Lecture 1: Intro & Overview

- Fundamental Problems in Information Theory
- Course Overview
- Logistics

Fundamental Problems in IT

- Q1: Is there a limit to how much data can be compressed?

- Q2: At what rates is reliable communication possible over a noisy channel?
Question 1

• Q1: Is there a limit to how much data can be compressed?

• A: \( H(X) \) bits/symbol

• For binary source, \( H(X) = \) true information, 1-\( H(X) = \) redundancy

Question 2

• Q2: At what rates is reliable communication possible over a noisy channel?

• A: \( C = \max_{p(x)} I(X;Y) \)

• At any rate \( R < C \), reliable communication is possible
Channel Definition

- Channel: Probabilistic relationship between input X and output Y: \( p(y|x) \)

\[
\begin{array}{c}
X \\
\xrightarrow{\text{Channel}} \\
p(y|x) \\
\xrightarrow{} \\
Y
\end{array}
\]

- Use channel multiple times (discrete-time)
  - Each use might correspond to a symbol period

Communication System

Message \( m \) from \( \{1, \ldots, M\} \) → Encoder → Codeword \( (x_1, \ldots, x_N) \) → Channel → RX Signal \( (y_1, \ldots, y_N) \) → Decoder → Estimate \( \hat{m} \)

**Rate** \( (R) = \frac{\log_2 M}{N} = \frac{\# \text{ of info bits in message}}{\# \text{ of channel uses}} = \text{bits/use} \)

**Block error rate** = \( P(e) = P(\hat{m} \neq m) \)
Example Channel

Binary Symmetric Channel with cross-over probability $\alpha < 1/2$

\[
\begin{array}{c}
0 \\
\alpha \\
1 \\
1-\alpha \\
\end{array}
\begin{array}{c}
0 \\
\alpha \\
1 \\
1-\alpha \\
\end{array}
\]

\[p(y = x) = 1 - \alpha, \quad p(y \neq x) = \alpha\]

Encoder/Decoder Design

- Encoder: Choose $M$ (# of codewords) length $N$ binary codewords
- Decoder: Given length $N$ received vector, choose message $m$ that TX most likely sent
Limits of Communication

• What is highest rate (for any N) of reliable communication, i.e. what is best any encoder/decoder can do?

• Zero-error capacity: Reliable <-> P(e) =0
  – For BSC, zero error capacity is zero because P(e) > 0 for any code
  – Generally very difficult problem
  – Not so interesting from practical/engineering standpoint

Channel Capacity

• Shannon’s Formulation:
  What is highest rate such that P(e) -> 0 as N goes to infinity?

• A:
  \[ C = \max_{p(x)} I(X;Y) \]

• For any R < C, there exist encoders/decoders for all N with P(e) -> 0 as N grows large
• For any R > C, P(e) -> 1 as N grows large
Source Channel Separation

- **Optimal** to do source and channel coding separately for single TX, single RX channel
- Can **reliably** transmit any source with \( H(X) < C \)

**Course Overview**

- **Information Theory Basics**
  - \( H(X), I(X;Y), \text{AEP}, \ldots \)

- **Single User Gaussian Channels**
  - \( \text{AWGN: } Y = X + N \)
  - Fading
  - MIMO
  - Freq-selective
Course Overview

- Multiple-access Channel

\[ m_1 \rightarrow X_1 \rightarrow \text{Channel} \ p(y|x_1, x_2) \rightarrow Y \rightarrow (\hat{m}_1, \hat{m}_2) \]

\[ m_2 \rightarrow X_2 \rightarrow \text{Channel} \ p(y|x_1, x_2) \rightarrow Y \rightarrow (\hat{m}_1, \hat{m}_2) \]

- Broadcast Channel

\[ (m_1, m_2) \rightarrow X \rightarrow \text{Channel 1} \ p(y_1|x) \rightarrow Y_1 \rightarrow \hat{m}_1 \]

\[ \text{Channel 2} \ p(y_2|x) \rightarrow Y_2 \rightarrow \hat{m}_2 \]

- Interference Channel

\[ m_1 \rightarrow X_1 \rightarrow \text{Channel 1} \ p(y|x_1, x_2) \rightarrow Y_1 \rightarrow \hat{m}_1 \]

\[ m_2 \rightarrow X_2 \rightarrow \text{Channel 2} \ p(y|x_1, x_2) \rightarrow Y_2 \rightarrow \hat{m}_2 \]

- Relay Channel

\[ m \rightarrow X \rightarrow Y_1 : X_1 \rightarrow \hat{m} \]

\[ \text{Direct} \ p(y|x_1) \rightarrow Y \rightarrow \hat{m} \]
Course Overview

• Rate Distortion Theory
  – Maximum compression such that reconstruction not perfect but meets distortion criteria (lossy source coding)

Course Overview

• Capacity of general (ad-hoc) multi TX/multi RX networks

• Includes relaying, routing, etc.
Course Overview

• Sensor Networks: Distributed Estimation/Detection, CEO Problem, Joint Source/Channel Coding

Course Overview

• Network Coding: Perform coding at routers instead of just multiplexing to increase performance and add robustness
Logistics

- Text: No required text, but info theory book is highly recommended (Cover & Thomas)
- Prerequisite: EE5581 or equivalent
- Homework: Approximately weekly for first half of course, ~7 total
- Midterm exam in middle of course
- Research Project: In-depth study, or original research topic
- Grading: 35% HW, 25% Midterm, 40% Project