Dynamics and control of wall-bounded shear flows

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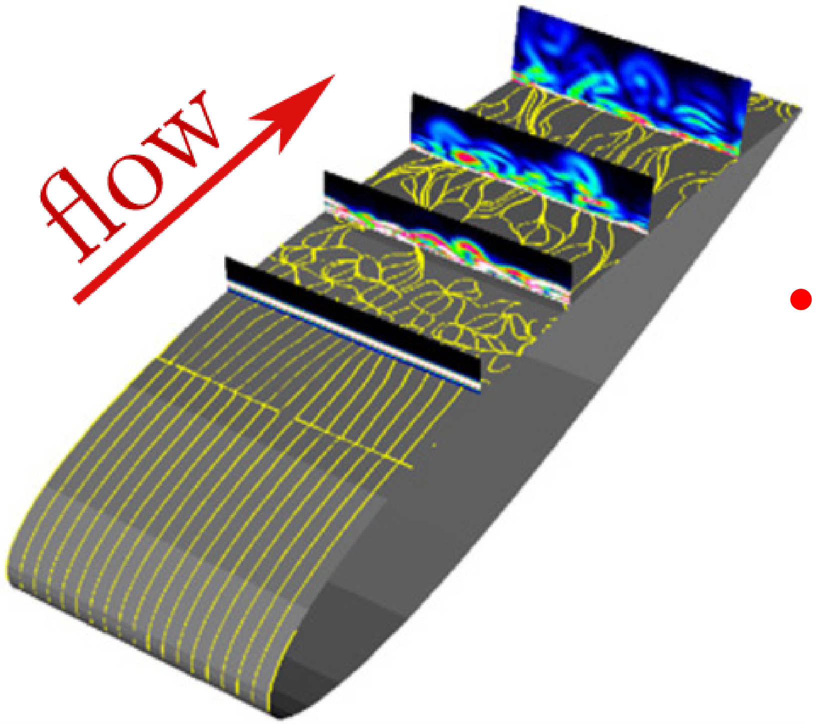
- **The early stages of transition**
  - ⋆ initiated by high flow sensitivity
- **Controlling the onset of turbulence**
  - ⋆ simulation-free design for reducing sensitivity

} **Key issue:** **high flow sensitivity**

IPAM Workshop on Turbulence in Engineering Applications

Transition to turbulence



- EXPERIMENTAL ONSET OF TURBULENCE
  - ⋆ **much before instability**
  - ⋆ **depends on experimental conditions**

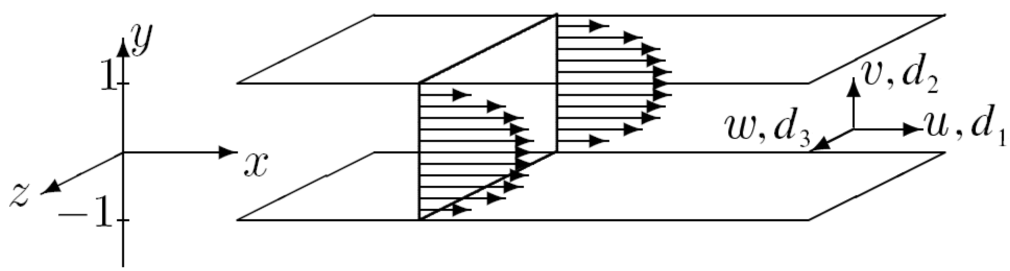
- BYPASS TRANSITION
  - ⋆ **triggered by high flow sensitivity** { large transient responses; large noise amplification; small stability margins }

Farrell, Ioannou, Henningson, Schmid, Trefethen, Kim, Luchini, Bewley, Bamieh, . . .

Input-output analysis

- TOOL FOR QUANTIFYING SENSITIVITY
  - ⋆ **spatio-temporal frequency responses**

$\mathbf{d}$ { free-stream turbulence / surface roughness } → **Linearized Dynamics** → fluctuating velocity field $\mathbf{v}$



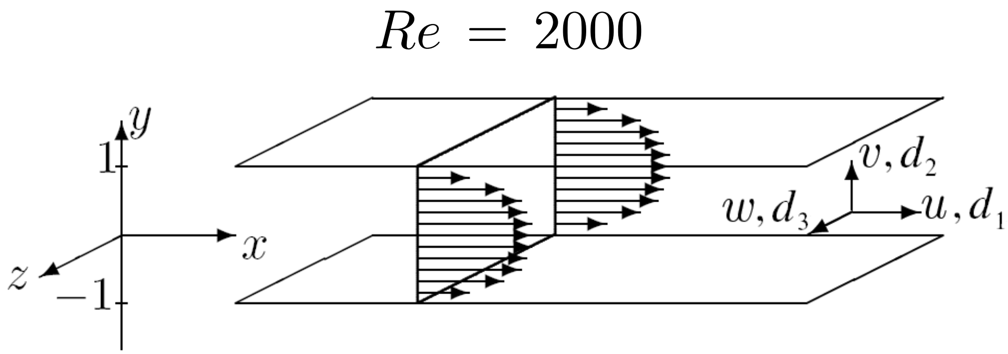
$$\underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{\text{amplification}} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\mathbf{v}}$$

IMPLICATIONS FOR:

transition: insight into mechanisms

control: control-oriented modeling

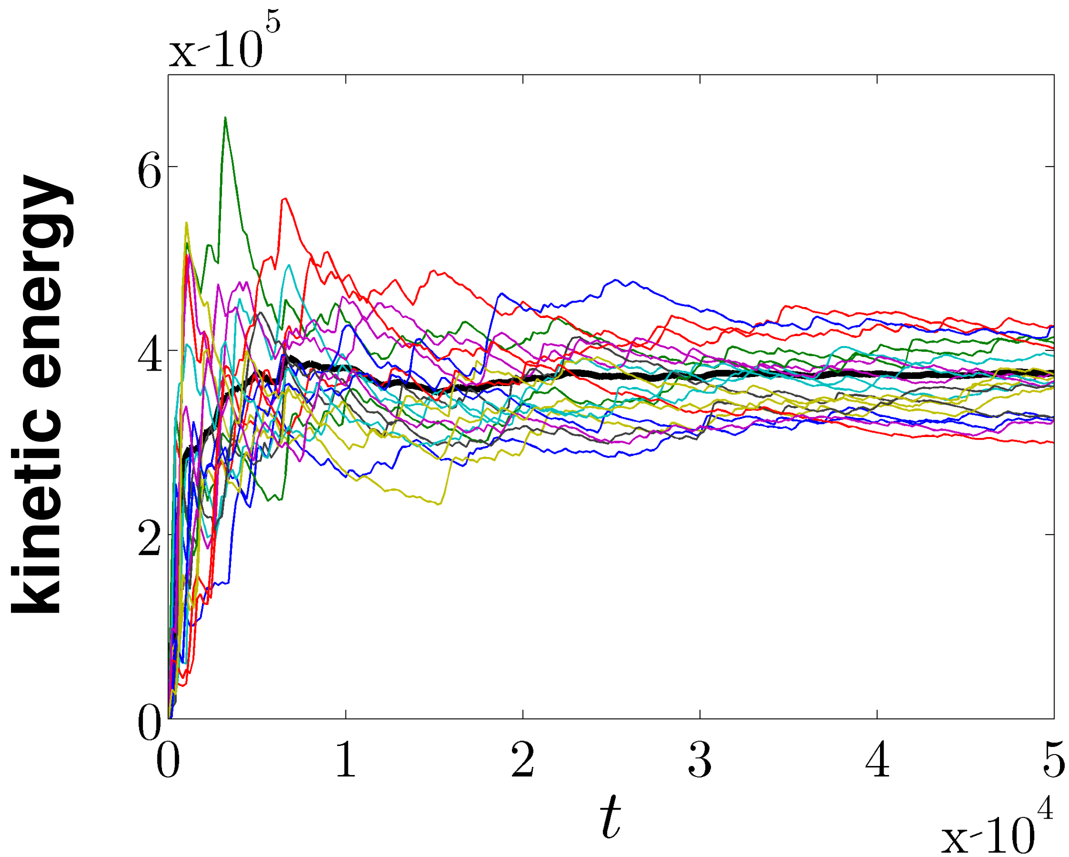
Response to stochastic forcing



forcing:

white	in t and y
harmonic	in x and z

$$\mathbf{d}(x,y,z,t) \;=\; \hat{\mathbf{d}}(k_x,y,k_z,t)\,\mathrm{e}^{\mathrm{i}k_x x}\,\mathrm{e}^{\mathrm{i}k_z z}$$

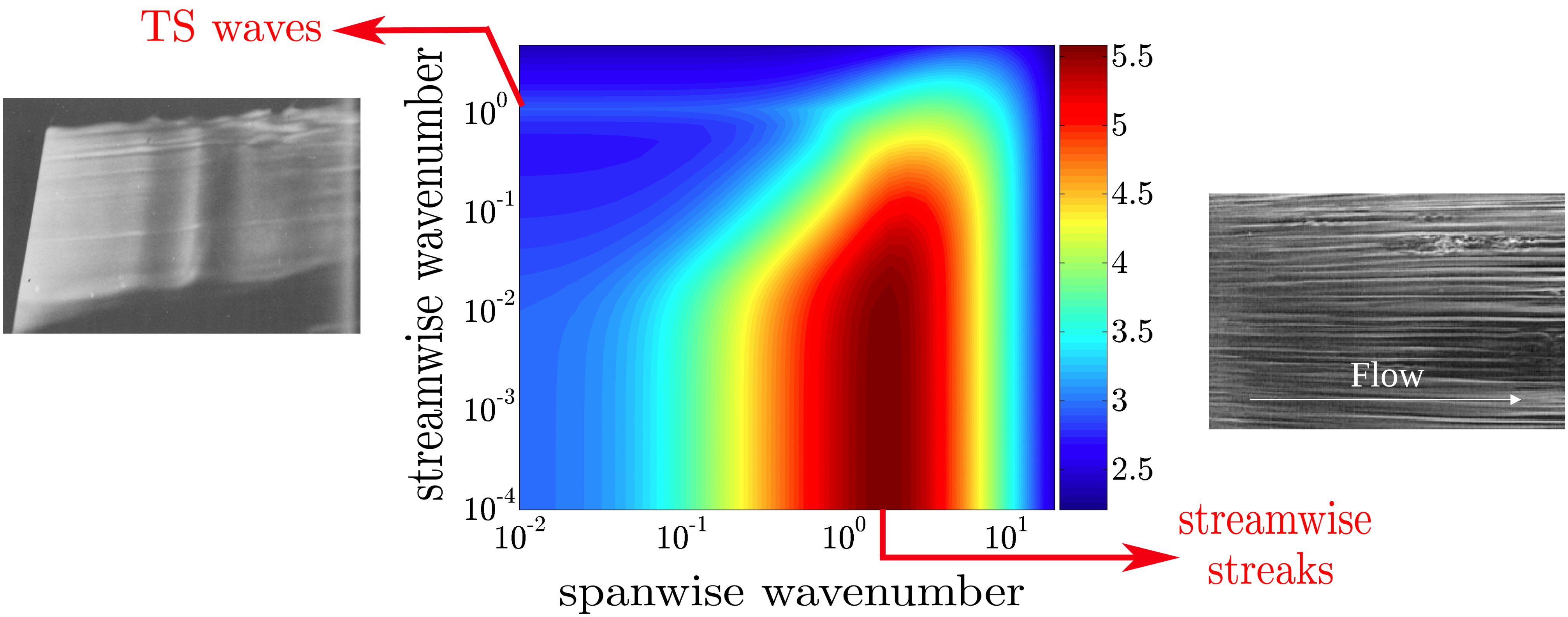


$$\hat{\psi}_t \;=\; \mathbf{A}(k_x,k_z)\,\hat{\psi} \;+\; \hat{\mathbf{d}}$$

20 simulations

Variance amplification

channel flow with $Re = 2000$:



- **Dominance of streamwise elongated structures**

  streamwise streaks!

