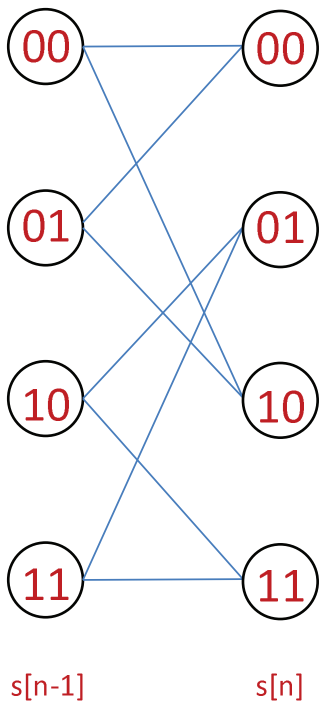
EE 5501 Prof. N. Jindal
Digital Communication Nov. 24, 2009

Midterm 2

1. Consider the $R = 1/2$ binary convolutional code with memory 2 that outputs the following two bits for each information bit:

$$u[k] + u[k-1] + u[k-2]$$
$$u[k]$$

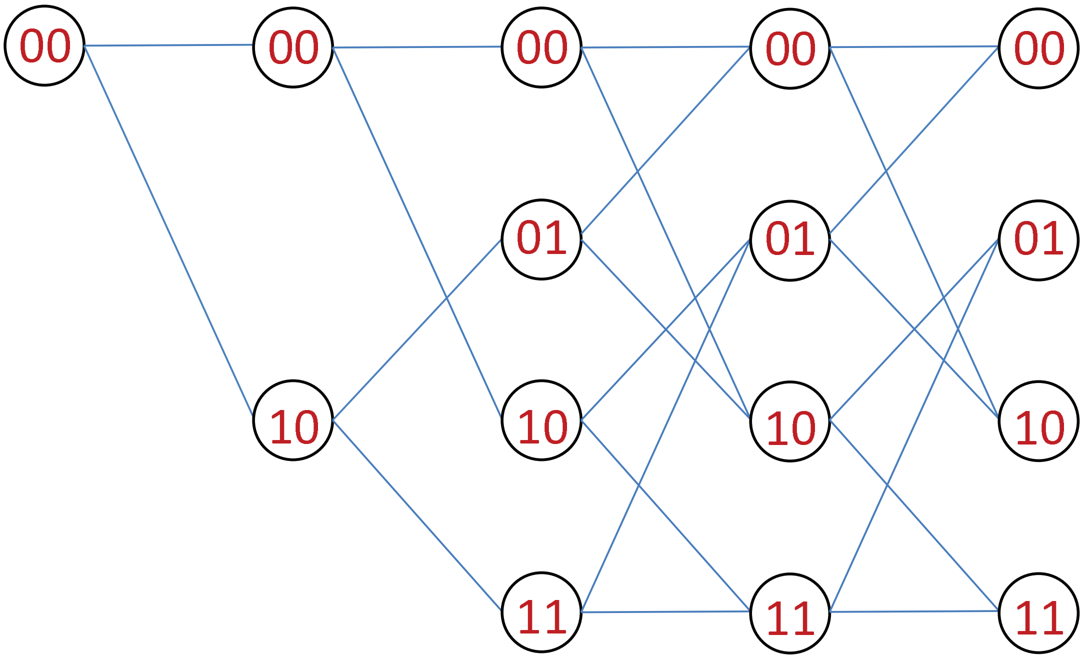
   (a) Label the branches in the trellis diagram. (10 pts)



   (b) Compute d_{free} for this code. Find all error events at d_{free} and indicate them on the extended trellis (below). For each error event at d_{free}, write down the corresponding input sequence, output sequence, and input weight. (10 pts)

   (c) Write down the expression for the nearest neighbor approximation. (10 pts)

   (d) On the following page you are given the Matlab code for the Viterbi decoder for the $[7, 5]$ (nonrecursive & nonsystematic) convolutional code that we studied in class. Modify the Matlab code so that it implements the Viterbi decoder for the convolutional code in this problem. The code is terminated in state 00 by setting two additional information bits to zero.
   Hint: You should only have to modify a small number of lines in the Matlab code. (10 pts)



2. In this problem we will study the BCJR algorithm for the systematic recursive $[7,5]$ binary convolutional code we studied in class. The trellis diagram for this code is provided below. (Recall that the branch labels are the two outputs bits, which are the information bit and the parity bit) You are also provided with extended trellises for the three parts.

   Assume there are three information bits followed by two terminating bits (such that the code terminates in state 00), and that $\sqrt{E_s} = 1$ and $\sigma^2 = 1$.

   (a) Compute the BCJR algorithm (by hand, and in the linear domain) for the following received sequence:

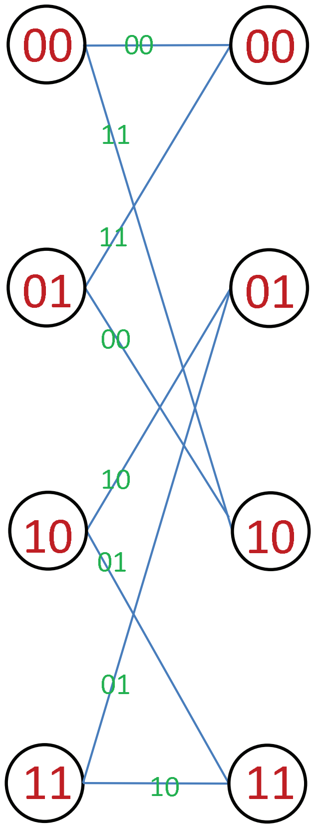
   $$\frac{\theta}{2}, \frac{\theta}{2}, 0, 0, 0, 0, 0, 0, 0, 0 \tag{1}$$

   where θ is a positive constant. Write down the values of α, β, and γ on the extended trellis on the following page, using the convention that α values are written above the state label and β values are written below. Compute the LLR's for the first and third information bit. (15 pts)

   (b) Consider the following input sequence

   $$1, 2, -\frac{\theta}{2}, -\frac{\theta}{2}, 0, 0, -\frac{\theta}{2}, -\frac{\theta}{2}, 3, 4 \tag{2}$$

   Determine what happens to the LLR of the third information bit as $\theta \to \infty$, and explain your answer.



Hint: You can solve this question without computing the actual LLR. (10 pts)

(c) Consider the following input sequence

$$\theta, \theta, -2.1, 3.4, -\theta, -\theta, 0, 0, 0, 0 \tag{3}$$

Determine what happens to the LLR of the second information bit as $\theta \to \infty$, and explain your answer.

Hint: You can solve this question without computing the actual LLR. (10 pts)

3. Consider a system where the impulse response of the TX filter is:

$$g_{\mathrm{TX}}(t) = \begin{cases} 1 & 0 \leq t \leq 2 \\ -1 & 2 < t \leq 4 \\ 0 & \text{else} \end{cases}$$

and the channel impulse response is:

$$g_C(t) = \delta(t) + 1.5\delta(t-2)$$

(a) Assuming binary modulation and a sample period of $T = 4$, how many states would the Viterbi implementation of MLSE have? (10 pts)

(b) Now consider the same TX filter but the following channel impulse response:

$$g_C(t) = 2\delta(t) - 0.5\delta(t-\tau)$$

You wish to use the Viterbi algorithm to implement MLSE, but due to complexity constraints you are limited to 16 states. Assuming binary modulation, what is the largest value of τ for which you can implement MLSE? What if you used 4-level modulation (e.g., $-3, -1, +1, +3$) instead of binary modulation - how would your answer change? (15 pts)