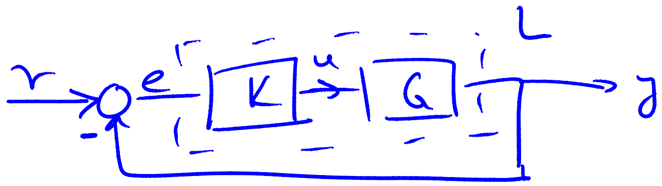
Lecture 4



⊙ We want to ascertain closed-loop performance based on open-loop characteristics of L.

⊙ <u>Tracking Requirement</u>

- Need to track all frequencies in $r(t)$ till a given bandwidth.
  - → Need to have the $r \mapsto e$ transfer function small in a given frequency range $\omega \in [0, \omega_B]$

$$(*) \quad |S(j\omega)| \leq 3\,dB \quad \forall\ \omega \in [0, \omega_B]$$

- $S(s) = \frac{1}{1+GK}$ is the transfer function from r to e.

- $\left|\frac{1}{1+L}\right|$ has to be small

- $|L|$ has to be large

Closed-loop Spec is $20\log_{10}\left|\frac{1}{1+L(j\omega)}\right| \leq 3 \quad \forall\ \omega \in [0, \omega_B]$

This is satisfied if $|L(j\omega)| \geq M_s$ for all $\omega \in [0, \omega_B]$.

S is the sensitivity transfer function

Noise Rejection:

- $n \mapsto y$ transfer function small in a frequency range $\omega \in [\omega_{BT}, \infty)$
- The transfer function from $n \mapsto y$ is the Complimentary Sensitivity transfer function. is

$$T = \frac{GK}{1+GK} = \frac{L}{1+L}$$

Ⓐ Spec is that

$$20\log|T(j\omega)| = 20\log\frac{|L(j\omega)|}{|1+L(j\omega)|} \leq -3$$

$$\forall\, \omega \in [\omega_{BT}, \infty)$$

$$\rightarrow |T(j\omega)| = \left|\frac{L(j\omega)}{1+L(j\omega)}\right| = \left|\frac{1}{1+\frac{1}{L(j\omega)}}\right|$$

$\therefore$ for $|T(j\omega)|$ to be small we need $|L(j\omega)|$ to be small

$\rightarrow \quad |L(j\omega)| \leq M_T$ for all $\omega \in [\omega_{BT}, \infty)$

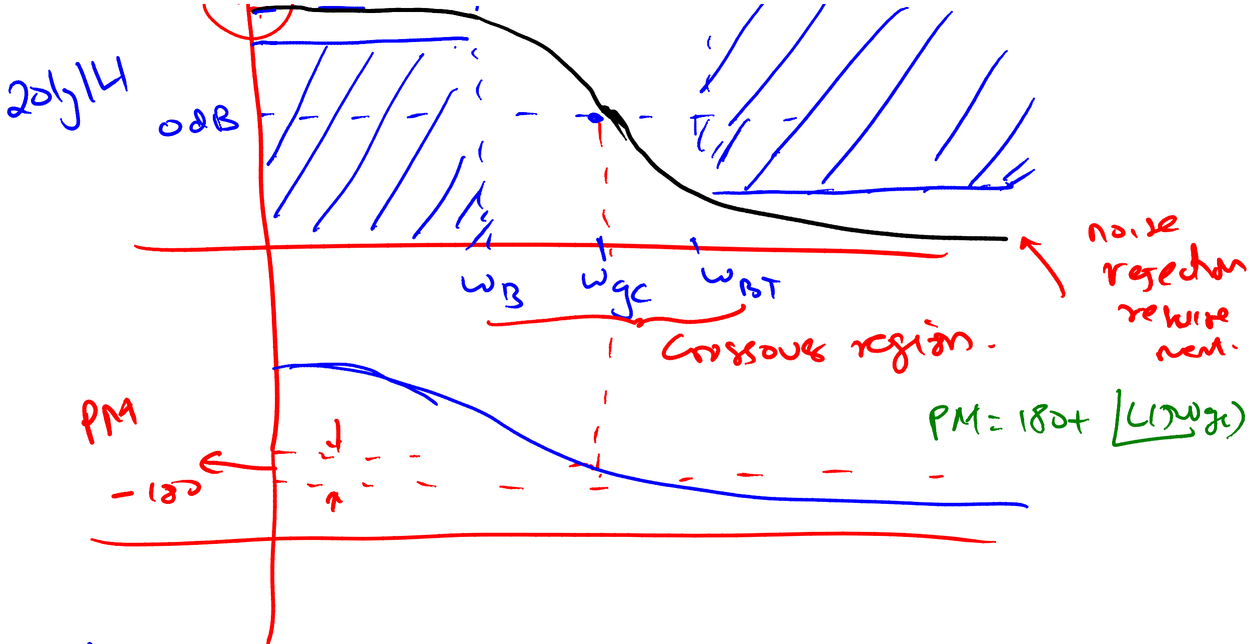
$\rightarrow$ Note that $S + T = 1$

$$\text{Ⓐ} \quad \underbrace{\frac{1}{1+L}}_{S} + \underbrace{\frac{L}{1+L}}_{T} = \frac{1+L}{1+L} = 1$$

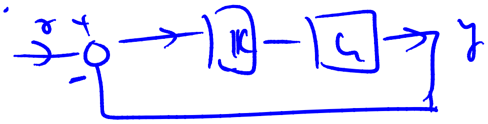
$$\text{Ⓐ} \quad S(j\omega) + T(j\omega) = 1 \qquad \forall\, \omega$$

clearly we want the intervals $[0, \omega_B)$ and $[\omega_{BT}, \infty)$ to be disjoint.

tracking requirement



Bode-Plot:



① Typical System G is stable and has no rhp zeros (minimum phase system)

→ (1+ KG)
↑ proportional

→ for very low gains K the closed-loop system has poles at the poles of G

→ as the gain K is increased, the poles migrate to the zeros of the plant G and if the relative degree of G is q, then q poles moves to have a magnitude ∞ with $K \to \infty$.