Farrell & Ioannou, Phys. Fluids A ’93

Bamieh & Dahleh, Phys. Fluids ’01

Jovanović & Bamieh, J. Fluid Mech. ’05

Amplification mechanism

non-normal

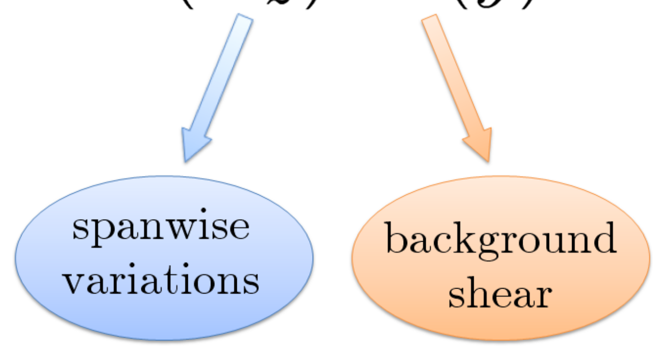
$$\begin{bmatrix} v_t \\ \eta_t \end{bmatrix} = \overbrace{\begin{bmatrix} A_{\rm os} & 0 \\ Re\, A_{\rm cp} & A_{\rm sq} \end{bmatrix}}^{\text{non-normal}} \begin{bmatrix} v \\ \eta \end{bmatrix}$$

- STREAMWISE-CONSTANT MODEL
  - ⋆ dynamics of normal vorticity η

$$\eta_t \;=\; \Delta \eta \;+\; Re\, A_{\rm cp}\, v \qquad\qquad A_{\rm cp} \;=\; -({\rm i} k_z)\; U'(y)$$

$\downarrow$

source



vortex tilting or **lift-up**

FLOW CONTROL

- **Objective**
  - ⋆ **controlling the onset of turbulence**

- **Transition initiated by**
  - ⋆ **high flow sensitivity**

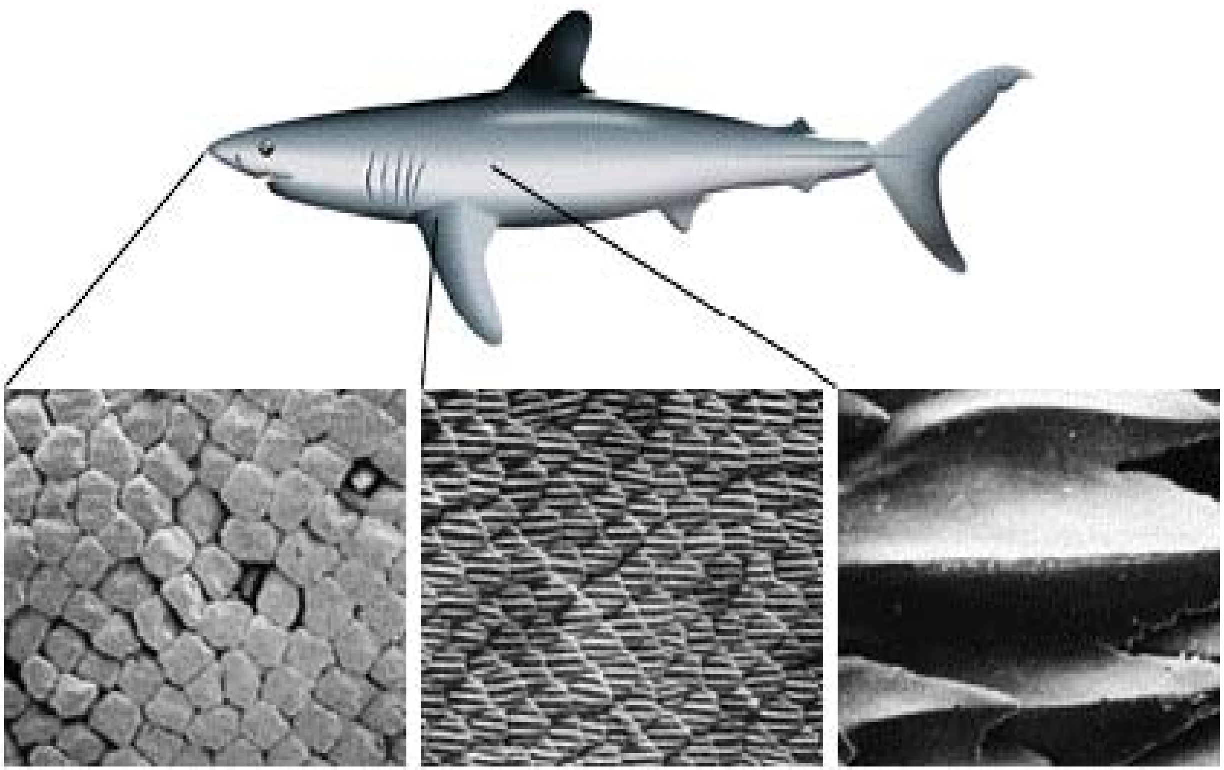
- **Control strategy**
  - ⋆ **reduce flow sensitivity**

Moarref & Jovanović, J. Fluid Mech. '10

Lieu, Moarref, Jovanović, J. Fluid Mech. '10

Sensor-free flow control

geometry modifications	surface oscillations	body forces
riblets super-hydrophobic surfaces	transverse oscillations	oscillatory forces traveling waves

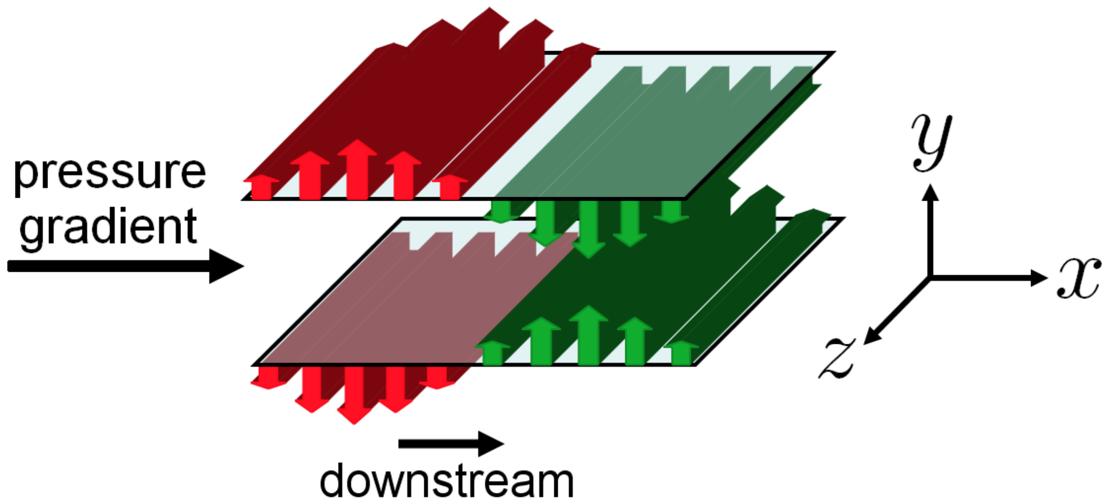


shark skin

lotus leaf

COMMON THEME: **PDEs with spatially or temporally periodic coefficients**

Blowing and suction along the walls



$c > 0$ downstream

$c < 0$ upstream

BOUNDARY CONDITION: $V(y = \pm 1) \;=\; \mp\, \alpha \cos\big(\omega_x (x \,-\, ct)\big)$

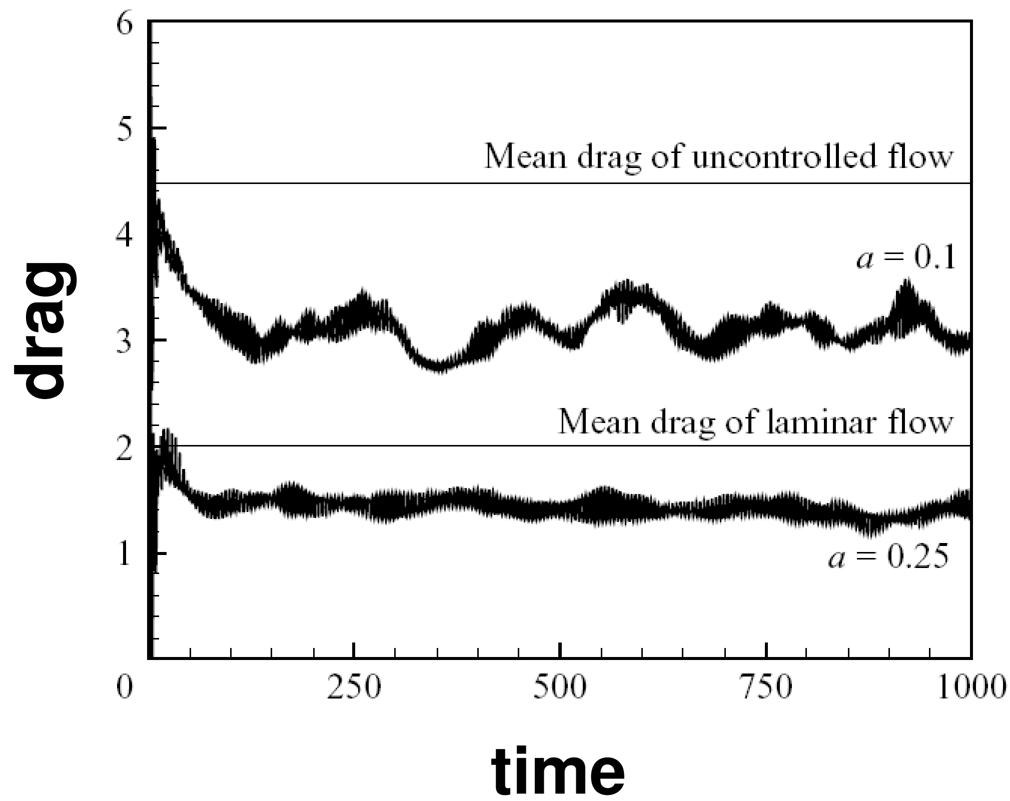
NOMINAL VELOCITY: $(U(\bar{x}, y),\; V(\bar{x}, y),\; 0)$

steady in a traveling wave frame

periodic in $\bar{x} \;:=\; x \,-\, ct$

Min, Kang, Speyer, Kim, J. Fluid Mech. '06

upstream $\Rightarrow$ SUSTAINED SUB-LAMINAR DRAG



CHALLENGE

selection of wave parameters

- THIS TALK
  - ⋆ **cost of control**
  - ⋆ **onset of turbulence**

• DESIRED EFFECTS OF CONTROL

⋆ **skin-friction** $\searrow$

⋆ **net efficiency** $\nearrow$

⋆ **fluctuations' energy** $\searrow$

SKIN-FRICTION ANALYSIS

relative to laminar flow:

