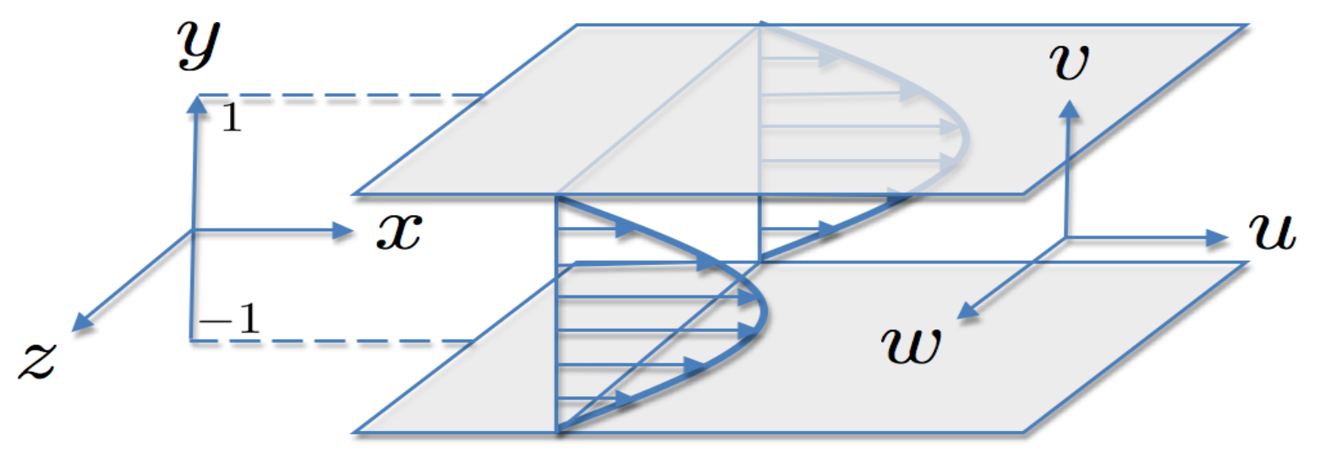
UNIVERSITY OF MINNESOTA

Dynamics and control of transitional and turbulent shear flows of Newtonian and viscoelastic fluids

MIHAILO R. JOVANOVIĆ (PI, CAREER AWARD CMMI-06-44793, CONTROL SYSTEMS), BINH K. LIEU, RASHAD MOARREF

PROBLEM FORMULATION



Pressure driven channel flow

- Governing equations

$$\begin{aligned}\mathbf{u}_t &= -(\mathbf{u}\cdot\nabla)\,\mathbf{u} - \nabla p + (\beta/Re)\,\Delta\mathbf{u}\\ &\quad + ((1-\beta)\,/Re)\,\nabla\cdot\boldsymbol{\tau}\\ 0 &= \nabla\cdot\mathbf{u}\\ \boldsymbol{\tau}_t &= \boldsymbol{\tau}\cdot(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\cdot\boldsymbol{\tau} - (\mathbf{u}\cdot\nabla)\boldsymbol{\tau}\\ &\quad + (1/We)\,(\nabla\mathbf{u} + (\nabla\mathbf{u})^T - \boldsymbol{\tau})\end{aligned}$$

$\mathbf{u}$ – velocity
p – pressure
$\boldsymbol{\tau}$ – polymer stress

- **Key parameters:**

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}}$$

$$We = \frac{\text{polymer relaxation time}}{\text{characteristic flow time}}$$

$$\beta = \frac{\text{solvent viscosity}}{\text{total viscosity}} \in [0,1]$$

$\beta = 1 \Rightarrow$ Newtonian fluids

$\beta \in [0,1) \Rightarrow$ viscoelastic fluids

TURBULENCE MODELING

1) Turbulent viscosity hypothesis

2) Turbulent viscosity: $\nu_T = c\,\dfrac{k^2}{\epsilon}$

k - turbulent kinetic energy
ϵ - rate of dissipation of k
ν_T - turbulent viscosity

