MIMO Channel Model

- $N_t$ TX antennas, $N_r$ RX antennas
- $X: N_t \times 1$, $Y: N_r \times 1$, $N: N_r \times 1$
  - $N$ has iid complex Gaussian components, zero-mean, variance one
- $H: N_r \times N_t$ complex matrix
  - Different models for distribution of entries of $H$

\[ Y = HX + N \]
Capacity: H Fixed

- Gaussian inputs optimal, have to optimize correlation Q

\[ C = \max_{X : E \| X^2 \| \leq P} I(X;Y) \]

\[ = \max_{Q : Q \geq 0, \text{Tr}(Q) \leq P} \log \left| I + HQH^H \right| \]

- Choose Q to be aligned with right eigenvectors of H, use waterfilling to determine eigenvalues

H Fading, Perfect TX/RX CSI

- Perform waterfilling over time and space
  - In each fading state, choose Q to be aligned with left eigenvectors of HH^H
  - Waterfill over time and parallel channels to determine eigenvalues in each state

- Same intuition as waterfilling for fading scalar channels, but with parallel channels in each state
Capacity: $H$ iid Rayleigh

- Entries of $H$ iid zero-mean complex Gaussian (Rayleigh), RX CSI, no TX CSI

\[
C = \max_{X : E \| X \|^2 \leq P} I(X;Y,H)
\]

\[
H = H(w)
\]

- Optimum $Q$: Scaled identity matrix
  - Transmit equal power, independent Gaussian codewords from each antenna

Capacity: $H$ Ricean

- Entries of $H$ independent complex Gaussian, non-zero mean (line-of-sight component), RX CSI, no TX CSI

\[
C = \max_{Q : Q \succeq 0, \text{Tr}(Q) \leq P} E_{H}[\log(1 + HQH^H)]
\]

- Optimum $Q$ aligned with eigenvectors of mean matrix, compute eigenvalues numerically (not waterfilling)
Capacity: H Correlated

- Entries of H correlated (separable) complex Gaussian, non-zero mean, RX CSI, no TX CSI

\[ H = \Sigma_r^{1/2} H(w) \Sigma_i^{1/2} \]

\[
C = \max_{Q \geq 0, \text{Tr}(Q) \leq P} \mathbb{E}_n \left[ \log \left( I + HQH^H \right) \right]
\]

- Optimum Q aligned with eigenvectors of transmit correlation matrix, compute eigenvalues numerically (not waterfilling)
- Eigenvectors of Q only depend on transmit eigenvectors, but RX eigenvectors affect capacity, eigenvalues