UPSTREAM:	**reduce** skin-friction ✓
DOWNSTREAM:	**increase** skin-friction

Min, Kang, Speyer, Kim, J. Fluid Mech. '06

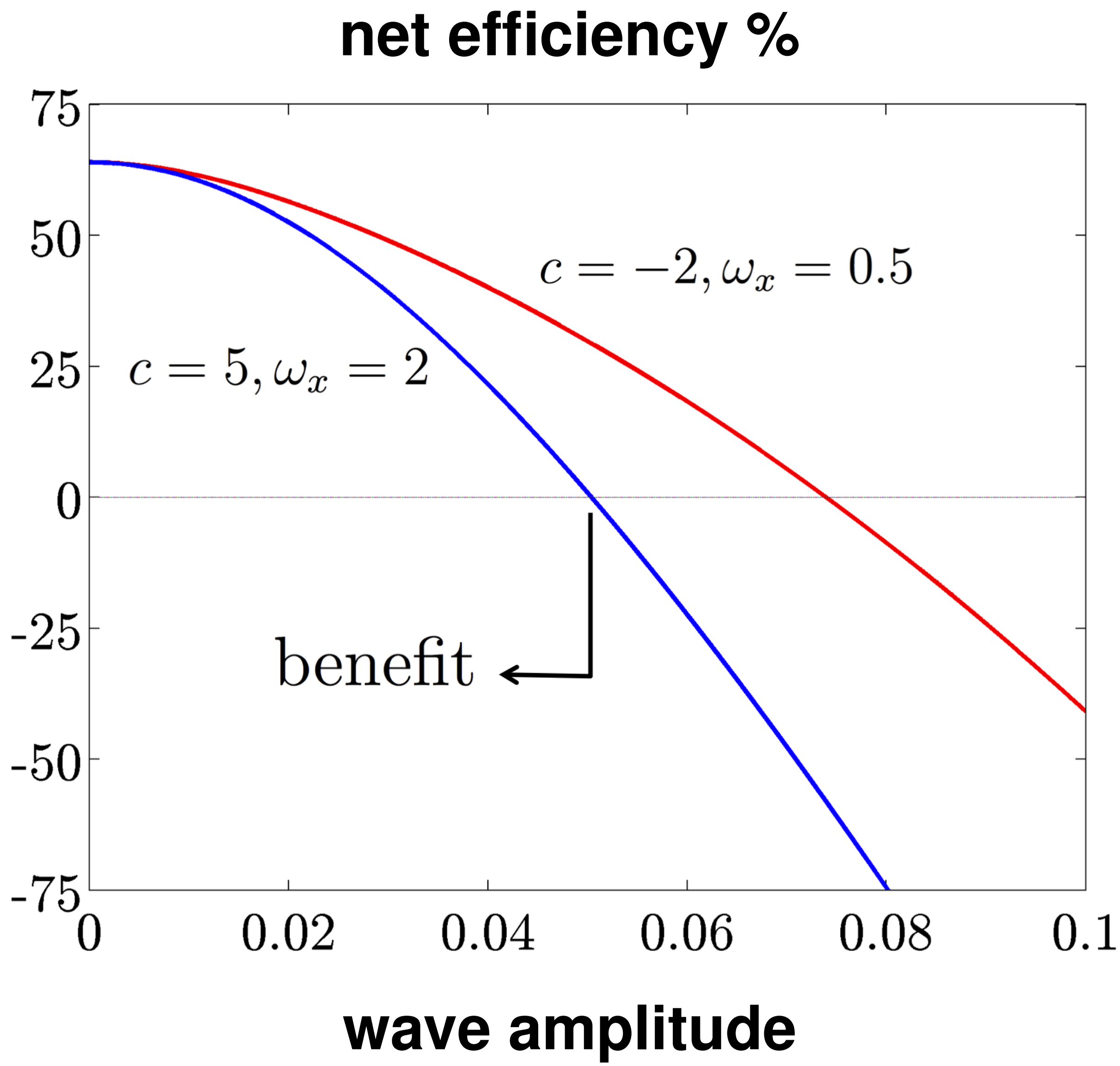
Hœpffner & Fukagata, J. Fluid Mech. '09

Net efficiency

RELATIVE TO

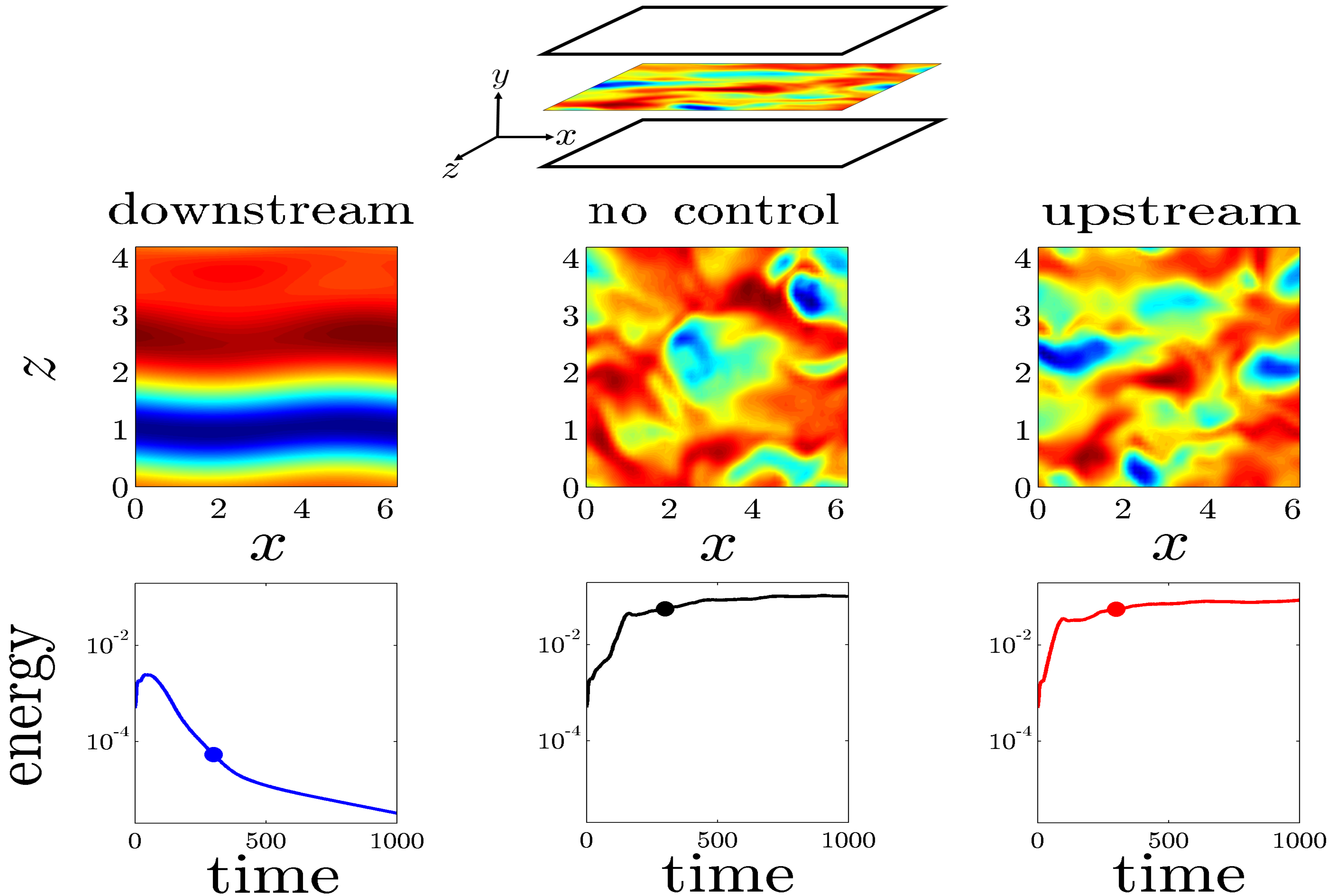
uncontrolled

turbulent flow



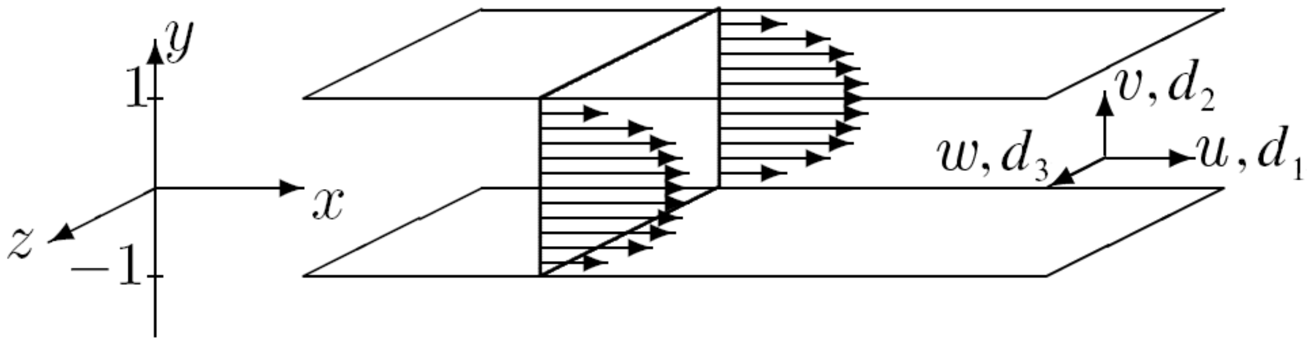
Moarref & Jovanović, J. Fluid Mech. '10

Velocity fluctuations: DNS preview



Lieu, Moarref, Jovanović, J. Fluid Mech. '10

Fluctuations' dynamics: controlled flow



EVOLUTION MODEL: **linearization around** $(U(\bar{x}, y),\; V(\bar{x}, y),\; 0)$

⋆ periodic coefficients in $\bar{x} \;:=\; x \;-\; ct$

$$\begin{aligned} \psi_t \;&=\; \mathbf{A}\,\psi \;+\; \mathbf{B}\,\mathbf{d} \\ \mathbf{v} \;&=\; \mathbf{C}\,\psi \end{aligned}$$

CONTROL OBJECTIVE: amplification reduction

- **Simulation-free approach to determining energy density**

Moarref & Jovanović, J. Fluid Mech. ’10

effect of small wave amplitude:

$$\text{energy density} = \text{energy density} + \underbrace{\alpha^2}_{\text{small}} E_2(\theta, k_z; Re; \omega_x, c) + O(\alpha^4)$$

$\downarrow$ **with control** $\quad\quad$ $\downarrow$ **w/o control**

$(\theta,\ k_z) \rightsquigarrow$ spatial wavenumbers

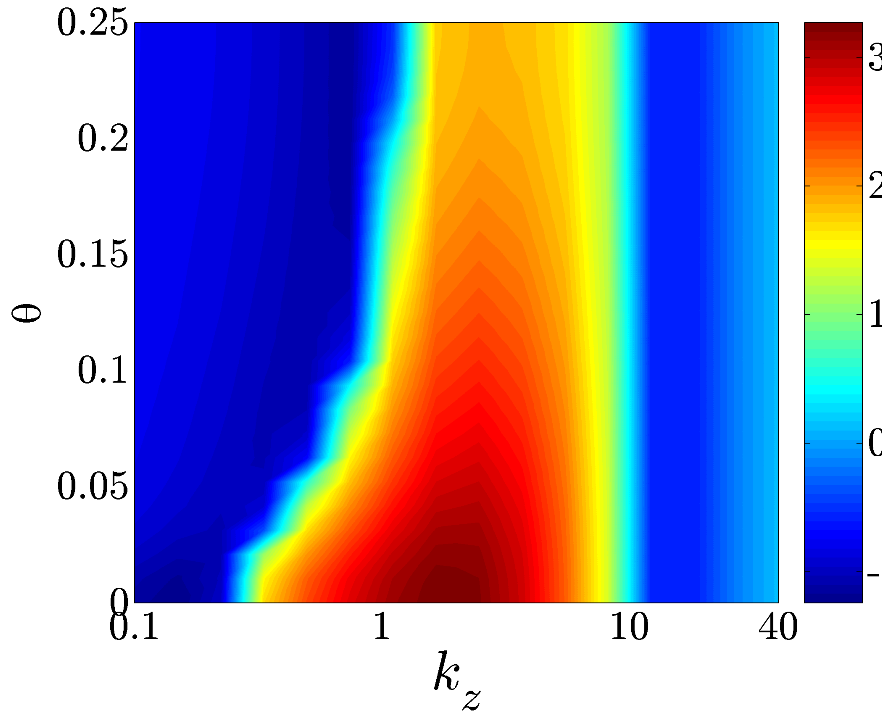
Variance amplification: controlled flow with $Re = 2000$

explicit formula:

$$\frac{\text{energy density with control}}{\text{energy density w/o control}} \approx 1 + \alpha^2 g_2(\theta, k_z; \omega_x, c)$$

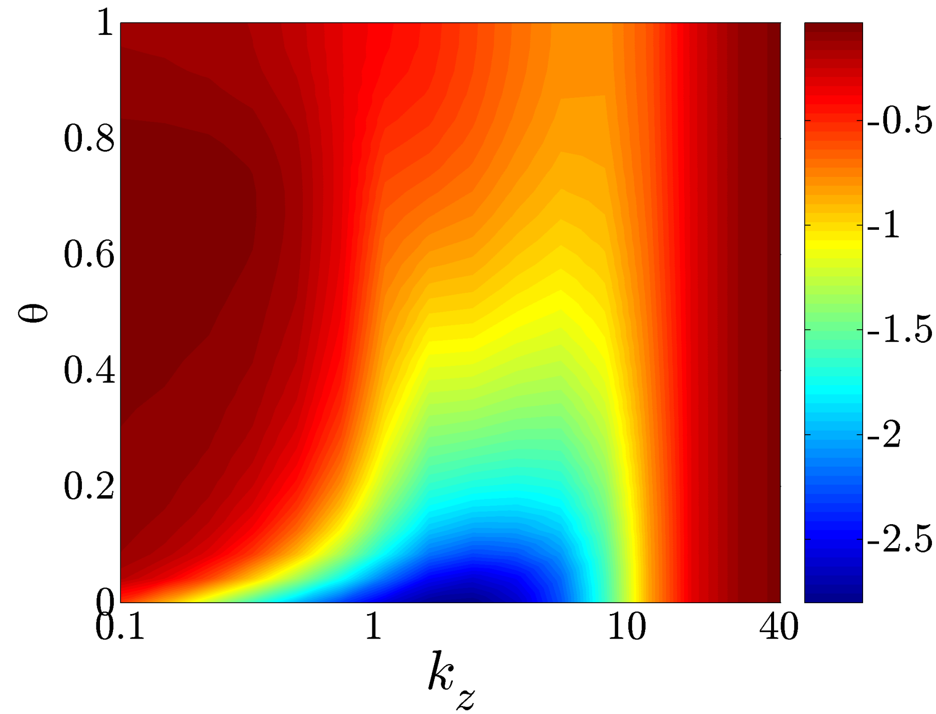
upstream

$(c = -2,\, \omega_x = 0.5)$



downstream

$(c = 5,\, \omega_x = 2)$



Recap

- CONTROLLING THE ONSET OF TURBULENCE

facts revealed by perturbation analysis:

UPSTREAM:	**reduce** skin-friction ✓	**promote** amplification
DOWNSTREAM:	**increase** skin-friction	**reduce** amplification ✓

