

### Homework 1

Due: Thursday, January 28, 11:15 AM

Please turn in your MATLAB scripts in addition to your solutions and plots.

1. 2.1
2. 2.8
3. 2.13
4. 2.19
5. Consider the downlink of a cellular system as shown in the figure. Each cell is square with a side of length  $2R$ , and the two base stations are located at the middle of each cell.

The signal from the desired base station as well as the interference from the other base station both follow the empirical path-loss model with  $\gamma = 3$ ,  $d_0 = 1\text{m}$ , and  $K = 1$ , and both links also suffer from log-normal shadowing with  $\sigma_{dB} = 6\text{ dB}$ .

In this question we ignore noise and only study the ratio between the power received from the desired base station and the power received from the interfering base station. We assume each cell transmits with the same power, so the actual value of  $P_t$  is unimportant.

- (a) Assume the mobile is on the line between the two base stations and is a distance  $d$  away from BS 1 (which means the mobile is a distance  $2R - d$  away from BS 2), as shown in the figure. Find an expression for the outage probability, where the outage probability is the probability that the ratio of signal power to interference power falls below some threshold  $P_{\min}$ , in terms of  $d$  and  $P_{\min}$  for  $0.05R \leq d \leq R$ . Plot outage probability versus  $d$  for  $P_{\min} = 0\text{ dB}$  and for  $P_{\min} = 5\text{ dB}$ .  
Hint: You should be able to derive a simple expression in terms of the  $Q$  function.
- (b) Plot outage probability as a function of  $d$  for the same setup as in part (a), except with  $\gamma = 4$ . Explain why your plots differs from the plot in part (a).
- (c) In this part we will numerically compute the outage probability for a mobile that is randomly located (according to a uniform distribution) in Cell 1, using the model parameters from part (a) and a threshold of  $0\text{ dB}$ . In order to do this, we perform the following experiment over and over again: randomly place the mobile in cell 1 (you should be able to easily do this using the `rand` command in MATLAB), generate realizations of the shadowing terms for the desired signal and for the interference (using the `randn` command), and compute the ratio of signal to interference power. By computing the fraction of times the ratio is larger than the  $0\text{ dB}$  threshold, we get a good estimate of the actual outage probability. Of course, to get a reliable estimate we should perform this experiment a very larger number of times (e.g.,  $10^4$  or greater)

- (d) In the previous parts of this question we have assumed that a mobile in Cell 1 treats BS 1 as the desired base station. However, due to shadowing, a mobile in Cell 1 may actually receive more power from BS 2 than from BS 1. In fact, a mobile device is generally able to measure the signal strength from each BS, and then chooses the desired BS to be the one from which it receives the most power. This is referred to as *macro-diversity*.

Reconsider the scenario of part (a) and write out an expression for the outage probability as a function of  $d$  assuming the desired BS is the stronger BS (you don't need to try to find an expression in terms of the Q-function; it is sufficient to provide an expression that is in terms of the maximum and minimum of a pair of random variables). Then

- i. Show that the outage probability is zero if  $P_{\min} = 0$  dB.
- ii. Numerically compute and plot outage probability versus  $d$  for  $P_{\min} = 5$  dB, and compare your results to the outage probability without macro-diversity.

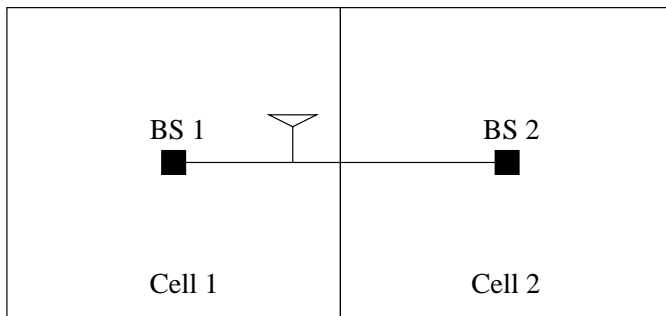


Figure 1: Two cell model