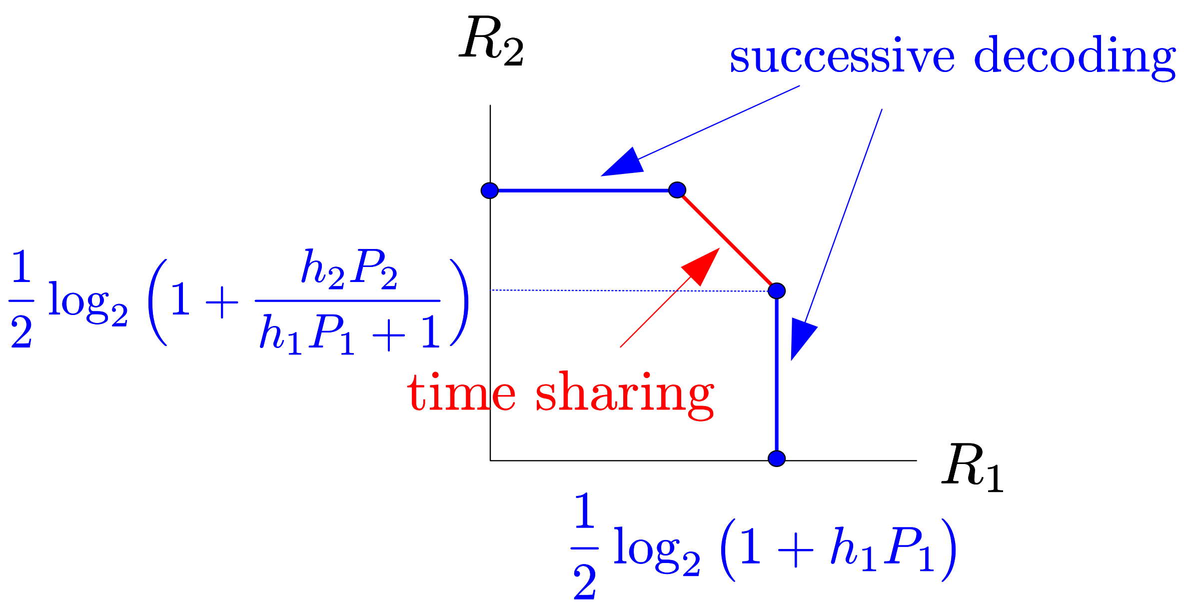


Multi-User Scalar Channel (3)



Multi-User Scalar Channel (4)

- Strategies to achieve the boundary point



Multi-User Scalar Channel (5)

- Generalization to Multi-user cases
  - The rate vector that maximizing the minimum capacity is determined where the straight line $r_1=r_2=...=r_N$ is touching the boundary of the capacity region.

- Implications
  - All users rates are the same.
  - That is, tell us what is the minimum possible rate to communicate with all users.

- Multi-user scalar channel

- Single-user vector channel
  - Show allocation strategy.

- Multi-user vector channel

Single-User Vector Channel

- Same as the parallel Gaussian channel

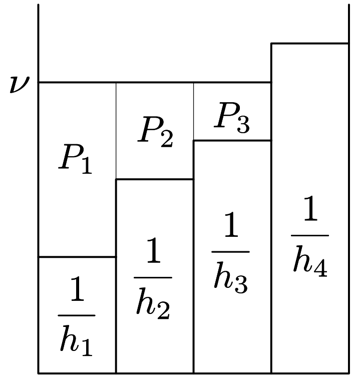
$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z} \qquad \mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

$$\mathrm{tr}\big(E[\mathbf{X}\mathbf{X}^T]\big) \leq P$$

$$\text{where } \mathbf{H} = \mathrm{diag}(\sqrt{h_1},\ \sqrt{h_2},\ \cdots,\ \sqrt{h_K}).$$

WLOG $\quad h_1 \geq h_2 \geq \cdots \geq h_K$, and $\mathbf{Q} := E[\mathbf{X}\mathbf{X}^T]$

