Problem 1

Consider the second order system given by

$$\ddot{p} + 2\xi\omega_0\dot{p} + \omega_0^2 p = \omega_0^2 f$$

1. Let the output be $y = p$. Determine the transfer function from f to y.
2. Determine the state space representation of the above input-output system in the form $\dot{x} = Ax + Bu, \quad y = Cx + Du$.
3. Sketch a Analog Computer Simulation model and implement it in Simulink
4. Assume that $p(0) = \dot{p}(0) = 0$ and simulate the step-response of the system with $\xi = 0.2, \ \omega_0 = 1 rad/s$.
5. Explore the command ss and **step** in Matlab. Note that the command **step** operates on an object of type sys. Use these commands to obtain the step response usin Matlab and compare it wil the step response obtained using Simulink

Problem 2

2. (a) Consider a system with the following input ($u(t)$)-output ($y(t)$) relation:

$$\ddot{y} + 10\dot{y} + 9y = u$$

   i. Find the transfer function that describes this system
   ii. Give a state space representation of this system.

(b) Consider a system with the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 0.1s + 4}.$$

   i. Give the ordinary differential equation that describes the relation between the input u and the output y of the system.
   ii. Give a state space representation of the same system.

(c) Consider a system with the following state space representation

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y &= (1 \;\; 1) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 2u \end{aligned}$$

   i. Find the ordinary differential equation that describes this system
   ii. Find a transfer function that describes this system (Hint: Find $X_1(s)$ and $X_2(s)$ in terms of $U(s)$ and substitute in terms of $Y(s)$).

Problem 3 and 4

3. (a) Consider the transfer function $\frac{V(s)}{U(s)} = \frac{1}{s^2+4s+1}$. Find a state space representation for this system.

   (b) Let $y = \ddot{v} + 3\dot{v} + v$.

      i. Write $y(t)$ in terms of the states in (3a)

      ii. Find the transfer function $\frac{Y(s)}{V(s)}$

   (c) Use parts (3a) and (3b) to determine the state space representation of the system described by

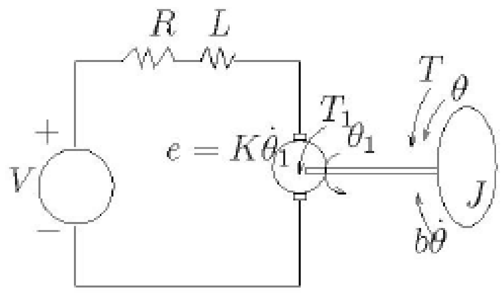
$$\ddot{y} + 4\dot{y} + y = \ddot{u} + 3\dot{u} + u.$$

4. The model used to design a cruise controller gives the following relation between throttle position u and velocity v

$$m\dot{v} + cv = ku.$$

At a particular driving condition with one driver (of weight 70 Kg) the parameters are $m = 1600$ [kg], $c = 32[kgm/s]$ and $k = 1200[N]$. Give the transfer function of the system. Also give the transfer function when there are four passengers 70 Kg each in the car.

Problem 5



5. Consider the schematic of a DC motor shown in the Figure above. A common actuator in control systems is a DC motor. The electric circuit of the armature and the body diagram of the rotor are shown in Figure. Consider as input the voltage $V(t)$ and as output the angular position of the load θ. The torque applied by the motor is $T_1 = K_e i$, whereas the emf is $e = K_e\dot{\theta}_1$, i designating current. We assume that the shaft is flexible and denote by θ_1 and θ the angular position of the two ends. We assume a simple "mass-less spring" type of model for the shaft, i.e., that the torque values T_1 applied to the shaft by the motor, and T applied to the load by the shaft are equal, i.e.,

$$T = T_1 \text{ and that } \theta_1 - \theta = \alpha T.$$

The electromechanical part of the system is modeled by the equations

$$L\frac{di}{dt} + Ri + K\dot{\theta}_1 = V$$

$$Ki = T_1$$
$$J\ddot{\theta} + b\dot{\theta} = T.$$

Find the transfer function representation of the DC motor system $\frac{\Theta(s)}{V(s)}$ in terms of J, b, K, L, R, α.