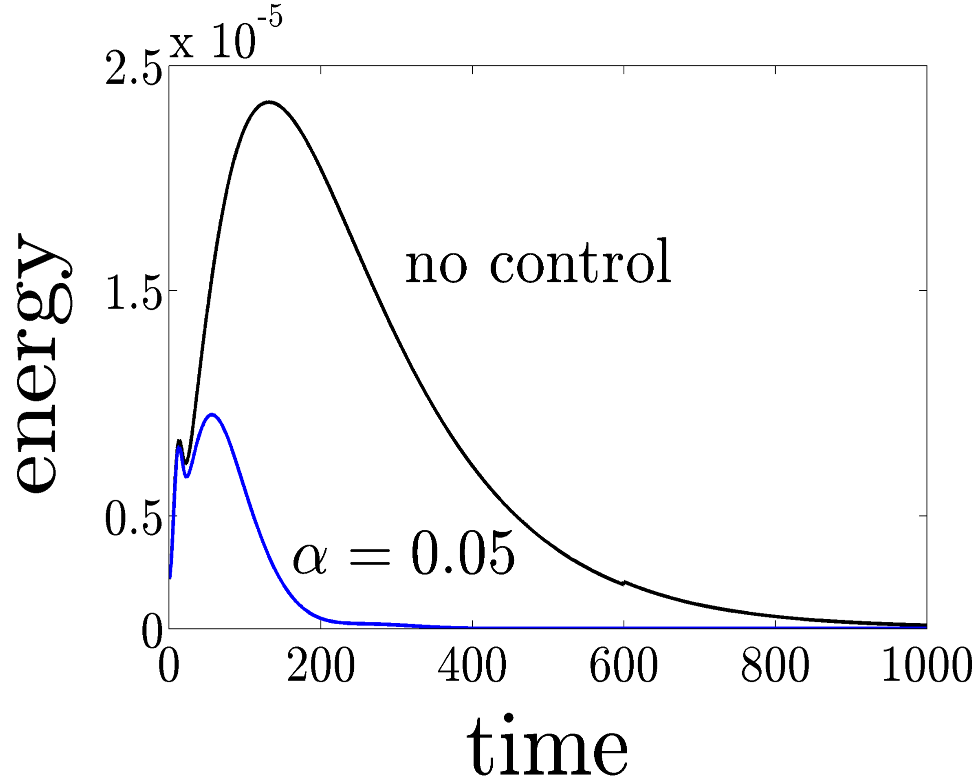
Moarref & Jovanović, J. Fluid Mech. ’10

DNS results: avoidance/promotion of turbulence

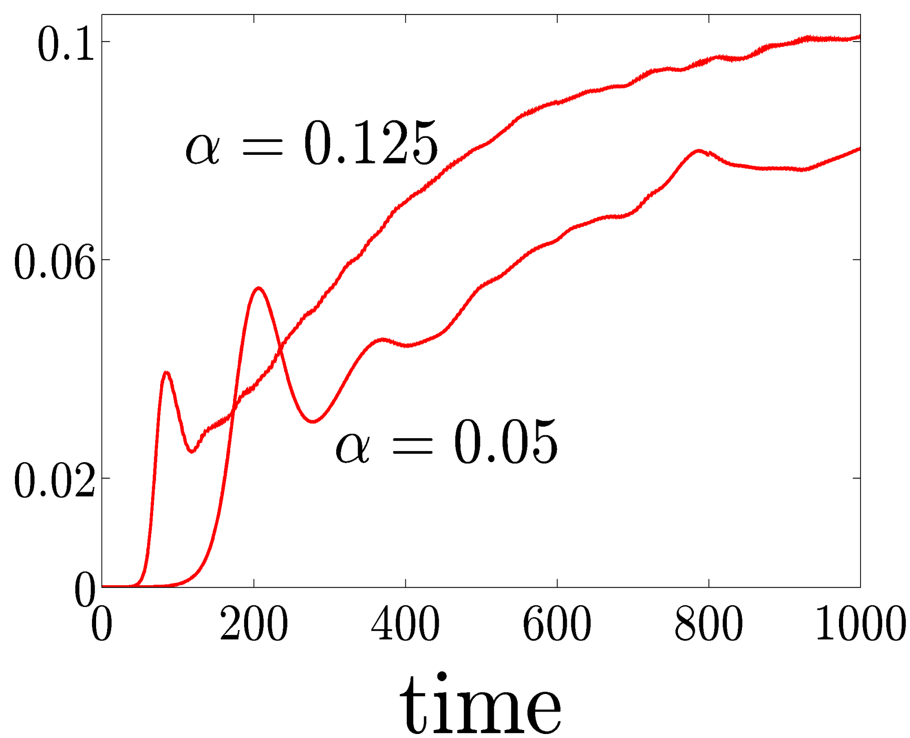
small initial energy

(flow with no control stays laminar)

downstream: no turbulence



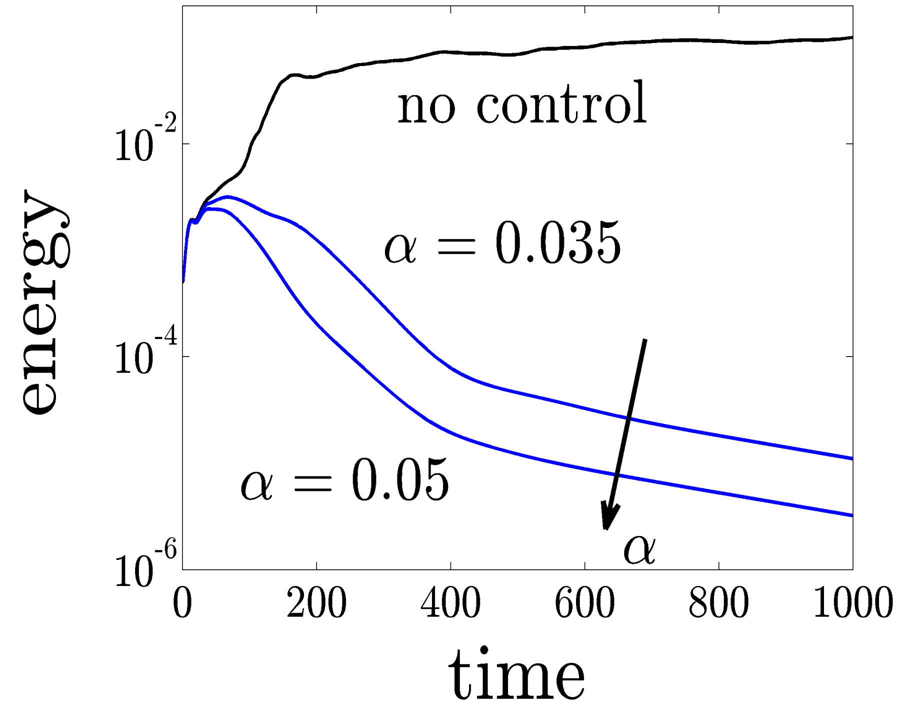
upstream: promotes turbulence

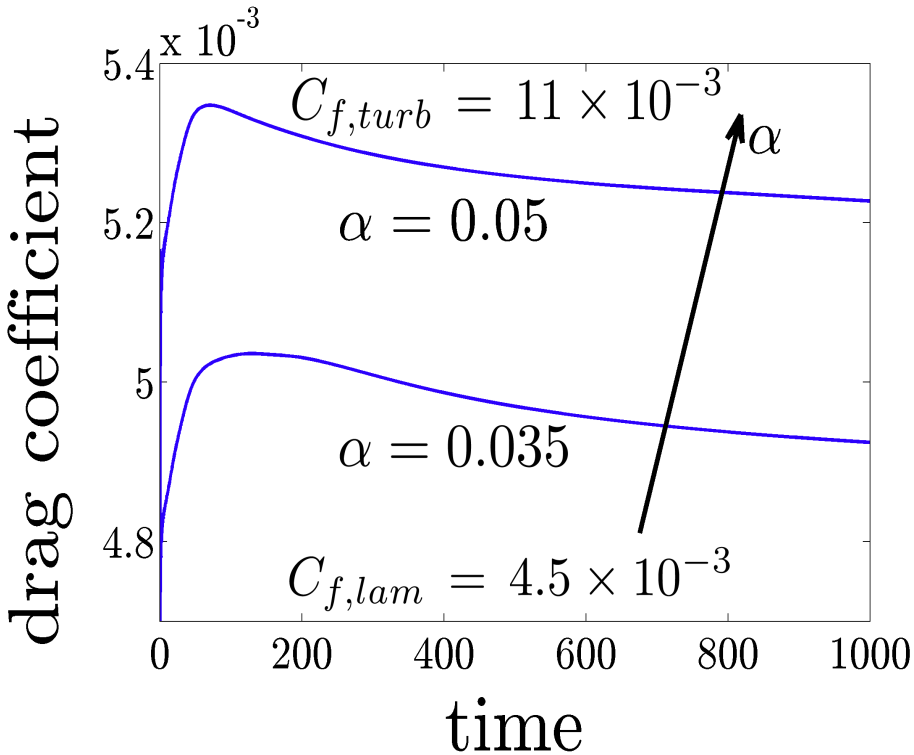


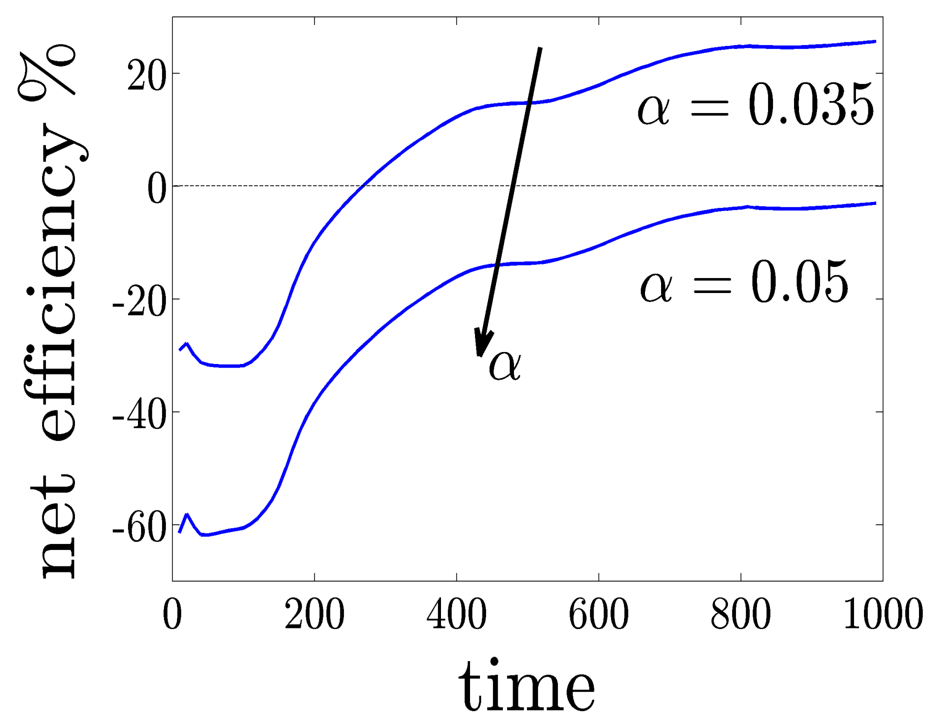
Lieu, Moarref, Jovanović, J. Fluid Mech. '10

NO turbulence:

