Homework 6
Due: Tuesday, March 30, 11:15 AM

1. **12.3**: Note that the corrected equation (12.17) is

\[
\text{DFT}\{x[n] \otimes h[n]\} = \sqrt{N} \cdot \text{DFT}\{x[n]\} \cdot \text{DFT}\{h[n]\}
\] (1)

(The $\sqrt{N}$ factor is missing in the book)

2. **12.7**

3. **12.9**: Replace $H$ with $\tilde{H}$ throughout the problem.

4. **12.11**: Replace $H$ with $\tilde{H}$ in part (b).

5. **12.10**

6. You are designing a 10 MHz OFDM system at a carrier frequency of 1 GHz. Assume that you are in an outdoor environment with a maximum delay spread of 15 microseconds.

   (a) What length cyclic prefix is needed for this system?

   (b) What is the efficiency of a system using 1024 subcarriers ($N = 1024$)?

   (c) Repeat parts (a) and (b) for an indoor environment with a maximum delay spread of 1 microsecond.

7. In this problem we study the PAPR of the transmit samples (i.e., the outputs of the IFFT at the TX). In the notation from lecture, we used $\tilde{d}[0], \ldots, \tilde{d}[N-1]$ to denote the QAM symbols, while $d[0], \ldots, d[N-1]$ are the IFFT of the QAM symbols (i.e., the transmit samples). In terms of these discrete-time samples, the PAPR is:

\[
PAPR = \max_{k=0,\ldots,N-1} \frac{|d[k]|^2}{(1/N) \sum_{k=0}^{N-1} |d[k]|^2}
\] (2)

This quantity depends on the values $d[0], \ldots, d[N-1]$, which are in turn determined by the $N$ data symbols $\tilde{d}[0], \ldots, \tilde{d}[N-1]$. Thus, we can think of PAPR as a random variable, where the randomness originates from the randomness in the data symbols.

In this problem you are to write a Matlab program that numerically generates the CCDF (one minus the CDF) of random variable PAPR. Assume that the $N$ data symbols are chosen in an iid fashion from a 4-QAM constellation.

   (a) Create a plot of the PAPR (in dB) versus the CCDF (i.e., the probability that PAPR is larger than the value on the x-axis) for $N = 16$, $N = 64$, and $N = 1024$. 


(b) Comment on how likely it is that the PAPR is the worst-case of $N$.

Note: In theory you could determine the exact distribution of PAPR by going through all $4^N$ different data symbol combinations. However, this is computationally impossible except when $N$ is very small. Thus, you should generate the CDF by trying a very large number of random data symbol combinations (i.e., use Monte Carlo simulation).