Challenge:

Determine the effect of control on ν_T

ACKNOWLEDGEMENTS

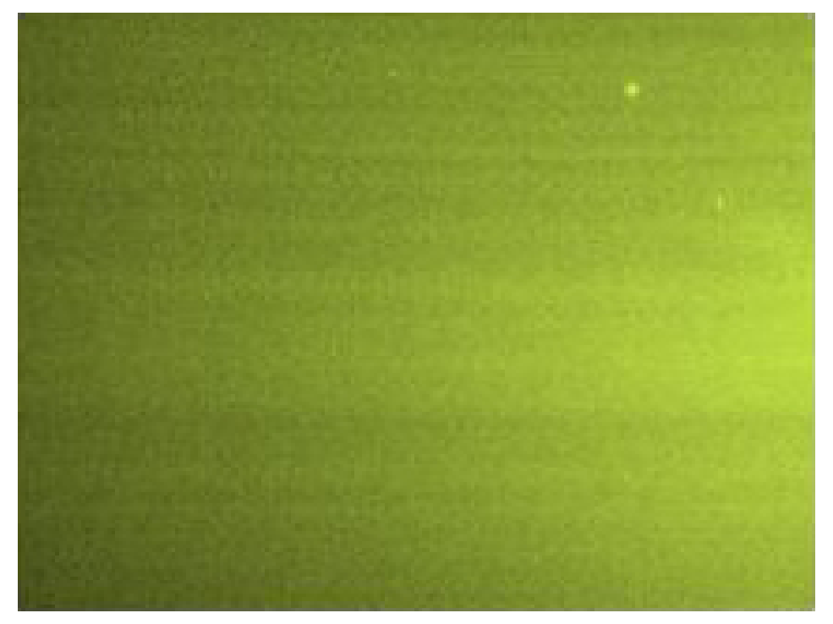
Supported by: National Science Foundation CAREER Award CMMI-06-44793

Computational resources: Minnesota Supercomputing Institute

MOTIVATION

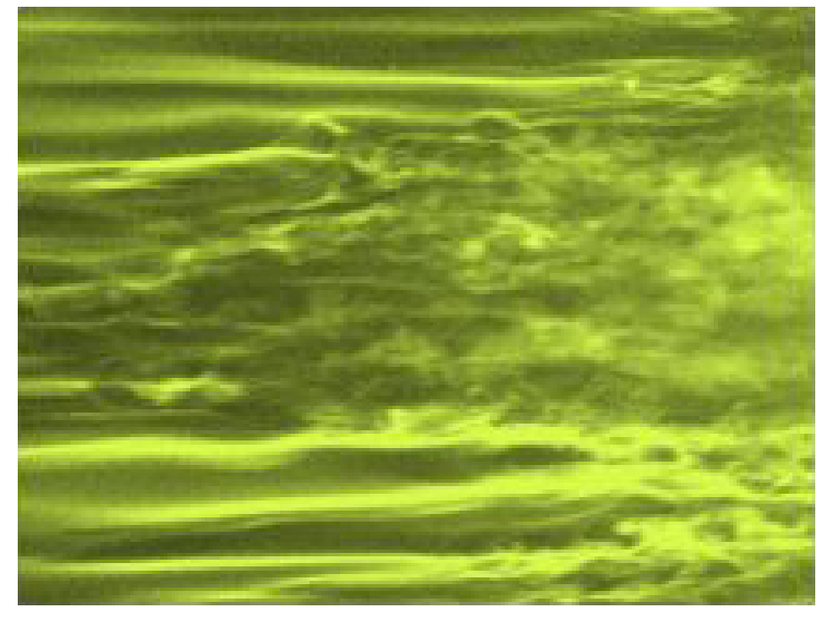
Turbulence: one of the most intriguing natural phenomena

Laminar
(smooth and ordered)



