Diversity-Multiplexing tradeoff in the Rayleigh Fading Relay Channel

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Relay Channel / Diversity-Mux

- Rayleigh fading relay channel
  \[ Y_D = \sqrt{h_{SD}} e^{j\phi_{SD}} X_S + \sqrt{h_{RD}} e^{j\phi_{RD}} X_R + Z_D \]
  \[ Y_R = \sqrt{h_{SR}} e^{j\phi_{SR}} X_S + Z_R \]

- SNR: \[ \gamma = \frac{P_S}{N_D} = \frac{P_S}{N_R} = \frac{P_R}{N_D} \]

- Diversity gain
  \[ d := - \lim_{\gamma \to \infty} \frac{\log[P_{\text{out}}(\gamma)]}{\log \gamma} \]

- Multiplexing gain
  \[ r := \lim_{\gamma \to \infty} \frac{R(\gamma)}{\log \gamma} \]
Ergodic Capacity

- Capacity depends on channel realization
  \[ C = C(h_{SD} e^{j\phi_{SD}}, h_{SR} e^{j\phi_{SR}}, h_{RD} e^{j\phi_{RD}}) \]

- Ergodic – all channel realizations in one packet
  \[ \bar{C} = \mathbb{E}[C(h_{SD} e^{j\phi_{SD}}, h_{SR} e^{j\phi_{SR}}, h_{RD} e^{j\phi_{RD}})] \]

- Max-flow min-cut
  \[ C < \max_{\rho \in [0,1]} \min \left\{ \log \left(1 + (h_{SD} + h_{SR} + 2\rho\sqrt{h_{SD}h_{RD}} |e^{j(\phi_{SD} + \phi_{RD})}|) \gamma \right), \log(1 + (h_{SD} + h_{RD})(1 - \rho^2)\gamma) \right\} \]

- Average over phases
  \[ \bar{C} < \mathbb{E} \left[ \min \left[ \log(1 + (h_{SD} + h_{SR})\gamma), \log(1 + (h_{SD} + h_{RD})\gamma) \right] \right] \]

- Prevents coherent superposition
Outage Capacity / Markov coding

- Prob. that a rate is not achievable
  \[ P_{out}(R_{out}) = \Pr\{C(h_{SD} e^{j\phi_{SD}}, h_{SR} e^{j\phi_{SR}}, h_{RD} e^{j\phi_{RD}}) < R_{out}\} \]

- Markov coding achievable rate (capacity lower bound)
  \[ C > I_{MC} = \min[\log(1 + h_{SR}\gamma), \log(1 + (h_{SD} + h_{RD})\gamma)] \]

- Outage probability upper bound:
  \[ P_{out}(R_{out}) < P_{out}^{MC}(R_{out}) := \Pr\{I_{MC} < R_{out}\} \]

- Channel power outage:
  \[ h_{out} = \frac{2^{R_{out}} - 1}{\gamma} \]

- Markov coding outage:
  \[ P_{out}^{MC}(h_{out}) = \Pr\{\min(h_{SR}, h_{SD} + h_{RD}) < h_{out}\} \]

- Large SNR behavior:
  \[ P_{out}^{MC}(h_{out}) \sim \frac{h_{out}}{h_{SR}} + \frac{h_{out}^2}{2h_{SD}h_{RD}} \sim \frac{h_{out}}{h_{SR}} \]

  No diversity gain
Adaptive Markov coding (AMC)

- Cooperate only if $h_{SR}$ is good

\[ C > I_{AMC} = \begin{cases} \log(1 + h_{SD} \gamma), & h_{SR} < h_{out} \\ \log(1 + (h_{SD} + h_{RD}) \gamma), & h_{SR} > h_{out} \end{cases} \]

- AMC outage probability

\[ P_{out}^{AMC}(h_{out}) = \Pr\{h_{SD} < h_{out}\} \Pr\{h_{SR} < h_{out}\} + \Pr\{h_{SD} + h_{RD} < h_{out}\} \Pr\{h_{SR} > h_{out}\} \]

- Large SNR behavior:

\[ P_{out}^{AMC}(h_{out}) \sim \left[ \frac{1}{h_{SD} h_{SR}} + \frac{1}{2h_{SD} h_{RD}} \right] h_{out}^2 \]

- Capacity upper bound:

\[ C < I_{TA} = \log(1 + (h_{SD} + h_{RD}) \gamma) \]

- Outage prob. lower bound:

\[ P_{out}^{TA}(h_{out}) \sim \left[ \frac{h_{out}^2}{2h_{SD} h_{RD}} \right] \]
Diversity-Mux tradeoff

- Upper bound diversity gain

\[ d := - \lim_{\gamma \to \infty} \frac{\log[P_{out}(\gamma)]}{\log \gamma} \leq - \lim_{\gamma \to \infty} \frac{\log[P_{out}^{TA}(\gamma)]}{\log \gamma} \]

- Use outage at high SNR behavior

\[ d \leq - \lim_{\gamma \to \infty} \frac{\log(h_{out}^2)}{\log \gamma} = -2 \lim_{\gamma \to \infty} \frac{R_{out} - \log(\gamma)}{\log \gamma} \]

- Recall multiplexing gain definition ( \( r := \lim_{\gamma \to \infty} \frac{R(\gamma)}{\log \gamma} \))

\[ d \leq 2(1 - r) \]

- Diversity gain not greater than 2
Diversity-Mux tradeoff

- Diversity-rate curve for MC: \( d_{MC}^r = (1 - r) \)
- Diversity-rate curve for AMC: \( d_{AMC}^r = 2(1 - r) \)

- AMC achieves tradeoff upper-bound
- Best possible tradeoff \( d^* = 2(1 - r) \)
- Achieved by AMC not by MC
Conclusions

- Studied diversity multiplexing tradeoff in Rayleigh fading relay channel

- Best achievable tradeoff: \( d^* = 2(1 - r) \)

- Achieved by Adaptive version of Markov coding
  - Relays cooperate only if \( h_{SR} \) is good enough