DOWNSTREAM
moderate initial energy



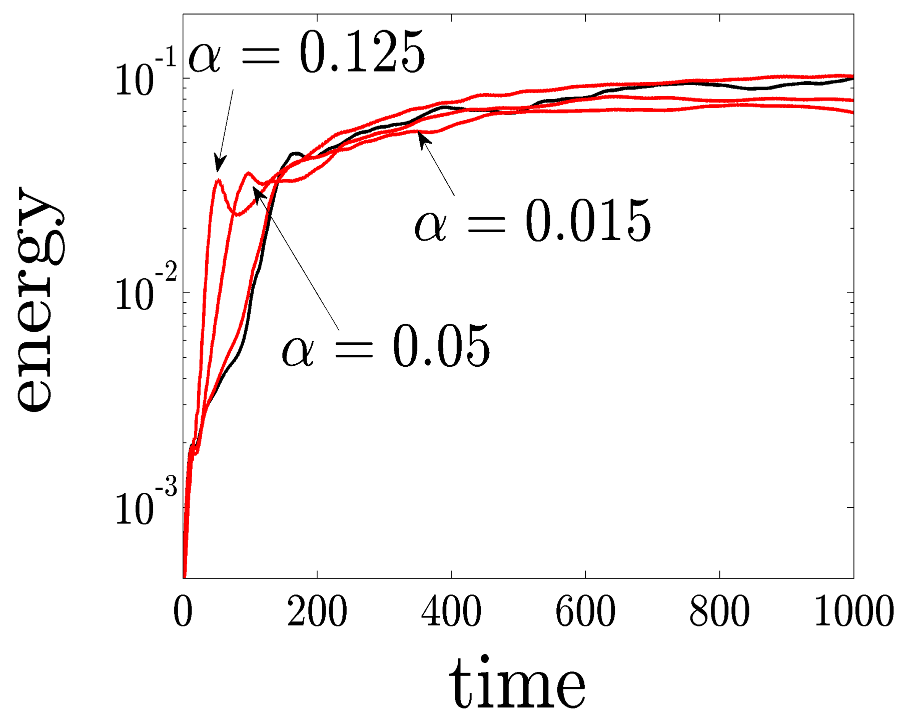


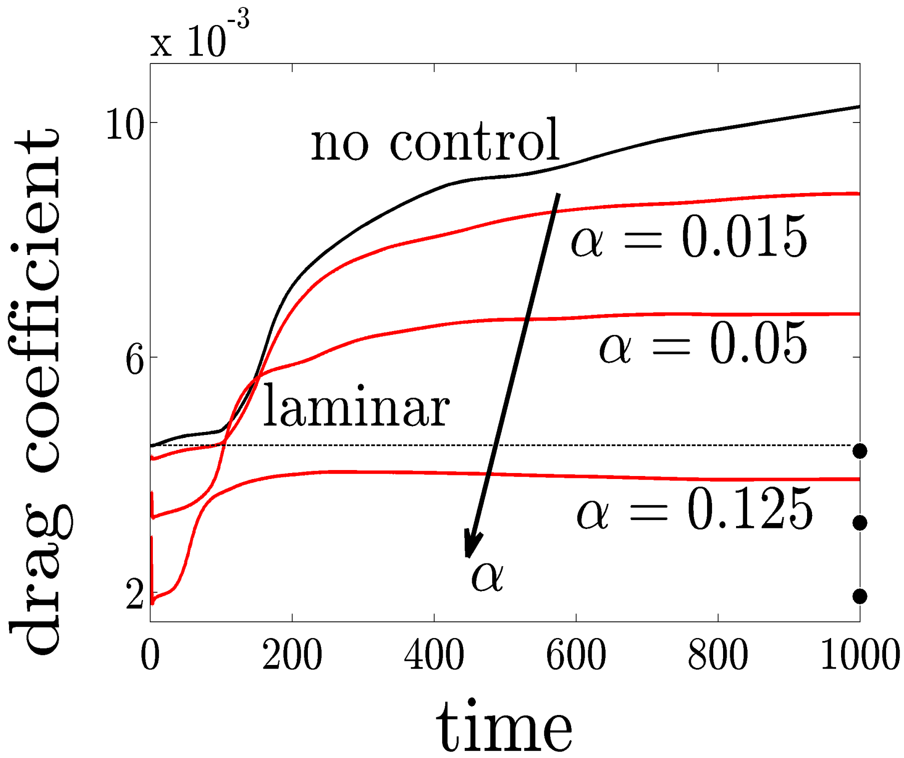


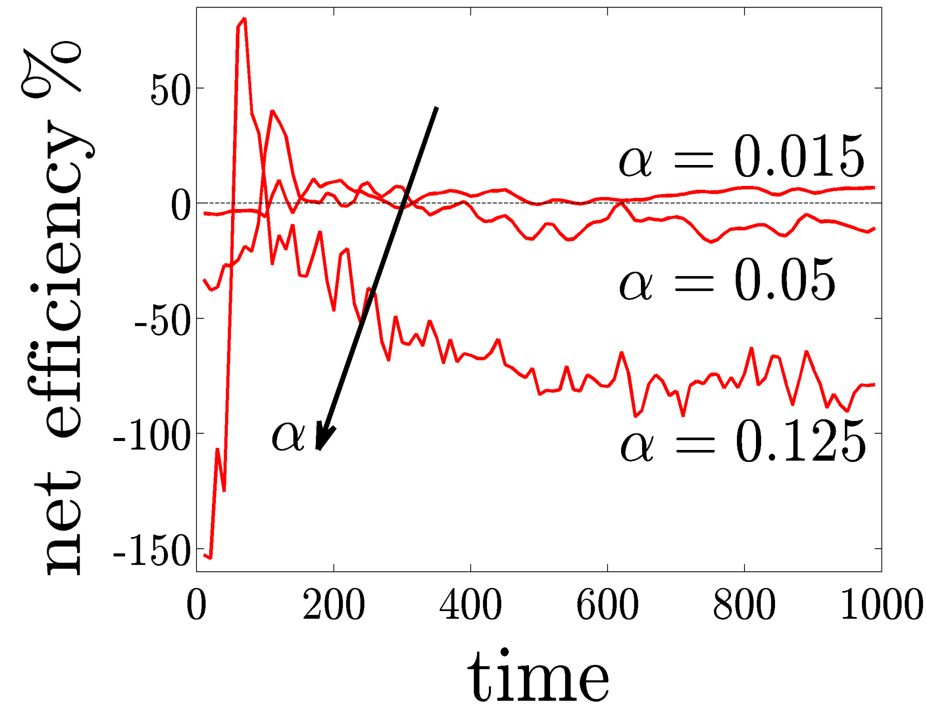
turbulence:

UPSTREAM

moderate initial energy







Lessons, opportunities, challenges

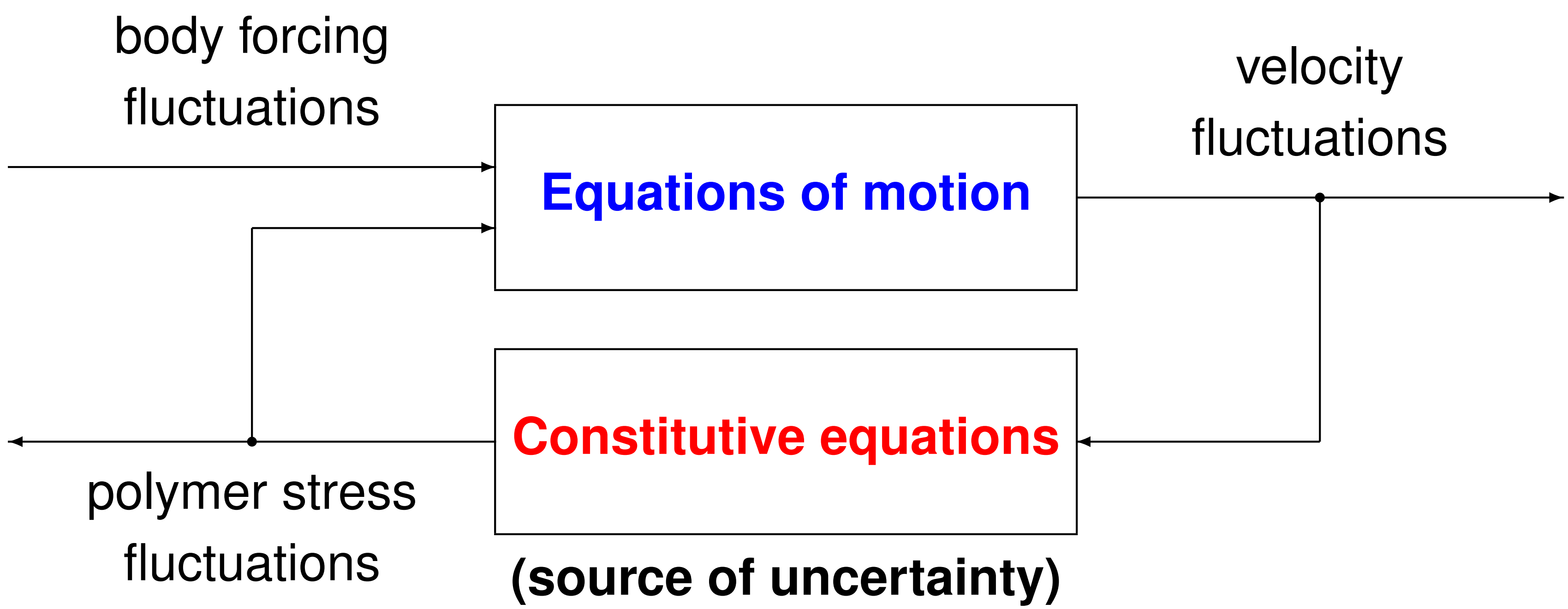
Summary: Early stages of transition

STABILITY	AMPLIFICATION
$\psi_t = \mathbf{A}\,\psi$	$\mathbf{v} = \mathbf{H}\,\mathbf{d}$
e-values of $\mathbf{A}$	**singular values** of $\mathbf{H}$

- OPPORTUNITIES AND CHALLENGES
  - ⋆ **Complex fluids**
  - ⋆ **Complex geometries**
  - ⋆ **Later stages of transition**
  - ⋆ **Control-oriented modeling of turbulent flows**

• COMPLEX FLUIDS

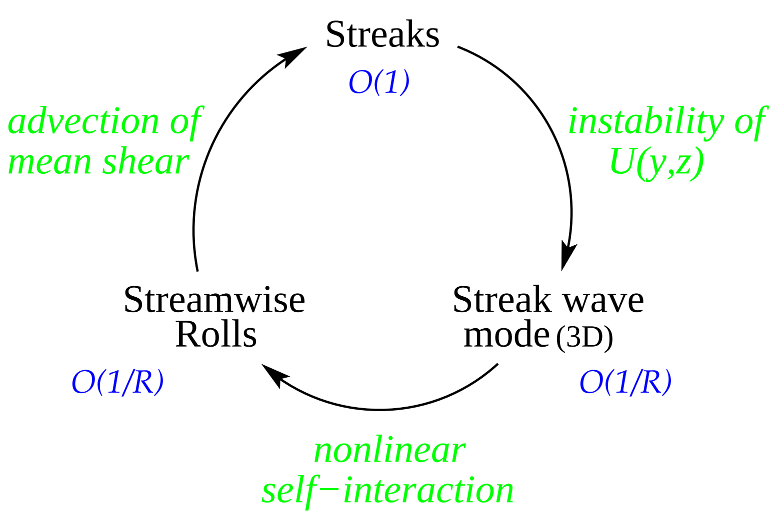
⋆ dynamics of viscoelastic fluids



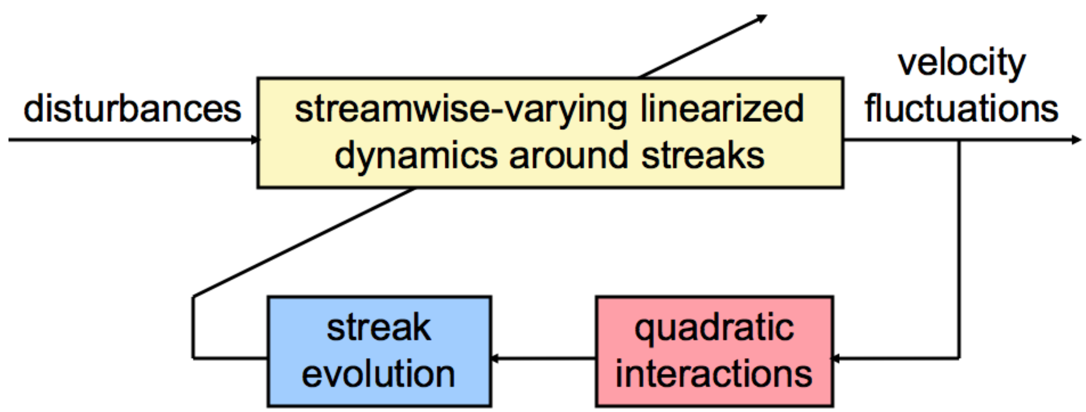
- ⋆ *Lieu, Jovanović, Kumar, J. Fluid Mech. '13*
- ⋆ *Jovanović & Kumar, J. Non-Newtonian Fluid Mech. '11*
- ⋆ *Jovanović & Kumar, Phys. Fluids '10*
- ⋆ *Hoda, Jovanović, Kumar, J. Fluid Mech. '08, '09*

- **COMPLEX GEOMETRIES**
  - ⋆ **iterative schemes for computing singular values**

- **LATER STAGES OF TRANSITION**
  - ⋆ **challenge: relative roles of flow sensitivity and nonlinearity**



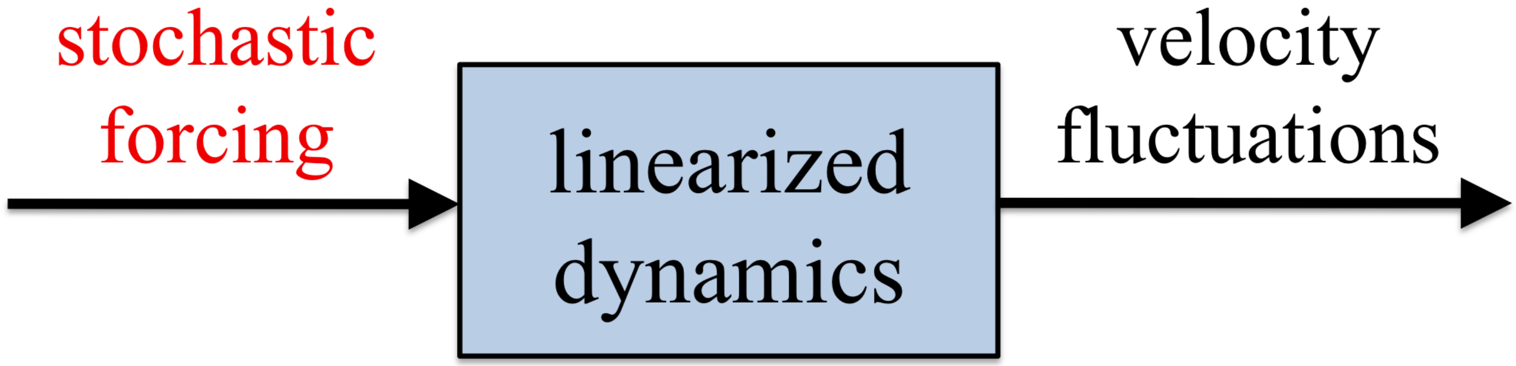
Waleffe, Phys. Fluids '97



Farrell, Ioannou, Gayme

CONTROL-ORIENTED MODELING OF TURBULENT FLOWS

Control-oriented modeling of turbulent flows



- MOTIVATION
  - ⋆ forcing statistics influence performance of flow estimators

    Chevalier, Hœpffner, Bewley, Henningson, J. Fluid Mech. '06

  - ⋆ embed observed statistical features in control-oriented models

- **PROPOSED APPROACH**
  - ⋆ **view second-order statistics as data for an inverse problem**

- **KEY QUESTIONS**
  - ⋆ **Can we identify statistics of forcing to reproduce available statistics?**
  - ⋆ **Can this be done by white in-time stochastic process?**