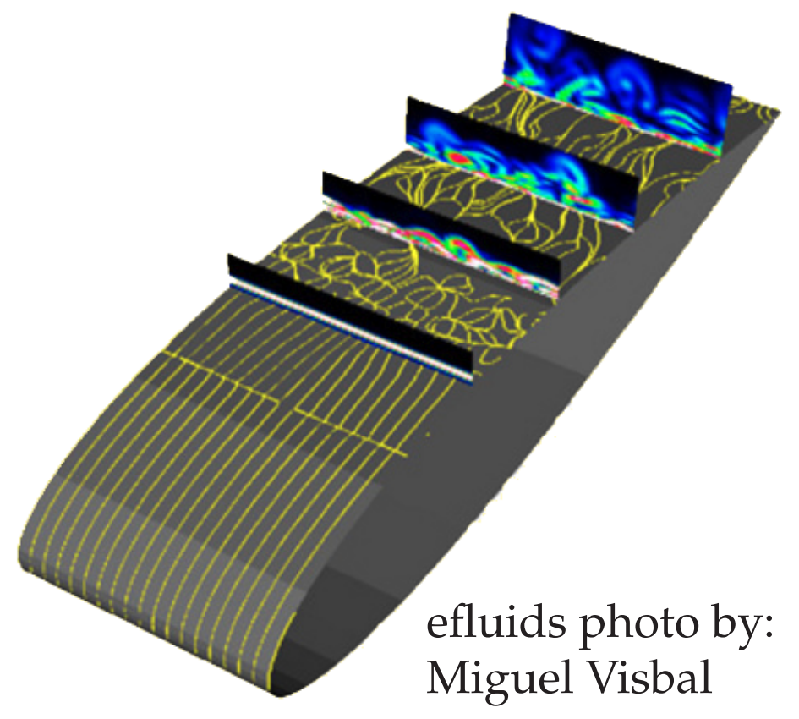
transition to turbulence

Turbulent
(complex and disordered)

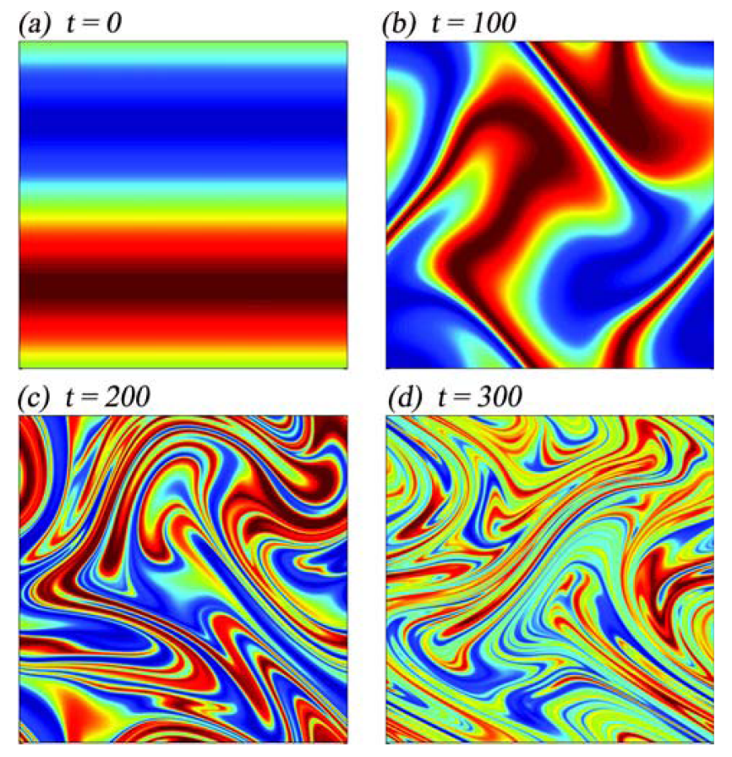


Boundary layer (Matsubara & Alfredsson, *J. Fluid Mech.* '01)

