EE 8510: Network Coding Notes

- Routing vs. Network Coding
- Min-Cut/Max-Flow
- Network Coding Examples
- Multi-cast Theorem
- Network Robustness

Traditional Network

- Routers typically act only as switches, i.e. decide which input stream to send on each output port
- Can networks be improved (in terms of capacity and/or robustness) by allowed routers to do some coding, e.g. combining of streams?

- Assumptions:
  - All links have capacity of one bit
  - No delays
  - All sources are independent
  - Links are perfect (i.e. no physical layer errors)
Min-Cut/Max-Flow

- Make cut with source, sink on opposite sides, add flow across cut (towards sink)
- Min-Cut/Max-Flow Bound: minimum of all such cuts

- Thm (Ford-Fulkerson): For single source, single sink networks, min-cut/max-flow bound is always achievable with routing
  - No coding necessary
  - Not always achievable if multiple sinks and sources

Min-Cut/Max-Flow for Multicast

- Single source (S), multiple destinations (Y,Z)
- Each destination wants to recover all data from source

- Compute min-cut/max-flow individually for each destination (i.e. for S-Y and for S-Z)
- Upper bound = minimum of min-cuts over all destinations
Canonical Example

- Source has two streams \((b_1, b_2)\), \(Y\) and \(Z\) want both
- Max-flow/min-cut = 2
- With routing \(W\) sends either \(b_1\) or \(b_2\) to \(X\) -> \(Y\) and \(Z\) cannot both recover \(b_1\) and \(b_2\)

- Network coding approach: \(W\) transmits \(b_1 + b_2\) (XOR operation on binary streams)
  - \(Y\) and \(Z\) can recover both bit streams
- Routing approach requires 2 bit link from \(W\) to \(X\)

Multicast Theorem

- Thm (Li, Yeung, & Cai and Koetter & Medard): Linear network coding achieves the min-cut/max-flow bound for any multicast network with a single source and multiple destinations
- Proof: Describe input-output relationship in terms of transfer matrix, show can choose coefficients such that transfer matrix has non-zero determinant

- Linear coding over \(GF(2^m)\): routers transmit linear combinations of inputs (weighted by coefficients in \(GF(2^m)\))
  - May require \(m > 1\) (i.e. operate on blocks of \(m\) bits)
  - No gain to using more complex coding
  - Can use algebraic properties to find linear code
Network Robustness

• Use network coding to make network robust to link failures
  – Want network code that works for any feasible set of link failures
• Thm: One network code works for all feasible link failures
• Proof: Treat each set of link failures as a different destination in a multicast network

• Interior nodes do not need to change operation (single code works)
• Destinations need to change coefficients

Extensions

• Multicast theorem can be extended to:
  – Networks with delay
  – Cyclic networks

• Multiple-sources/multiple destinations:
  – Min-cut/Max-flow not necessarily achievable, but still an upper bound
  – Non-linear coding can be better than linear coding