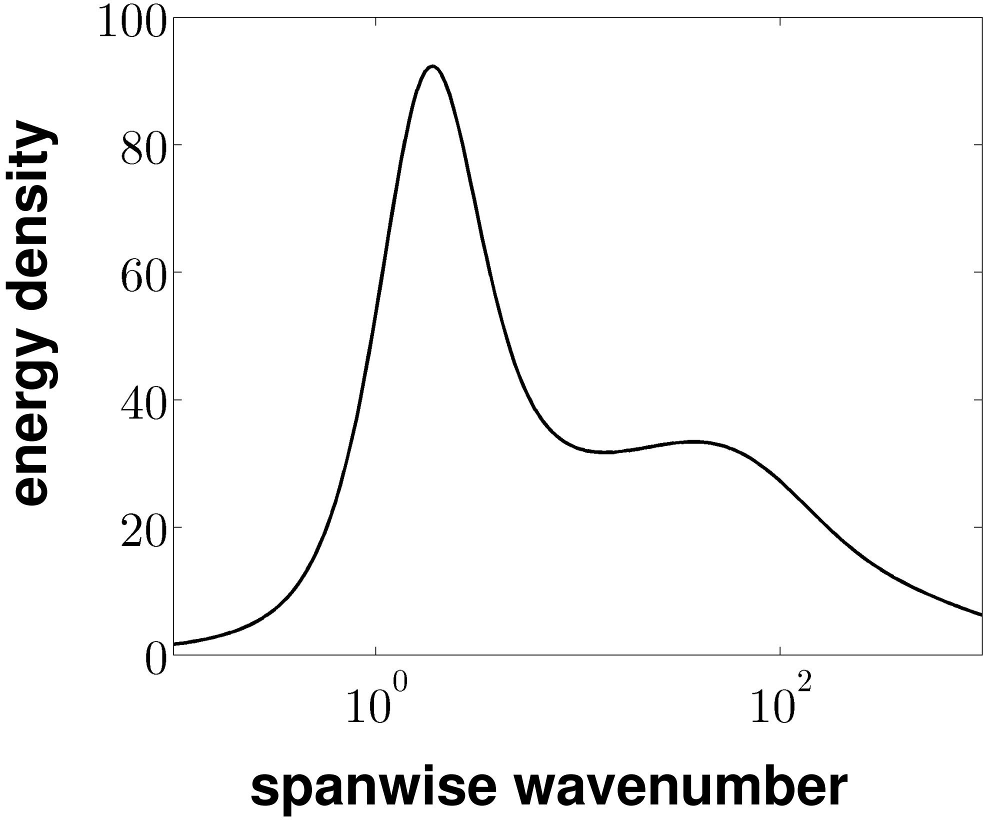
Jovanović & Bamieh, IEEE CDC ’01

- **OUR CONTRIBUTION**
  - ⋆ **systematic way of turbulence modeling as an inverse problem**

Input-output analysis of turbulent flows

- STREAMWISE CONSTANT FLUCTUATIONS

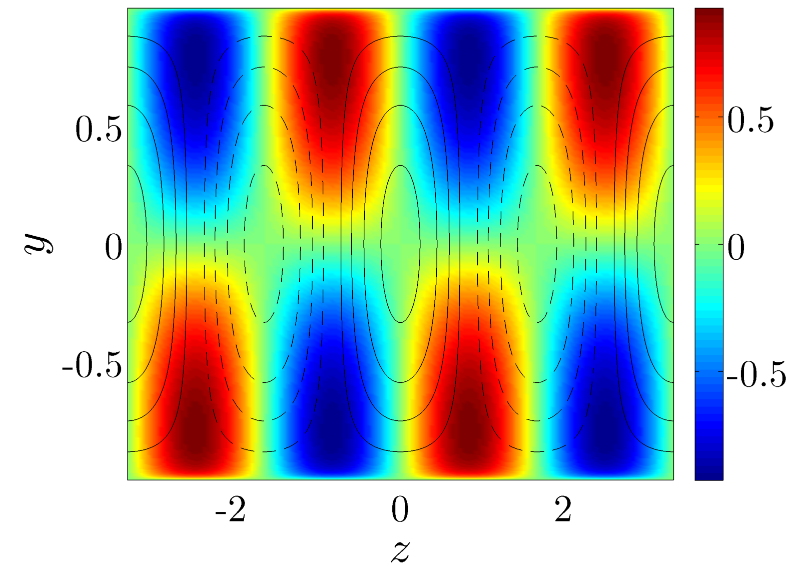
channel flow with $R_\tau = 547$



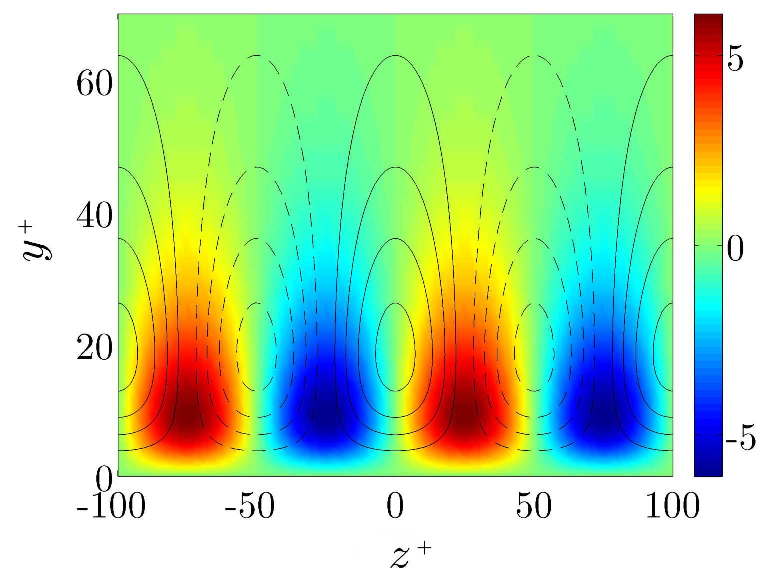
del Álamo & Jiménez, JFM '06

Cossu & coworkers

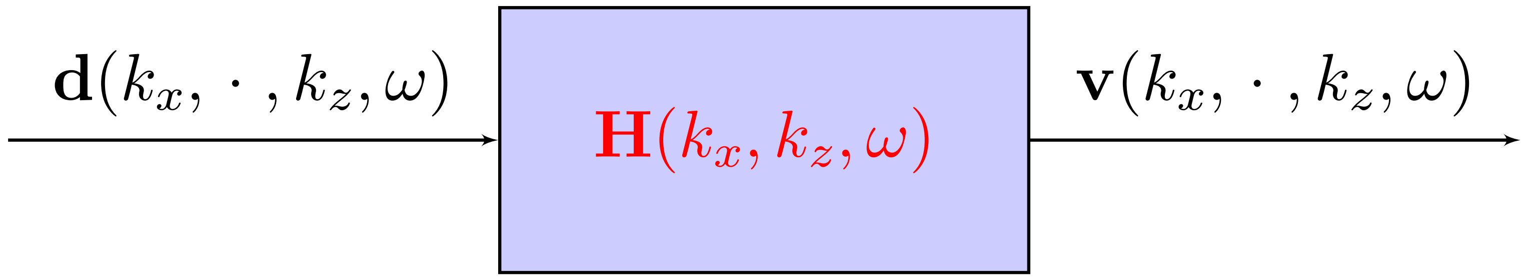
channel-wide streaks



near-wall streaks

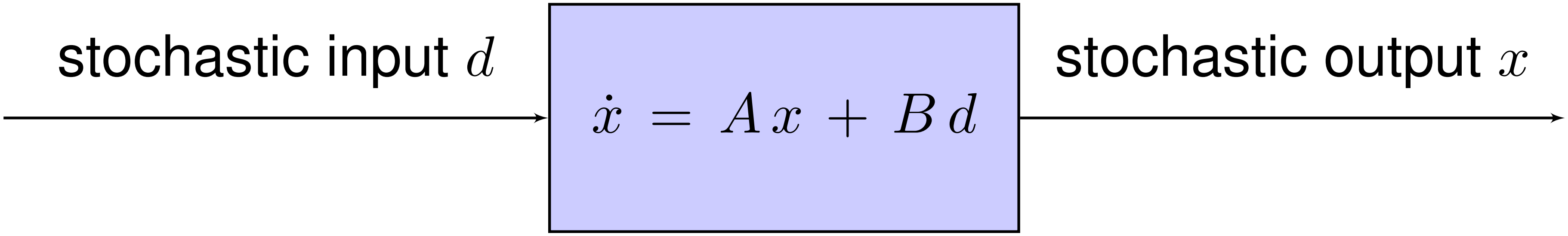


RESOLVENT ANALYSIS



McKeon, Sharma, Moarref

Response to stochastic forcing



- **Lyapunov equation**

  ⋆ propagates white correlation of d into colored statistics of x

$$A\,X \;+\; X\,A^* \;=\; -\,B\,W\,B^*$$

  ⋆ colored-in-time d

$$A\,X \;+\; X\,A^* \;=\; -\,\overbrace{(B\,H^* \;+\; H\,B^*)}^{Q}$$

white forcing: $\quad H \;=\; (1/2)\,B\,W$

Georgiou, IEEE TAC '02

discrete-time dynamics: $x_{t+1} = A\, x_t + B\, d_t$

white-in-time forcing: $\langle d_t\, d_\tau^* \rangle = W\, \delta_{t\,-\,\tau}$

- **LYAPUNOV EQUATION**

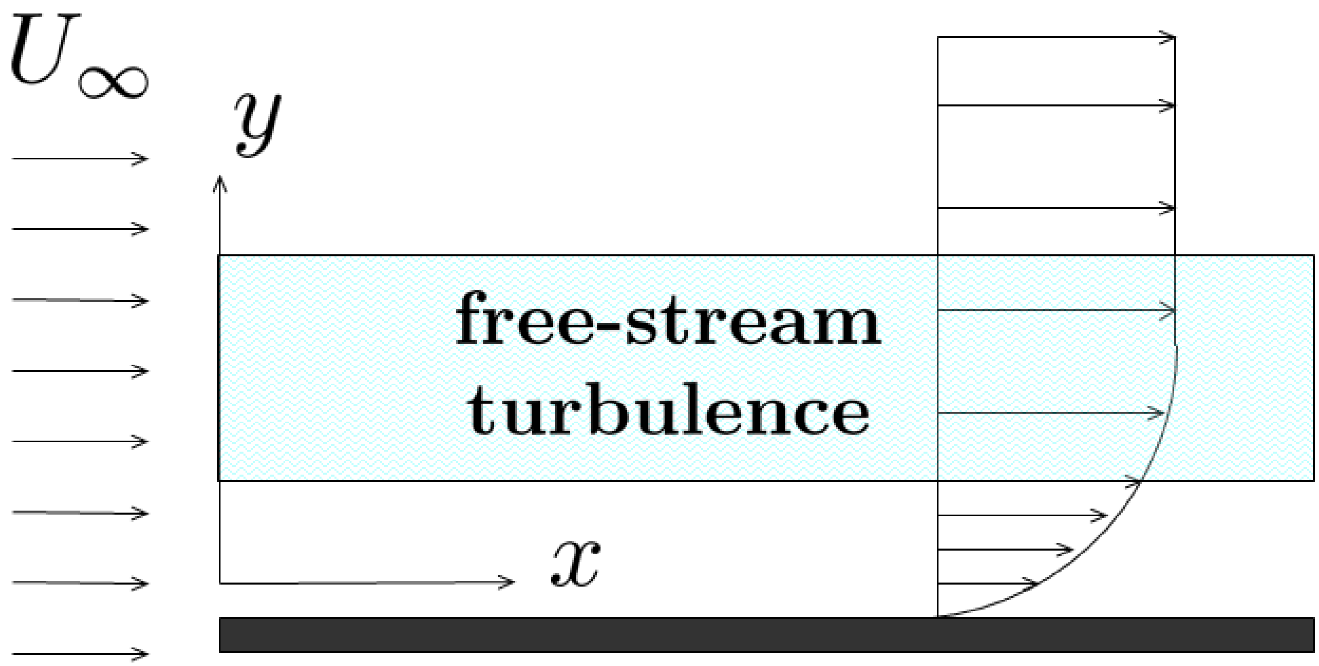
$$
\begin{aligned}
X_{t\,+\,1} \;&:=\; \langle x_{t\,+\,1}\, x_{t\,+\,1}^* \rangle \\
&=\; \langle (A\, x_t + B\, d_t)\, (x_t^* A^* + d_t^* B^*) \rangle \\
&=\; A\, \langle x_t\, x_t^* \rangle\, A^* + B\, \langle d_t\, d_t^* \rangle\, B^* \\
&=\; A\, X_t\, A^* + B\, W\, B^*
\end{aligned}
$$

⋆ continuous-time version

$$
\frac{\mathrm{d}\, X_t}{\mathrm{d}\, t} = A\, X_t + X_t\, A^* + B\, W\, B^*
$$

