Lectures 9-10

- MIMO Channel Model
- Capacity: H Fixed
- Capacity: H iid Rayleigh, RX CSI
- Capacity: H Ricean, RX CSI
- Capacity: Correlated H, RX CSI

MIMO Channel Model

- N_t TX antennas, N_r RX antennas

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{N}$$

- X: N_t x 1, Y: N_r x 1, N: N_r x 1
  - N has iid complex Gaussian components, zero-mean, variance one
- H: N_r x N_t complex matrix
  - Different models for distribution of entries of H

Capacity: H Fixed

- Gaussian inputs optimal, have to optimize correlation Q

$$C = \max_{X : E\|X^2\| \leq P} I(X;Y)$$
$$= \max_{Q : Q \geq 0, Tr(Q) \leq P} \log\left|I + HQH^H\right|$$

- Choose Q to be aligned with right eigenvectors of H, use waterfilling to determine eigenvalues

H Fading, Perfect TX/RX CSI

- Perform waterfilling over time and space
  - In each fading state, choose Q to be aligned with left eigenvectors of HH^H
  - Waterfill over time and parallel channels to determine eigenvalues in each state

- Same intuition as waterfilling for fading scalar channels, but with parallel channels in each state

Capacity: H iid Rayleigh

- Entries of H iid zero-mean complex Gaussian (Rayleigh), RX CSI, no TX CSI

$$C = \max_{X : E\|X^2\| \le P} I(X;Y,H)$$

$$H = H(w) \qquad = \max_{Q : Q \ge 0, Tr(Q) \le P} E\left[\log\left|I + HQH^H\right|\right]$$

$$= E\left[\log\left|I + \frac{P}{N_t} HH^H\right|\right]$$

- Optimum Q: Scaled identity matrix
  - Transmit equal power, independent Gaussian codewords from each antenna

Capacity: H Ricean

- Entries of H independent complex Gaussian, non-zero mean (line-of-sight component), RX CSI, no TX CSI

$$H = \overline{H} + H(w)$$

$$C = \max_{Q : Q \ge 0, Tr(Q) \le P} E_H\left[\log\left|I + HQH^H\right|\right]$$

- Optimum Q aligned with eigenvectors of mean matrix, compute eigenvalues numerically (not waterfilling)

Capacity: H Correlated

- Entries of H correlated (separable) complex Gaussian, non-zero mean, RX CSI, no TX CSI

$$H = \Sigma_r^{1/2} H(w) \Sigma_t^{1/2}$$

$$C = \max_{Q : Q \geq 0, Tr(Q) \leq P} E_H\left[\log\left|I + HQH^H\right|\right]$$

- Optimum Q aligned with eigenvectors of transmit correlation matrix, compute eigenvalues numerically (not waterfilling)
- Eigenvectors of Q only depend on transmit eigenvectors, but RX eigenvectors affect capacity, eigenvalues