The optimal $\mathbf{Q}$ is given by the water-filling algorithm:



$$[\mathrm{diag}(\mathbf{Q})]_{ii} = \left(\nu - \frac{1}{h_i}\right)^+$$

where ν is chosen so that

$$\sum \left(\nu - \frac{1}{h_i}\right)^+ = P$$

- Multi-user scalar channel
- Single-user vector channel
- Multi-user vector channel
  - Multi-user diversity
  - capacity region
  - optimization problem
  - one simple example

Multi-User Vector Channel (1)

- Multi-user diversity
  - A deep faded channel for a user may appear to be good for some other user. ➔ We can strategically allocate resources to maximize the system performance or minimize the cost.

- Two-user vector channel model

$$\mathbf{Y} = \mathbf{H}_1\mathbf{X}_1 + \mathbf{H}_2\mathbf{X}_2 + \mathbf{Z} \qquad \mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

$$\mathrm{tr}\big(E[\mathbf{X}_1\mathbf{X}_1^T]\big) \leq P_1$$
$$\mathrm{tr}\big(E[\mathbf{X}_2\mathbf{X}_2^T]\big) \leq P_2$$

Multi-User Vector Channel (2)

- Capacity region

$$
\begin{aligned}
R_1 &\le I(\mathbf{X}_1;\mathbf{Y}|\mathbf{X}_2)\\
R_2 &\le I(\mathbf{X}_2;\mathbf{Y}|\mathbf{X}_1)\\
R_1+R_2 &\le I(\mathbf{X}_1,\mathbf{X}_2;\mathbf{Y})
\end{aligned}
$$

