Nonlinear Systems

Lecture 28

05/09/13

Last few lectures,

- Input-output linearization
- Relative degree
- Zero dynamics
- Normal form [make z-d. transparent]

\[
\begin{align*}
\dot{z} &= f(z, \xi) & \text{zero dynamics} \\
\dot{\xi} &= A_{\xi} \xi
\end{align*}
\]

→ after input-output linearization is done.

Linearization:

\[
A = \begin{bmatrix}
\frac{\partial f}{\partial z} \big|_0 & \frac{\partial f}{\partial \xi} \\
0 & A_{\xi}
\end{bmatrix}
\]

\[
\begin{align*}
\text{LAS} \iff \begin{cases}
\frac{\partial f}{\partial z} \big|_0 \\
A_{\xi}
\end{cases} & \text{Hermitez}
\end{align*}
\]
Q. How about global asymptotic stability

After I/O linearization

\[ \dot{x} = A_x \dot{x} \rightarrow \dot{z} = P_{\theta}(z, \dot{z}) \rightarrow z \]

We need some additional assumptions on \( P_{\theta} \)
(e.g. Input to state stability, from input to state \( z \))

Ex.

\[ \dot{z} = -z + z^2 \]

\[ \dot{\dot{x}} = A_x \dot{x} \]

\( z \)-subsystem is not ISS (input to state stable)

In fact, finite escape time can happen for large enough initial conditions (inspite of local asymptotic stability)

strength of nonlinearity can beat decay of linear term
Recall example from last time:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \alpha x_3 + u \\
\dot{x}_3 &= \beta x_3 - u \\
y &= x_1
\end{align*}
\]

\[
\begin{align*}
\dot{z} &= (\alpha + \beta) z - (\alpha + \beta) \xi \\
\dot{\xi}_1 &= \dot{\xi}_2 \\
\dot{\xi}_2 &= \alpha z - \alpha \xi + u \\
\end{align*}
\]

\[
\begin{align*}
Z := x_2 + x_3 = \xi_2 + x_3
\end{align*}
\]

Transfer function (from u to y):

\[
H(s) = \frac{s - (\alpha + \beta)}{s^2 (s - \beta)}
\]

Note! Zero @ \( s = \alpha + \beta \)

2 poles at zero \(\rightarrow\) unstable system

A quick note: System is feedback linearizable if there is an output s.t. relative degree is equal to n (order of your system: # of states)

More info: Khalil