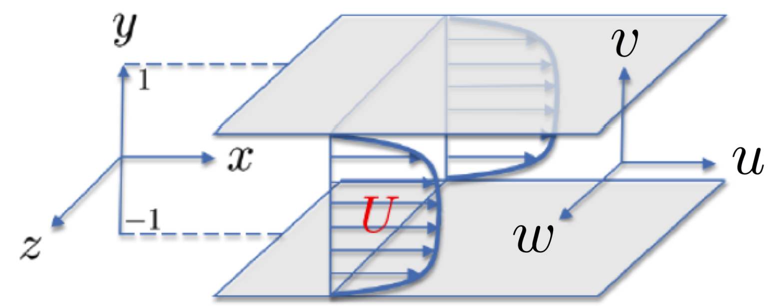
An example

- Response of a **boundary layer** to free-stream turbulence

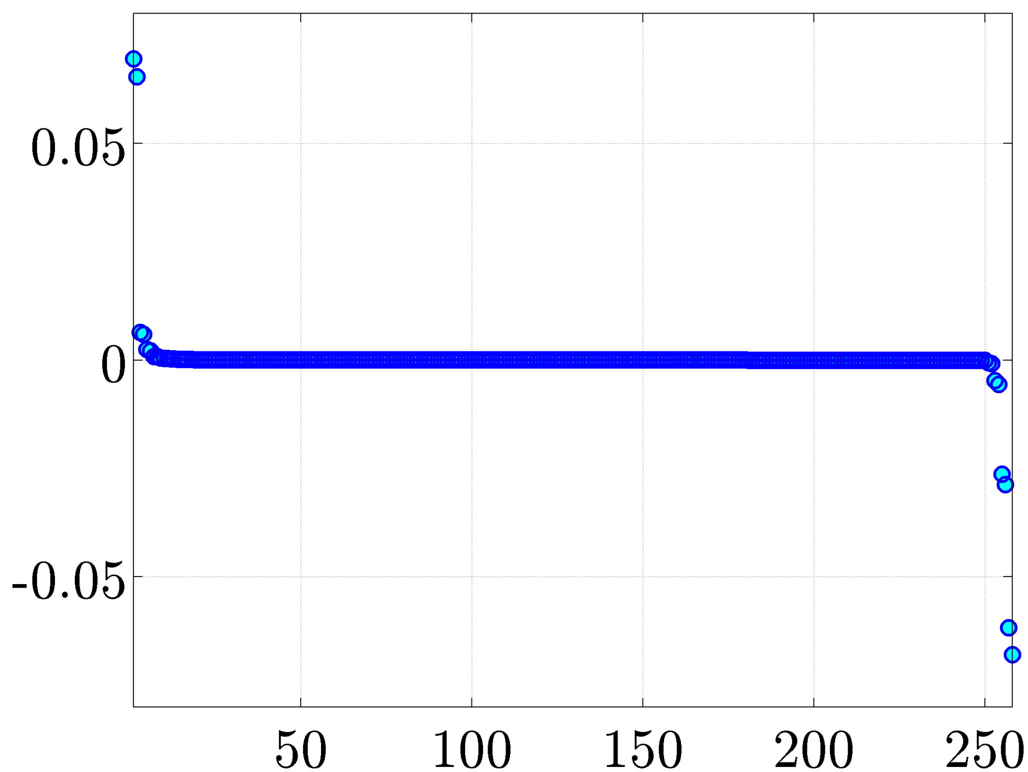


Turbulent channel flow

- KEY OBSERVATION
  - ⋆ **white-in-time forcing: too restrictive**



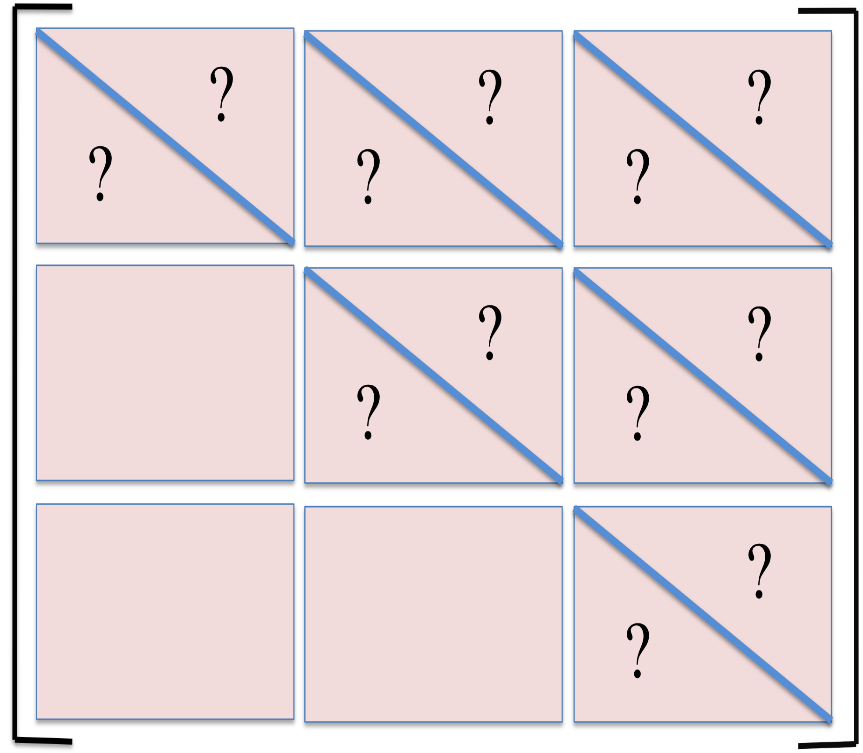
$$\lambda_i \left(A\, X_{\text{dns}} \; + \; X_{\text{dns}}\, A^* \right)$$



Jovanović & Georgiou, APS ’10

Problem setup

- **Problem data**
  - ⋆ **dynamical generator A**
  - ⋆ **partial second-order statistics**



$\mathcal{L}(X_{\text{dns}})$

- **Unknowns**
  - ⋆ **unavailable statistics of x**
  - ⋆ **disturbance dynamics** { **input matrix B** / **input power spectrum** }

Inverse problem

- CONVEX OPTIMIZATION PROBLEM

$$\begin{array}{ll} \underset{X,\,Q}{\text{minimize}} & \|Q\|_* \\ \text{subject to} & A\,X \;+\; X A^* \;+\; Q \;=\; 0 \\ & \mathcal{L}(X) \;-\; \mathcal{L}(X_{\mathrm{dns}}) \;=\; 0 \\ & X \;\succeq\; 0 \end{array}$$

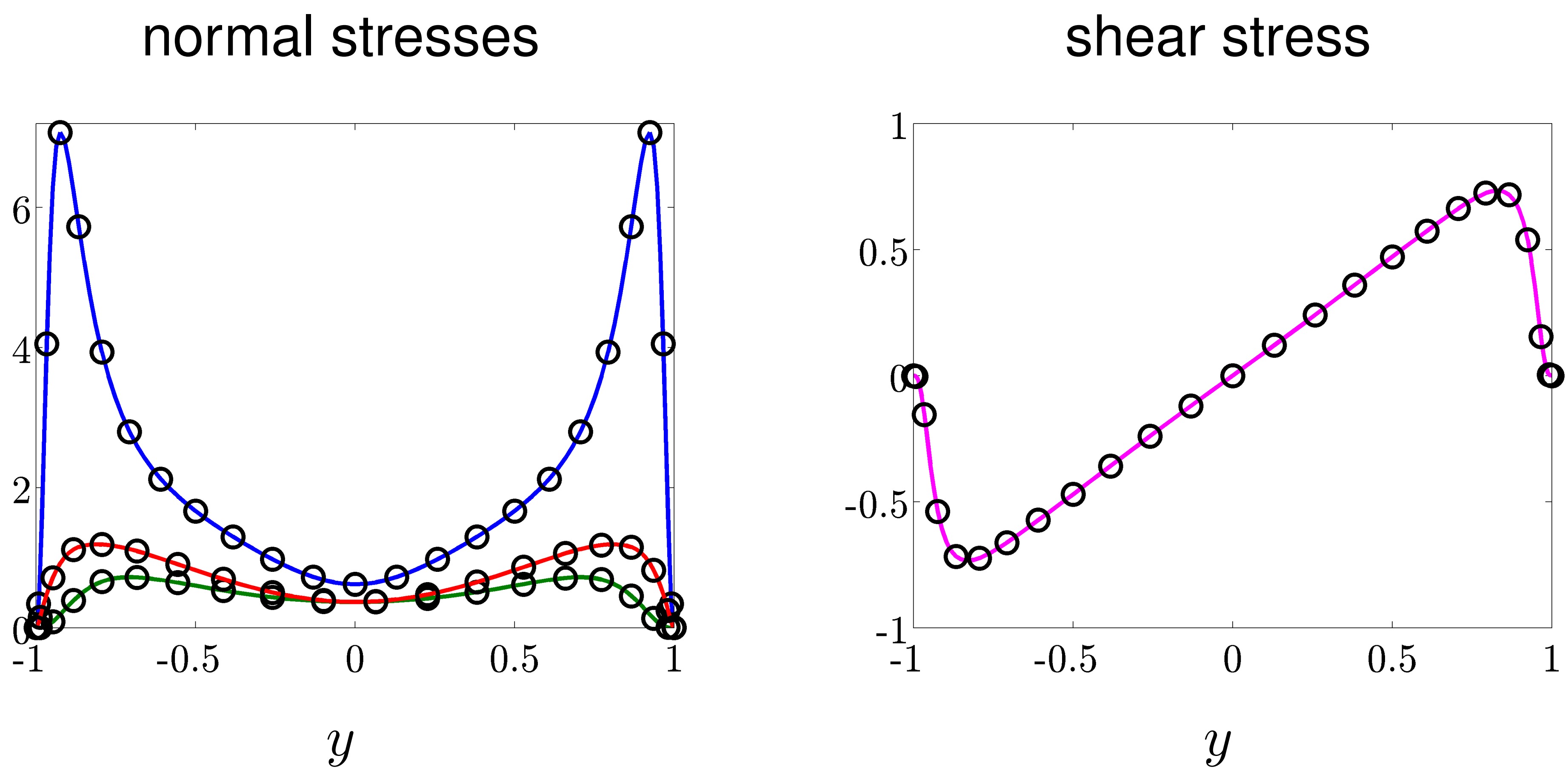
⋆ nuclear norm: proxy for rank minimization

$$\|Q\|_* \;:=\; \sum \sigma_i(Q)$$

Fazel, Boyd, Hindi, Recht, Parrilo, Candès, Chandrasekaran, . . .