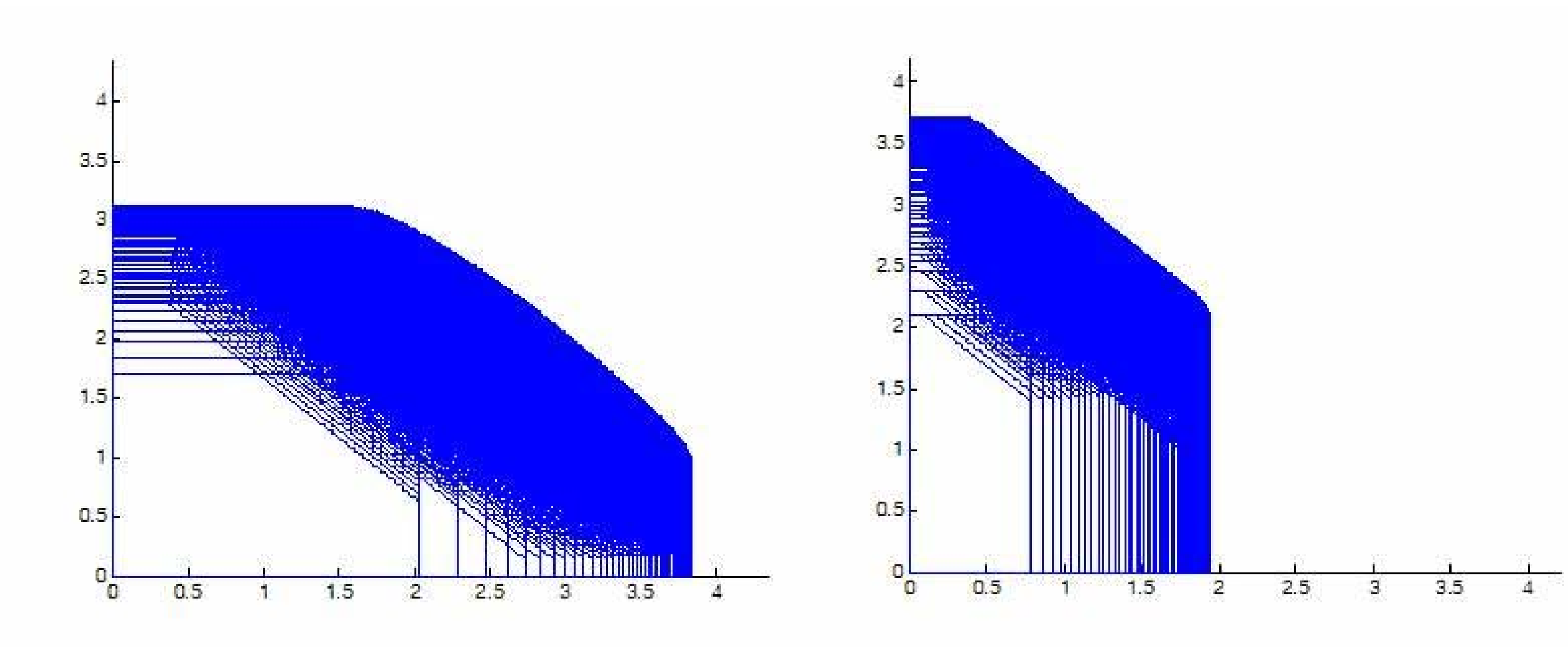
- For Gaussian channel

$$
\begin{aligned}
I(\mathbf{X}_1;\mathbf{Y}|\mathbf{X}_2) &\le \frac{1}{2}\log\det\left(\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^T+\mathbf{I}\right)\\
I(\mathbf{X}_2;\mathbf{Y}|\mathbf{X}_1) &\le \frac{1}{2}\log\det\left(\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T+\mathbf{I}\right)\\
I(\mathbf{X}_1,\mathbf{X}_2;\mathbf{Y}) &\le \frac{1}{2}\log\det\left(\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^T+\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T+\mathbf{I}\right)
\end{aligned}
$$

Note that $\mathbf{Q}_1 := E[\mathbf{X}_1\mathbf{X}_1^T]$ and $\mathbf{Q}_2 := E[\mathbf{X}_2\mathbf{X}_2^T]$

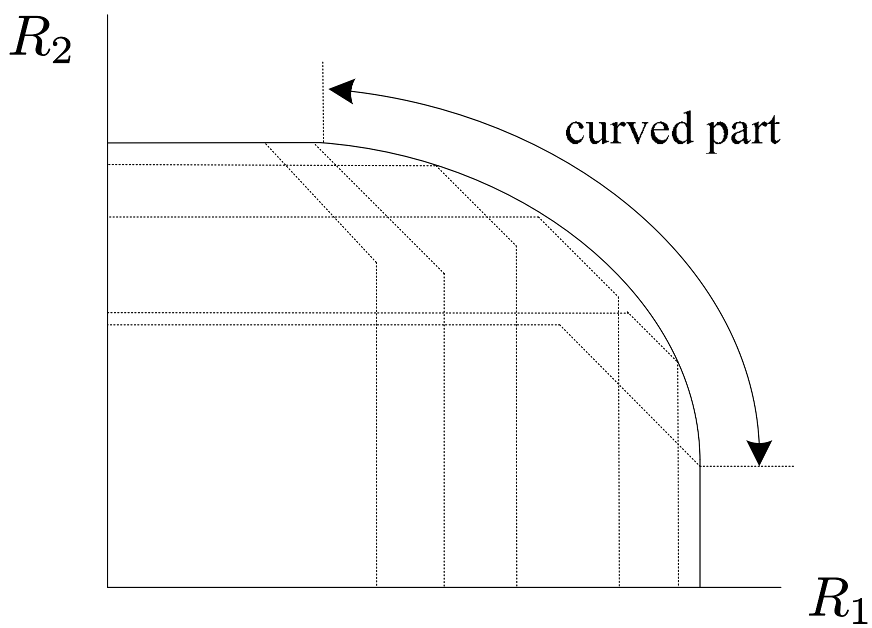
Multi-User Vector Channel (3)

