Sparsity-promoting wide-area control of power systems

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Electro-mechanical oscillations in power systems

• Local oscillations
  ★ single generators swing relative to the rest of the grid
  ★ typically damped by Power System Stabilizers (PSSs)

• Inter-area oscillations
  ★ groups of generators oscillate relative to each other
  ★ associated with dynamics of power transfers
Inter-area oscillations

- Blackout of Aug. 10, 1996
  - resulted from instability of the 0.25 Hz mode

western interconnected system: California-Oregon power transfer:
Slow coherency theory

- WHERE ARE THE INTER-AREA MODES COMING FROM?
  - slow coherency theory

*slow coherency theory*

Chow, Kokotović, et al. ’78, ’82

RTS 96 power system:

linearized swing equation:

![Graph showing generator angles over time]
Conventional control

- **Blue layer**: generators with transmission lines

- **Fully decentralized controller**
  - effective against local oscillations
  - ineffective against inter-area oscillations
Wide-area control

- **Blue layer**: generators with transmission lines

**KEY CHALLENGE**: identification of a **signal exchange network**

**performance vs sparsity**
A diagram illustrating the components of a wide-area power network dynamics system, labeled as follows:

1. **Wide-Area Controller**
2. **Remote Control Signals** $u_{wac}(t)$
3. **Wide-Area Measurements** (e.g., PMUs)
4. **Local Control Loops** $u_{loc}(t)$
5. **PSS & AVR**
6. **Generator**
7. **FACTS**
8. **Transmission Line**
9. **System Noise** $\eta(t)$

The diagram also includes a feedback loop indicating the channel and measurement noise.
Outline

1. **SPARSITY-PROMOTING WIDE-AREA CONTROL**
   - Safeguard against inter-area oscillations
   - Performance vs sparsity

2. **CASE STUDY**
   - IEEE New England power grid model

3. **SUMMARY AND OUTLOOK**
**Model Features**

- detailed sub-transient generator models
- exciters
- carefully tuned PSS data

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**Case study: IEEE New England Power Grid**

![Diagram of the IEEE New England Power Grid with numbers and connections marked.](image-url)
single wide-area comm link

single long range interaction \( \Rightarrow \) nearly centralized performance
Optimal wide-area control

linearized dynamics: \[ \dot{x} = Ax + B_1 d + B_2 u \]

objective function: \[ J = \lim_{t \to \infty} \mathcal{E} \left( x^T(t) Q x(t) + u^T(t) R u(t) \right) \]

memoryless controller: \[ u = -K x \]

* no structural constraints

globally optimal controller:

\[
\begin{array}{c}
A^T P + PA - PB_2 R^{-1} B_2^T P + Q = 0 \\
K_c = R^{-1} B_2^T P
\end{array}
\]
Sparsity-promoting optimal control

minimize \( J(K) \) + \( \gamma \sum_{i,j} W_{ij} |K_{ij}| \)

\[\downarrow\] variance amplification \[\downarrow\] sparsity-promoting penalty function

\( \star \gamma > 0 \) – performance vs sparsity tradeoff

\( \star W_{ij} \geq 0 \) – weights (for additional flexibility)

Parameterized family of feedback gains

\[ K(\gamma) := \arg\min_K (J(K) + \gamma g(K)) \]

**Algorithm:** alternating direction method of multipliers
Performance index

- Energy of power network without inter-area modes
  - inspired by slow coherency theory

\[ J := \lim_{t \to \infty} \mathcal{E} \left( \theta^T(t) Q_{\theta} \theta(t) + \dot{\theta}^T(t) \dot{\theta}(t) + u^T(t) u(t) \right) \]

\[ Q_{\theta} := \epsilon I + \left( I - \frac{1}{N} \mathbf{1} \mathbf{1}^T \right) \]

- other choices possible
Open-loop dynamics

- Dominant inter-area modes with local PSSs

Mode 1

- 10
- all others

Mode 2

- 1, 2, 3, 8, 9
- 4, 5, 6, 7

Mode 3

- 4, 5, 6, 7, 9
- 2, 3

Mode 4

- others
- 4, 5
- 6, 7

Mode 5

- others
- 1, 8
Performance vs sparsity

\[
\frac{(J - J_c)}{J_c} \quad \text{versus} \quad \frac{\text{card}(K)}{\text{card}(K_c)}
\]

\(\gamma = 1\) relative to \(K_c\) → \(\begin{cases} 
1.6\% \text{ performance loss} \\
5.5\% \text{ non-zero elements in } K
\end{cases}\)
• Signal exchange network

\[ \gamma = 0.0289, \text{card}(K) = 90 \]

\[ \gamma = 1, \text{card}(K) = 37 \]
Robustness?

Wide-area control

\[ K^*_\gamma \rightarrow u_{\text{wac}}(t) \]

System noise

\[ \eta(t) \]

Multiplicative uncertainty

\[ \Delta m \]

Gain uncertainty

\[ \Delta g \]

Dynamics with local control

Local control loops

Power network dynamics

\[ x(t) \]
multivariable phase margin

multivariable gain reduction margin

multivariable gain amplification margin
Summary and outlook

- **SPARSITY-PROMOTING OPTIMAL CONTROL**
  - Performance vs sparsity tradeoff
  - Software
    - www.umn.edu/~mihailo/software/lqrsp/

- **WIDE-AREA CONTROL OF POWER NETWORKS**
  - Remedy against inter-area oscillations
  - IEEE New England power grid model

- **OPEN QUESTIONS**
  - Extension to structure-preserving descriptor models
  - Theoretic analysis of robustness degradation
  - Exploit the rotational symmetry of the models