turbulence suppression:
increased energy efficiency



promotion of turbulence:
enhanced mixing

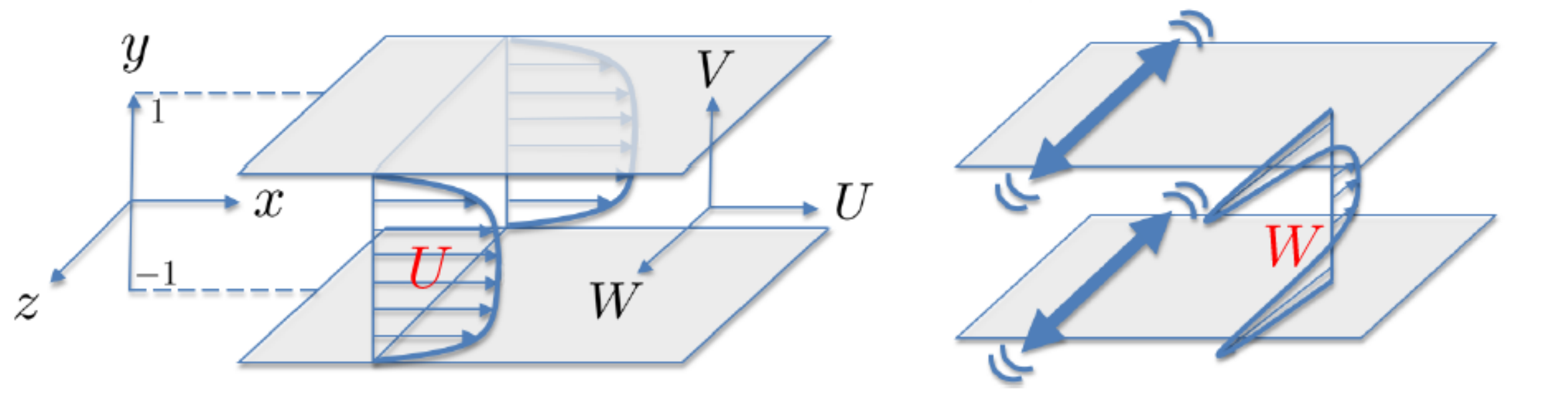


Saintillan & Shelley, *Phys. Fluids* '08

DRAG REDUCTION BY TRANSVERSE OSCILLATIONS

Sensor-free vibrational control

$W(y = \pm 1, t) = 2\alpha \sin(\frac{2\pi}{T} t)$ α - oscillation amplitude, T - oscillation period