- Capacity region examples



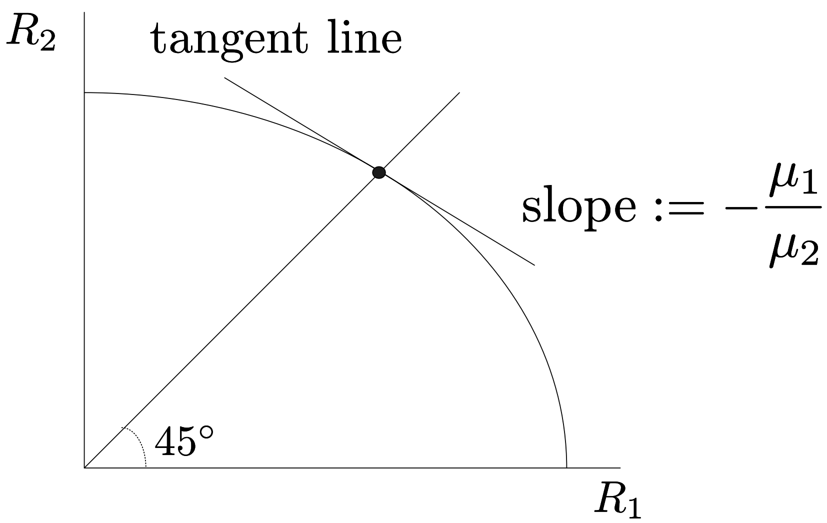
Multi-User Vector Channel (4)

- The capacity region is known to be as the union of pentagons [Tse, Hanly 98]



Multi-User Vector Channel (5)

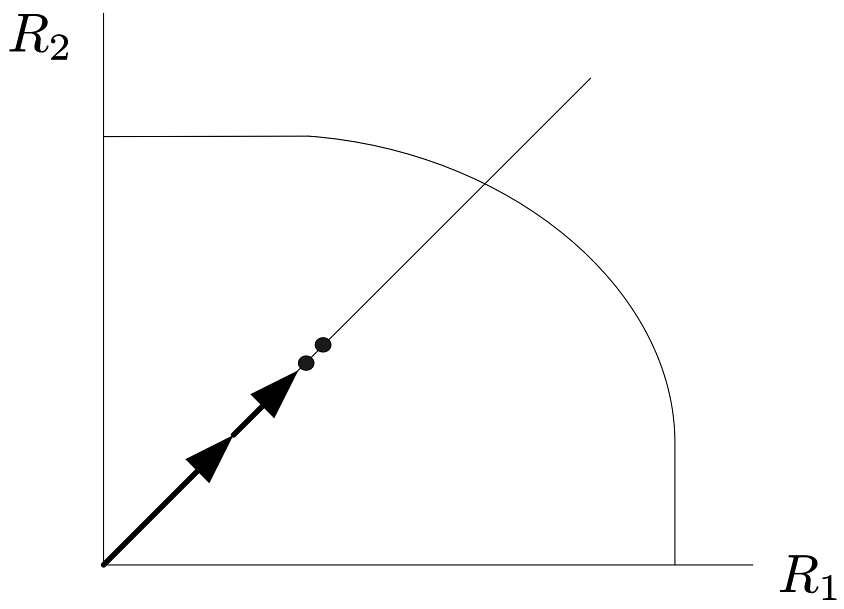
- re-casting to a sum-rate capacity



$$\begin{aligned} \text{maximize} \quad & \mu_1 R_1 + \mu_2 R_2 \\ \text{subject to} \quad & \text{tr}\{\mathbf{Q}_1\} \le P_1 \\ & \text{tr}\{\mathbf{Q}_2\} \le P_2 \end{aligned}$$

Multi-User Vector Channel (6)

