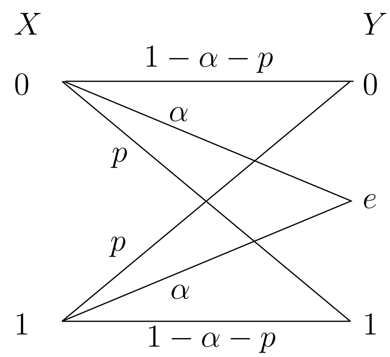
EE 5581 Prof. Jindal
Information Theory & Coding November 18, 2005

Midterm 2

You have 50 minutes to complete this exam. You must show your work to receive credit.

1. **Combined BSC & Erasure Channel** (40 pts) - 2 parts

   Consider a combination of the binary symmetric channel and the erasure channel, where the input bit is either received correctly (with probability $1 - \alpha - p$), erased (with probability α), or flipped (with probability p).

   (a) Find the capacity and the capacity-achieving input distribution for this channel. (25 pts)

   (b) Do erasures always reduce capacity? Let $C(\alpha, p)$ denote the capacity of the combination BSC-erasure channel with erasure probability α and cross-over probability p. If the cross-over probability p is fixed, is capacity always maximized when $\alpha = 0$? In other words, is the following statement true:

   $$C(0, p) \geq C(\alpha, p) \ \ \forall \ 0 \leq \alpha \leq 1,$$

   for all $0 \leq p \leq 1$? Prove this is true, or provide a counter-example. (15 pts)

2. **Channels with Feedback** (40 pts) - 3 parts

   Consider a BSC with feedback with cross-over $p = 1/2$.

   Consider the following feedback code for this channel with block-length $n > 2$. Let $M = 2$ (i.e. two messages), and let $x_i(k)$ indicate the i-th symbol of the codeword for message k. Let $x_1(1) = 0$ and $x_1(2) = 1$ (i.e., if message 1 is selected, a 0 is transmitted during the first channel use, and if message 2 is selected, a 1 is transmitted during the first channel use).

   For $2 \leq i \leq n$, $x_i(W, Y^{i-1}) = y_{i-1}$, i.e., the i-th channel *input* is equal to the $(i-1)$-th channel *output*. Assume the message W is chosen equiprobably from $\{1, 2\}$.

(a) What is the capacity of this channel? (5 pts)

(b) Compute $I(W; Y^n)$. (15 pts)
(Hint: There is a very easy method to compute this)

(c) Compute $I(X^n; Y^n)$. (20 pts)

3. **Differential Entropy** (20 pts)

Consider a continuous random variable X with infinite support ($f(x) > 0$ for all x) and with a symmetric density function ($f(x) = f(-x)$ for all x). Assume that $h(X)$ is finite. Let $Y = |X|$. Derive an expression for $h(Y)$ in terms of $h(X)$.