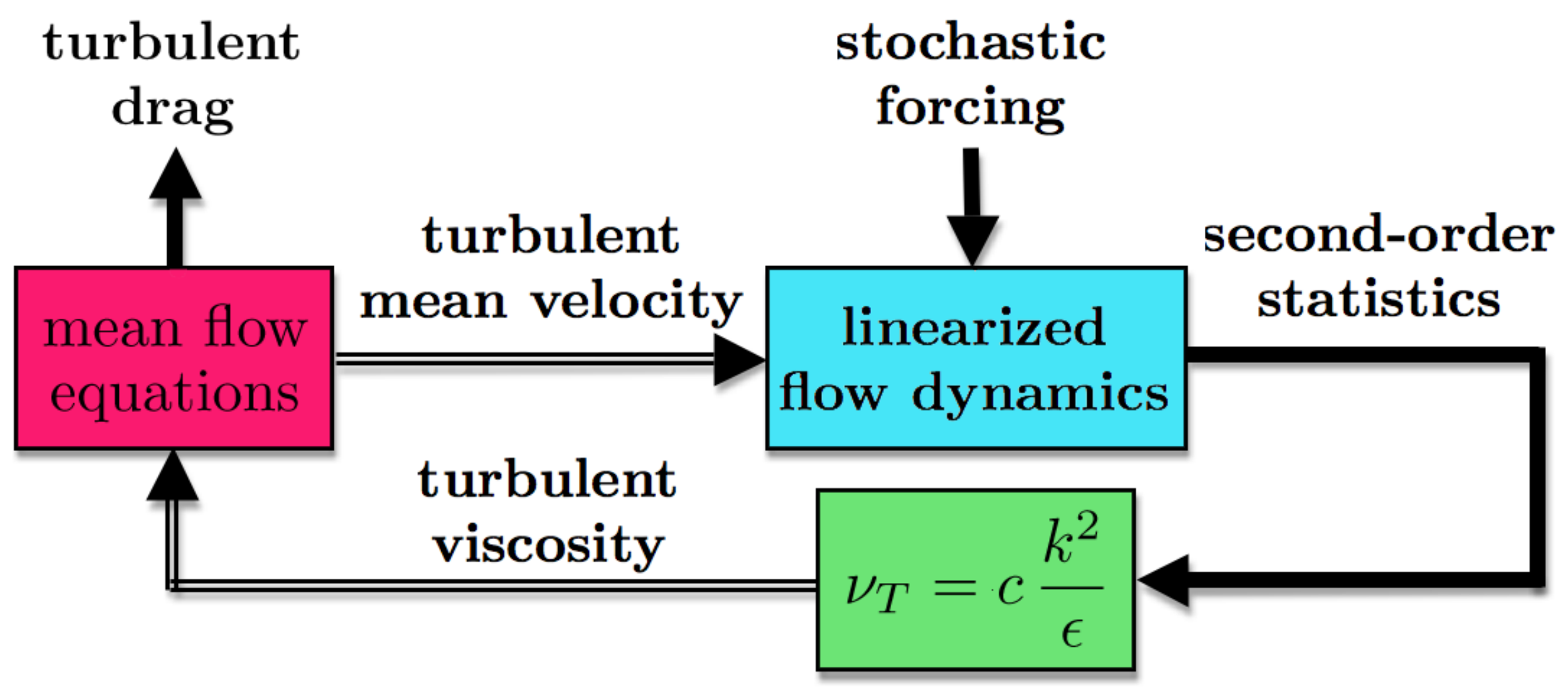


drag reduction: $f_1(U)$

control effort: $f_2(W)$

net efficiency: $f_1(U) - f_2(W)$

Control-oriented turbulence modeling



Model-based design of small-amplitude oscillations

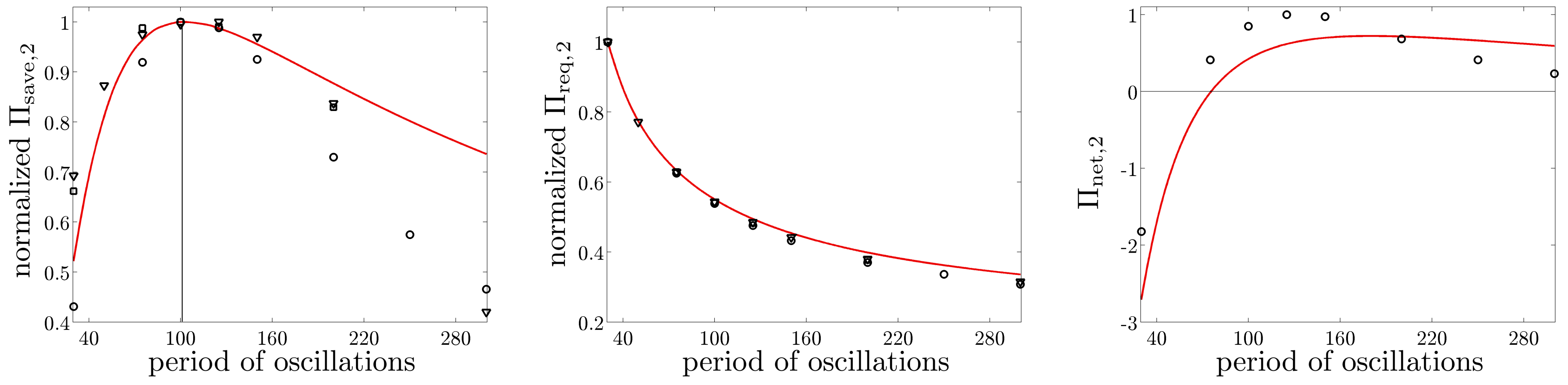
$$\begin{aligned}k &= k_0 + \alpha^2\,k_2 + \mathcal{O}(\alpha^4)\\ \epsilon &= \epsilon_0 + \alpha^2\,\epsilon_2 + \mathcal{O}(\alpha^4)\\ &\Downarrow\\ \nu_T &= \nu_{T0} + \alpha^2\,\nu_{T2} + \mathcal{O}(\alpha^4)\end{aligned}$$

