EE 8510: Network Coding Notes

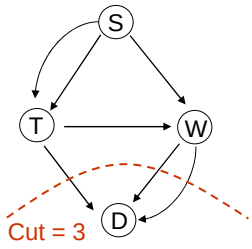
- Routing vs. Network Coding
- Min-Cut/Max-Flow
- Network Coding Examples
- Multi-cast Theorem
- Network Robustness

Traditional Network

- Routers typically act only as switches, i.e. decide which input stream to send on each output port
- Can networks be improved (in terms of capacity and/or robustness) by allowed routers to do some coding, e.g. combining of streams?

- Assumptions:
  - All links have capacity of one bit
  - No delays
  - All sources are independent
  - Links are perfect (i.e. no physical layer errors)

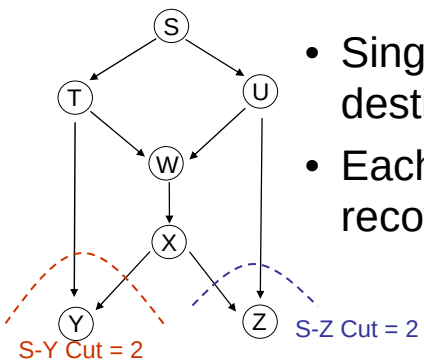
Min-Cut/Max-Flow



- Make cut with source, sink on opposite sides, add flow across cut (towards sink)
- Min-Cut/Max-Flow Bound: minimum of all such cuts

- Thm (Ford-Fulkerson): For single source, single sink networks, min-cut/max-flow bound is always achievable with routing
  - No coding necessary
  - Not always achievable if multiple sinks and sources

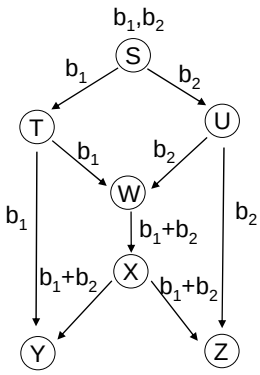
Min-Cut/Max-Flow for Multicast



- Single source (S), multiple destinations (Y,Z)
- Each destination wants to recover all data from source

- Compute min-cut/max-flow individually for each destination (i.e. for S-Y and for S-Z)
- Upper bound = minimum of min-cuts over all destinations

Canonical Example



- Source has two streams (b_1,b_2), Y and Z want both
- Max-flow/min-cut = 2
- With routing W sends either b_1 or b_2 to X -> Y and Z cannot both recover b_1 and b_2

- Network coding approach: W transmits $b_1 + b_2$ (XOR operation on binary streams)
  - Y and Z can recover both bit streams
- Routing approach requires 2 bit link from W to X

Multicast Theorem

- Thm (Li, Yeung, & Cai and Koetter & Medard): Linear network coding achieves the min-cut/max-flow bound for any multicast network with a single source and multiple destinations
- Proof: Describe input-output relationship in terms of transfer matrix, show can choose coefficients such that transfer matrix has non-zero determinant

- Linear coding over GF(2^m): routers transmit linear combinations of inputs (weighted by coefficients in GF(2^m))
  - May require m > 1 (i.e. operate on blocks of m bits)
  - No gain to using more complex coding
  - Can use algebraic properties to find linear code

Network Robustness

- Use network coding to make network robust to link failures
  - Want network code that works for any feasible set of link failures
- Thm: One network code works for all feasible link failures
- Proof: Treat each set of link failures as a different destination in a multicast network

- Interior nodes do not need to change operation (single code works)
- Destinations need to change coefficients

Extensions

- Multicast theorem can be extended to:
  - Networks with delay
  - Cyclic networks

- Multiple-sources/multiple destinations:
  - Min-cut/Max-flow not necessarily achievable, but still an upper bound
  - Non-linear coding can be better than linear coding