Zare, Jovanović, Georgiou, ACC '14; CTR SP '14

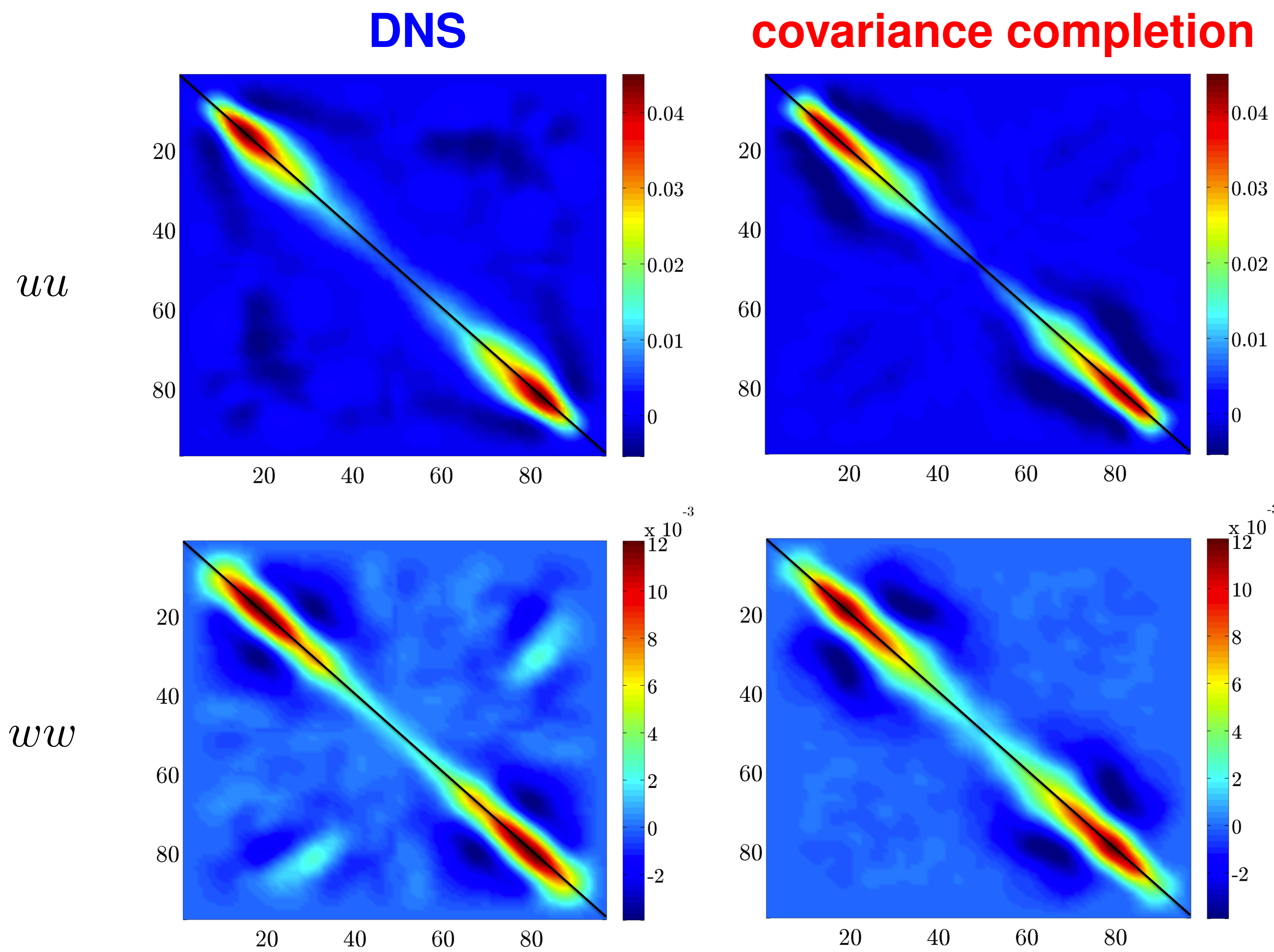
One-point correlations



Direct Numerical Simulations —

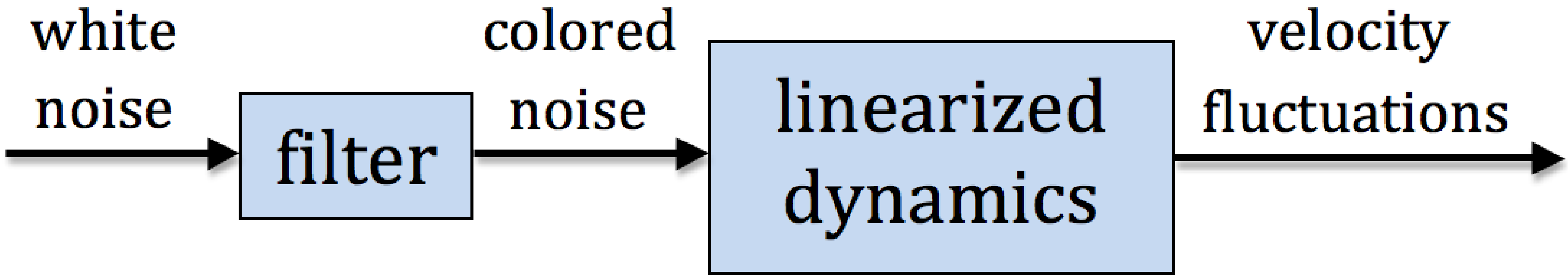
Solution to Inverse Problem ○

Two point correlations



$R_\tau = 180;\ k_x = 2.5,\ k_z = 7$

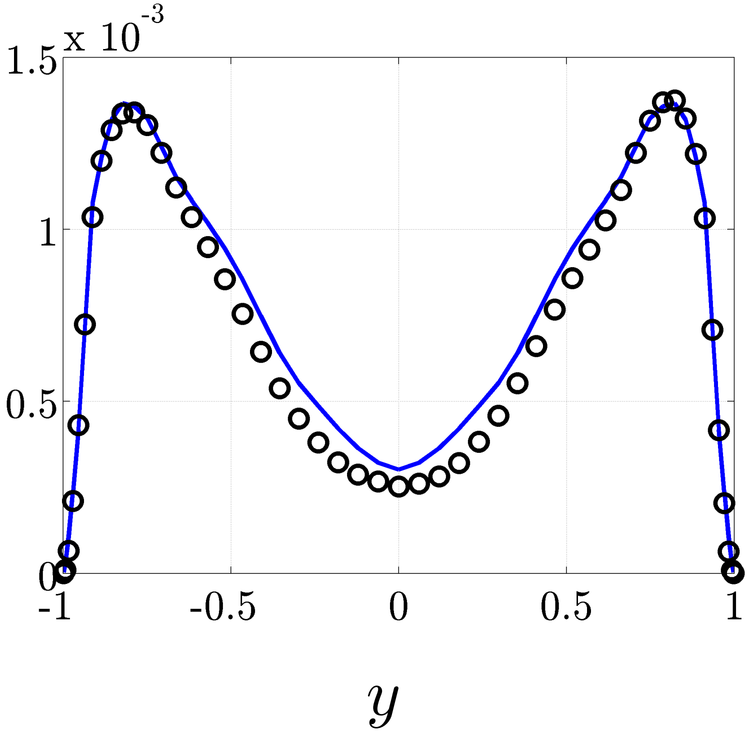
Colored-in-time forcing

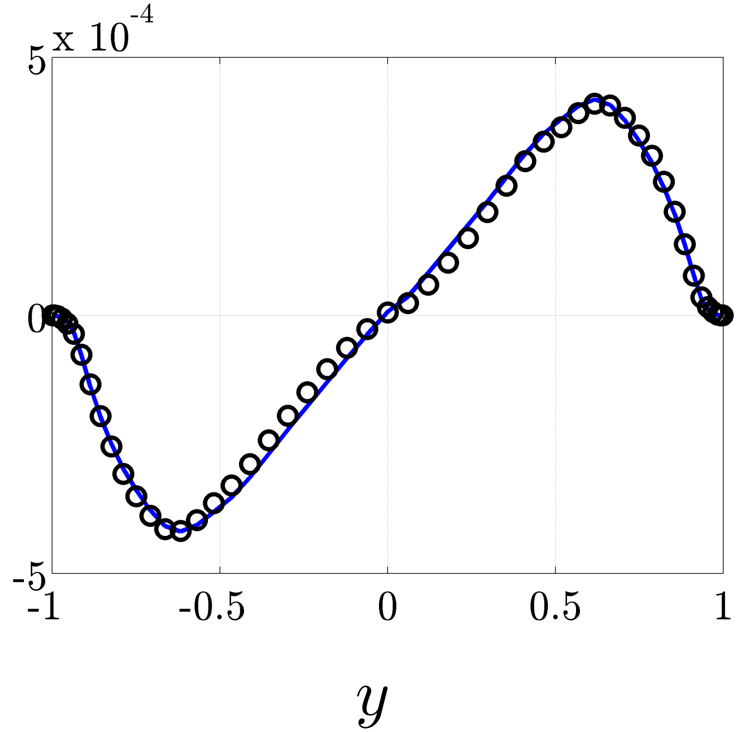


key result:

filter design using { **linearized dynamics** / **completed correlations** }

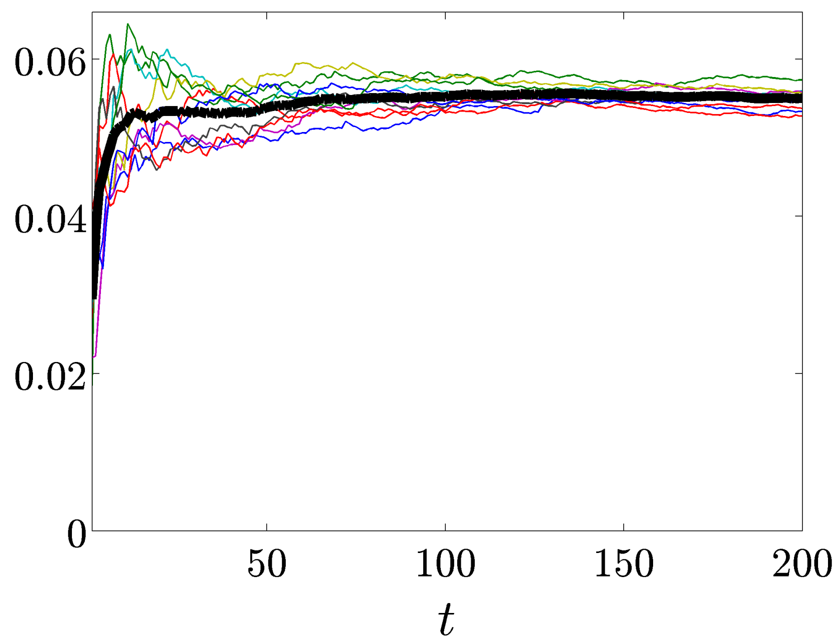
Verification in stochastic simulations



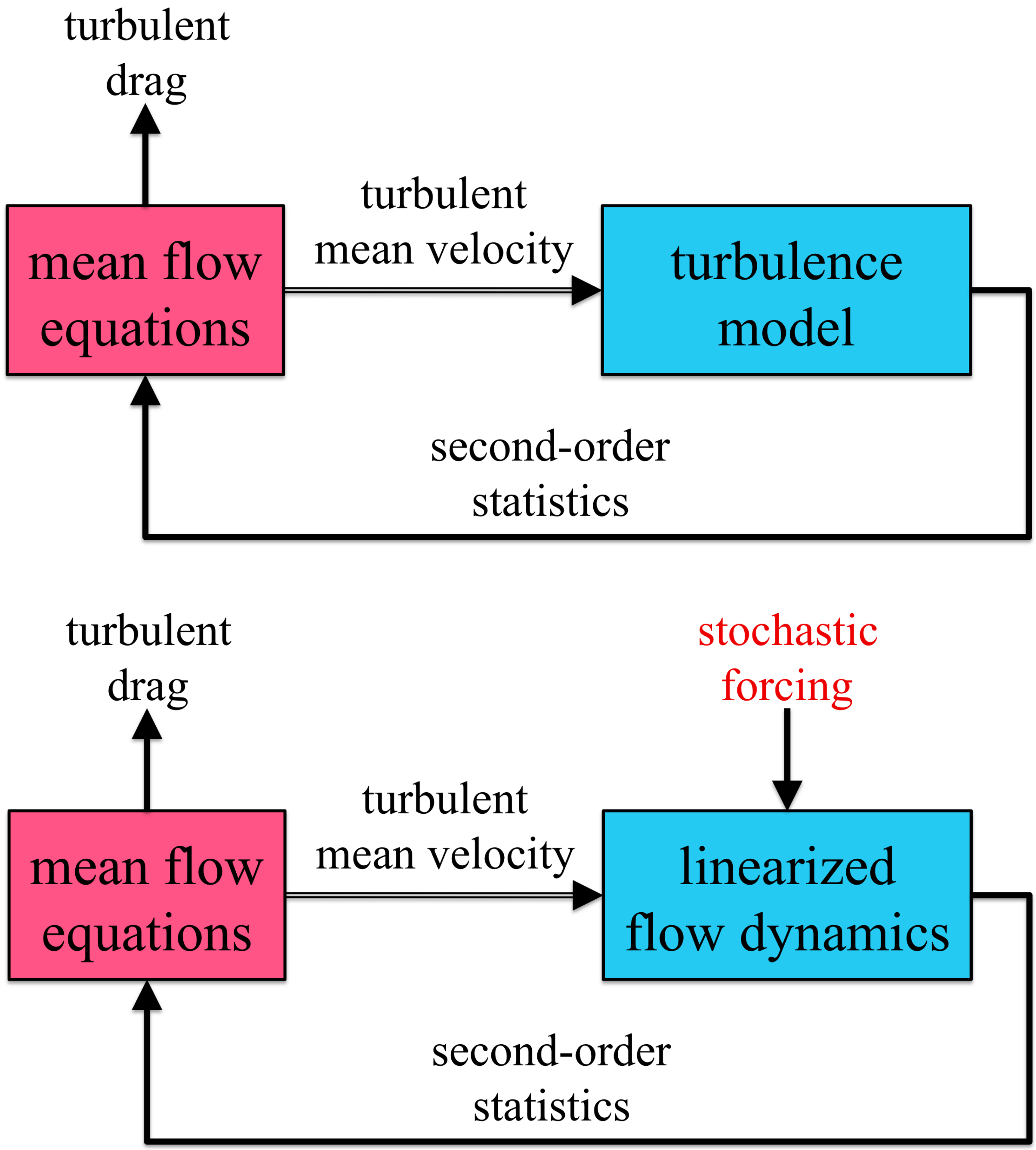


Direct Numerical Simulations —

Linear Stochastic Simulations ○



New class of stochastically-forced closure models?



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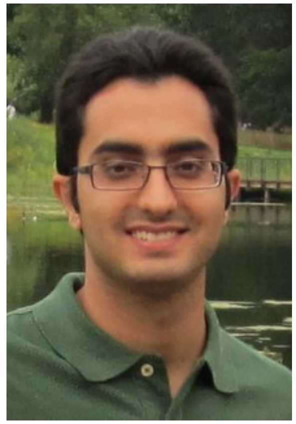
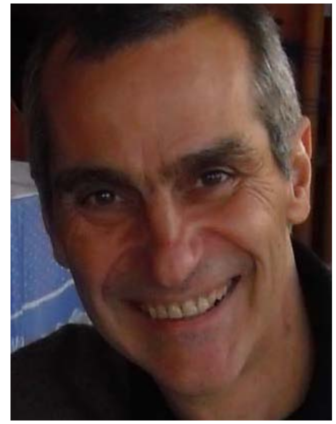


Armin Zare
U of M



Tryphon Georgiou
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Bassam Bamieh
UCSB

SUPPORT

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NSF Award CMMI-13-63266

U of M Informatics Institute Transdisciplinary Faculty Fellowship

CTR Summer Programs ’06, ’10, ’12, ’14

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