Saved power: $\Pi_{\rm save} \approx \alpha^2\,\Pi_{\rm save,2}$

Required power: $\Pi_{\rm req} \approx \alpha^2\,\Pi_{\rm req,2}$

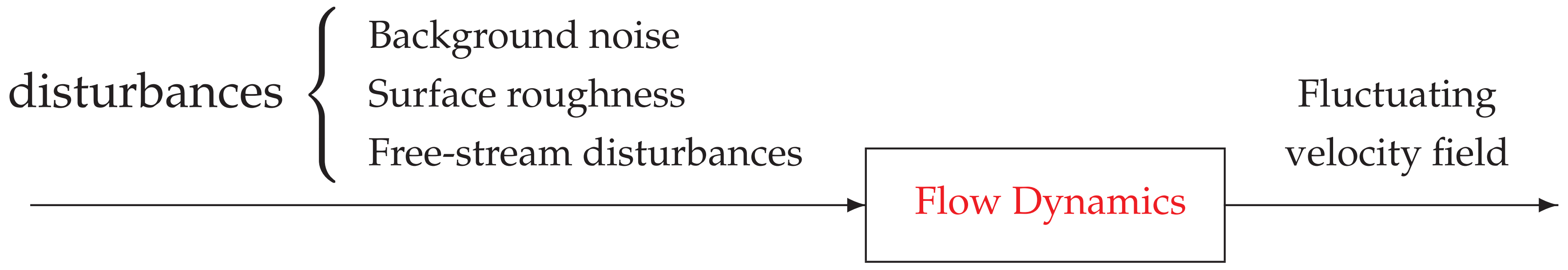
Net power: $\Pi_{\rm net} \approx \alpha^2\,\Pi_{\rm net,2}$

perturbation analysis (red), simulations (Quadrio & Ricco '04; symbols)