→ Typical Scenarios as the proportional gain K is increased ~~the~~ some of the closed-loop poles migrate from the lhp to the rhp.

→ [illegible]

- There exists a critical proportional gain where there are unstable poles only on the $j\omega$ axis.

at this critical gain

$$1 + \underbrace{K_{cr}\, G(j\omega)}_{L} = 0 \quad \text{for some } \omega.$$

at this K_{cr}: $|L| = 1$

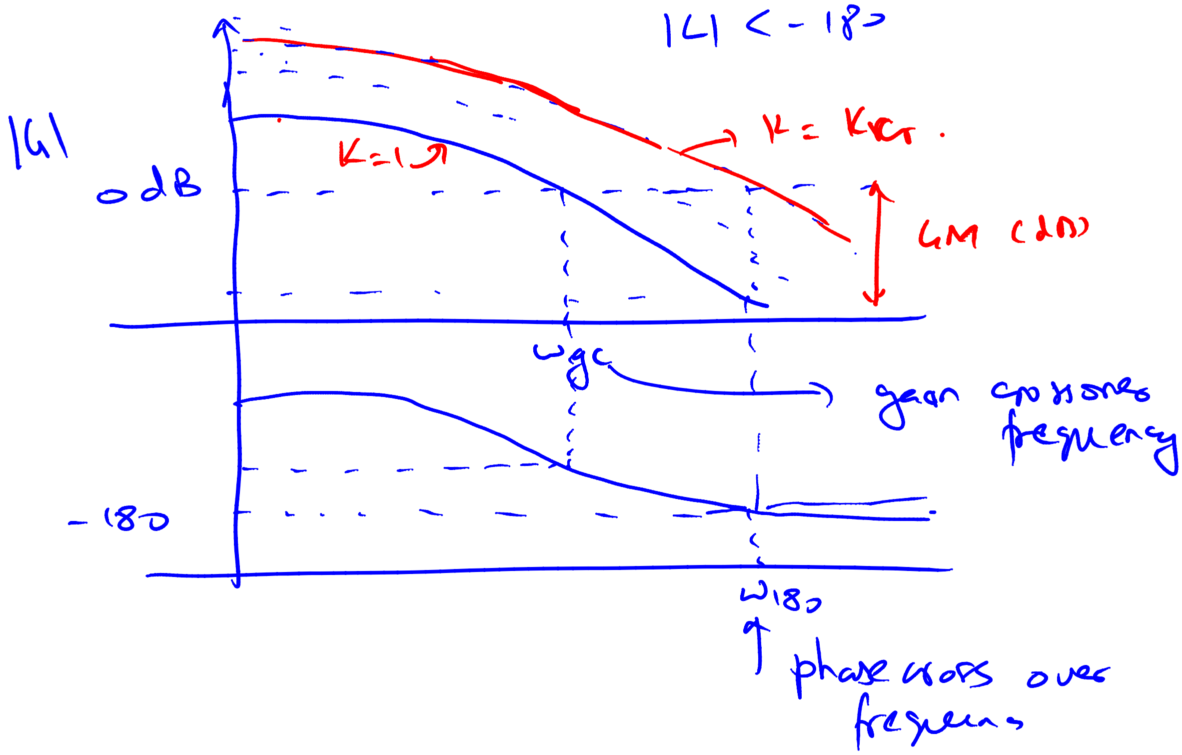
and $\angle L = -180$

for $K < K_{cr}$: $|L| < 1$ and

$\angle L > -180$

→ $K > K_{cr}$: $|L| > 1$

$|L| < -180$



⊙ <u>Steady-state error Requirements</u>:

- The behavior of

$$e_{ss} = \lim_{t\to\infty} e(t) \quad \text{due to specific } r(t).$$

① The key tool to asses steady-state error requirements is the Final Value theorem (FVT) :- Under ~~certain~~ stability conditions

$$\boxed{\lim_{t\to\infty} p(t) = \lim_{s\to 0} s\, P(s).}$$

① Suppose ~~r(t) is a~~ unit step

$$r(S) = \left(\frac{1}{1+L}\right)$$

② Suppose $r(t)$ is a unit step

$$r(S) = \frac{1}{s}$$

and $e(S) = \left(\frac{1}{1+L}\right)\frac{1}{s}$

and $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\left(\frac{1}{1+L}\right)\cdot\frac{1}{s}$

$$= \lim_{s\to 0} \frac{1}{1+L}$$

$$= \frac{1}{1+\lim_{s\to 0} L}$$

K. $k_p = \lim_{s\to 0} L(s)$

$\therefore \quad e_{ss} = \dfrac{1}{1+k_p}.$

$k_p = \infty$ for zero error in tracking steps.

$\therefore$ $L(0)$ has to be ∞ if steady state error for tracking steps has to be 0.

— $K_p = \infty$ are type I systems.

→ $K_v = \lim s\,L(s)$; $K_a = \lim s^2 L(s)$

$s \to 0 \qquad\qquad\qquad s \to 0$

$K_v = \infty$ then type II ; $K_a = \infty$ we have type III.

→ Internal-model principle: given a stable interconnection, the L has to have a model of the unstable part of the reference for tracking the reference with zero error. (Look at the undergraduate notes)