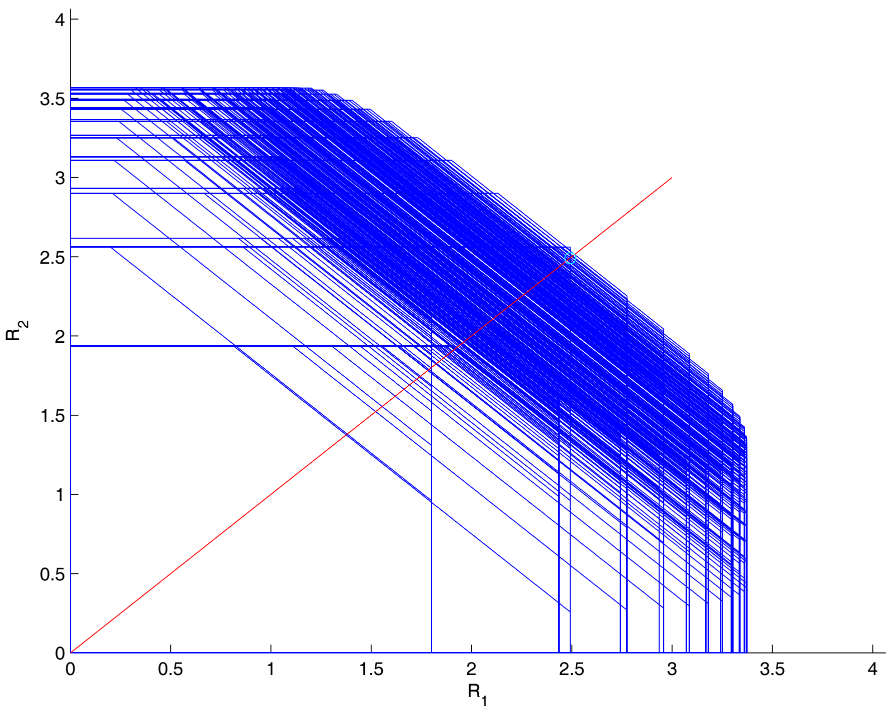
- One simple way: follow the 45-degree line



Multi-User Vector Channel (7)

$$
\begin{array}{l}
\mathbf{W} \leftarrow \mathbf{I} \qquad \text{noise} + \text{interference} \\
\text{while} \quad p_1 \leq P_1, \;\; p_2 \leq P_2 \\
\qquad p_1 \leftarrow p_1 + \Delta_1 \\
\qquad \mathbf{Q}_1 \leftarrow \text{waterfill}(\mathbf{H}_1, p_1, \mathbf{W}) \\
\qquad R_1 \leftarrow \dfrac{1}{2} \log_2 \dfrac{\det(\mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^T + \mathbf{W})}{\det(\mathbf{W})} \\
\qquad \mathbf{W} \leftarrow \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^T + \mathbf{I} \\
\qquad \text{while} \quad R_1 - R_2 > tol \\
\qquad\qquad p_2 \leftarrow p_2 + \Delta_2 \\
\qquad\qquad \mathbf{Q}_2 \leftarrow \text{waterfill}(\mathbf{H}_2, p_2, \mathbf{W}) \\
\qquad\qquad R_2 \leftarrow \dfrac{1}{2} \log_2 \dfrac{\det(\mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^T + \mathbf{W})}{\det(\mathbf{W})} \\
\qquad \text{end} \\
\text{end}
\end{array}
$$

Multi-User Vector Channel (8)



Condition

$$\mathbf{H}_1 = \mathrm{diag}(2.0918, 1.2608)$$
$$\mathbf{H}_2 = \mathrm{diag}(1.1688, 1.839)$$
$$\mathrm{tr}(\mathbf{Q}_1) \leq 7$$
$$\mathrm{tr}(\mathbf{Q}_2) \leq 10$$

$$\Longrightarrow (R_1, R_2) = (2.4923, 2.4913)$$

Conclusion

- Remarks:
  - Show the rate for of maximizing the worst-user's rate is determined at the maximal equal rate.
  - Multi-user vector channel problem can be re-casted into a sum-rate capacity problem.
  - Show one simple way to close to the optimal point.
- Future Works:
  - Understand the characteristics of the curved part of the capacity region.
  - Find a way to reach the intersection of the 45-degree-line and the curve.

Thank You