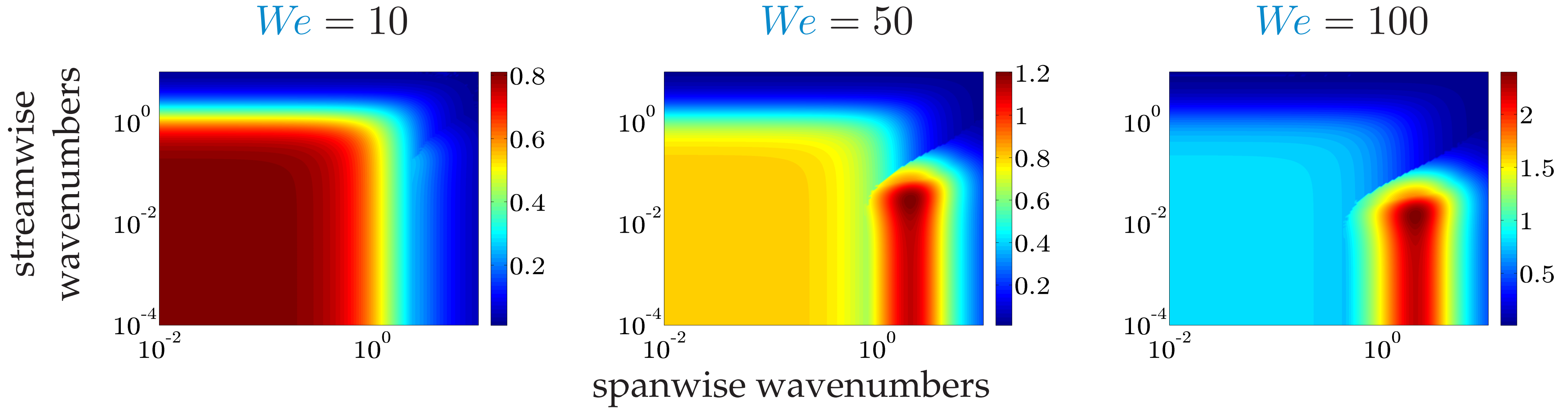


INERTIALESS FLOWS OF VISCOELASTIC FLUIDS

- **Objective:** study the mechanisms triggering "elastic turbulence"
- **Approach:** uncertainty quantification (worst-case amplification of disturbances)

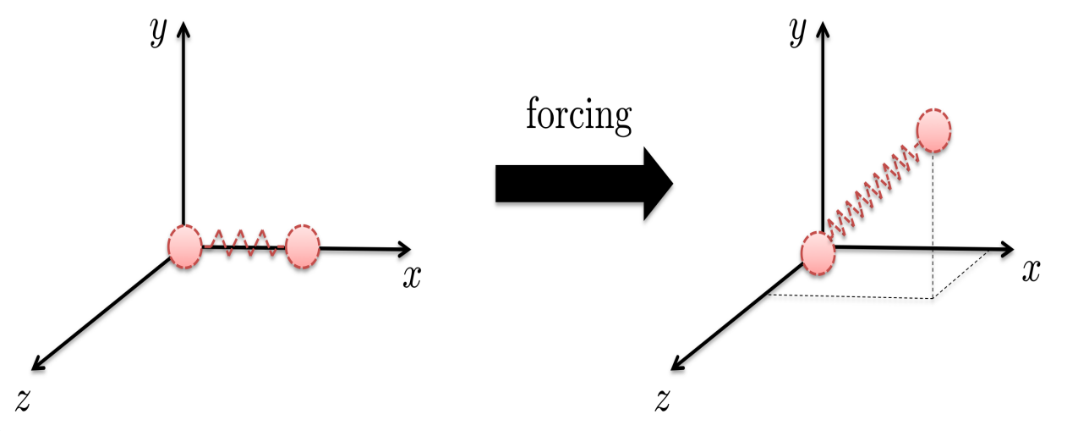


- Worst-case amplification in flows with $\beta = 0.5$:

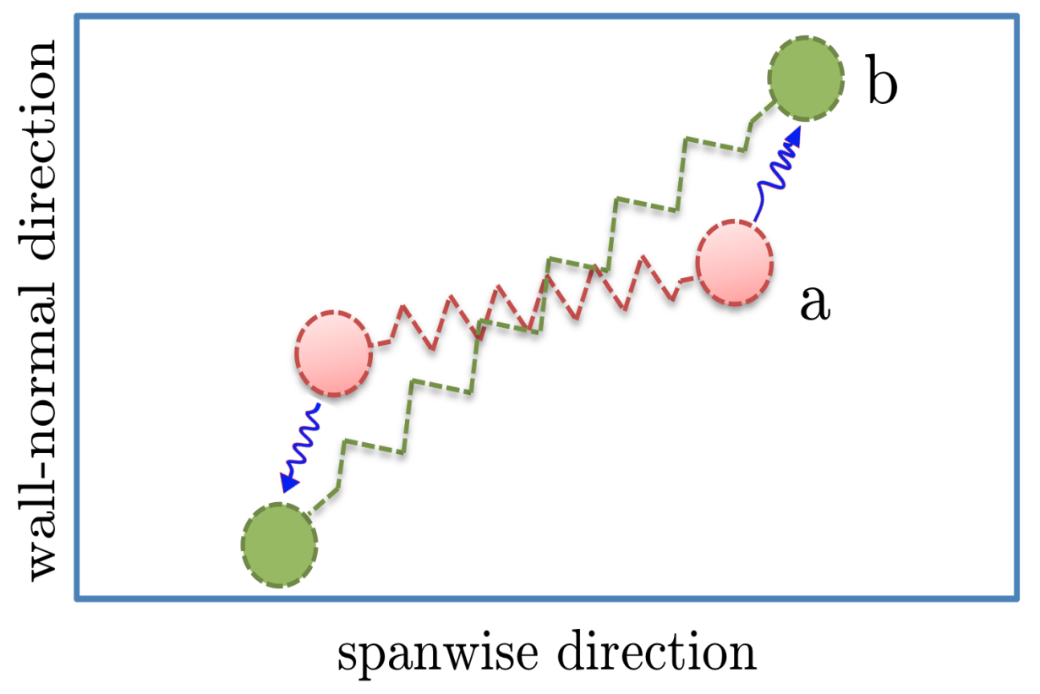


- Flows with high We: dominance of streamwise-elongated flow structures

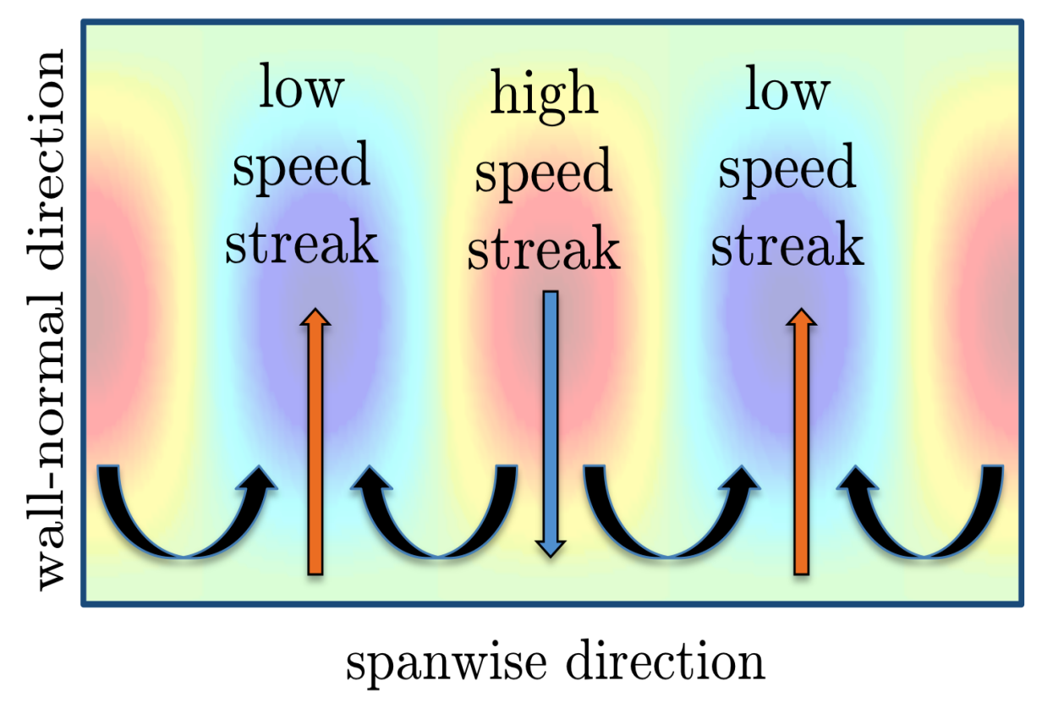
influence of disturbances:



polymer stretching:



lift-up mechanism:



PUBLICATIONS

[1] R. Moarref and M. R. Jovanović "Model-based design of transverse wall oscillations for turbulent drag reduction", *J. Fluid Mech.*, doi:10.1017/jfm.2012.272, 2012.

[2] M. R. Jovanović and S. Kumar "Nonmodal amplification of stochastic disturbances in strongly elastic channel flows", *J. Non-Newtonian Fluid Mech.*, 166(14-15):755-778, 2011.

[3] B. K. Lieu, M. R. Jovanović, and S. Kumar "Worst-case amplification of disturbances in inertialess flows of viscoelastic fluids", *Preprints of the 18th IFAC World Congress*, 14458-14463, 2011.