Due Monday 05/13/13 (noon, Xiaofan's office)

1. Khalil, Problem 13.27. (In part (b), do simulations but skip the performance comparison question.)

2. In class, we used the PR Lemma to show that a positive real linear system,

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

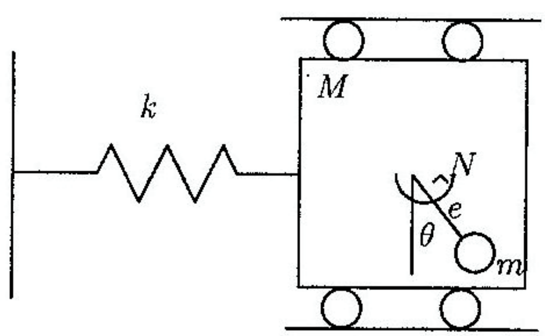
has relative degree one, that is, $CB \neq 0$. Show that positive realness also implies a minimum phase property. (Hint: Write the system equations in *normal form* and apply Positive Real Lemma.)

3. The dynamics of the *translational oscillator with rotating actuator* (TORA) are described by:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{-x_1 + \epsilon {x_4}^2 \sin x_3}{1 - \epsilon^2 \cos^2 x_3} + \frac{-\epsilon \cos x_3}{1 - \epsilon^2 \cos^2 x_3} u$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{1}{1 - \epsilon^2 \cos^2 x_3} \left( \epsilon \cos x_3 (x_1 - \epsilon x_4^2 \sin x_3) \; + \; u \right)$$

where $x_1$ and $x_2$ are the displacement and the velocity of the platform, $x_3$ and $x_4$ are the angle and angular velocity of the rotor carrying the mass $m$, and $u$ is the control torque applied to the rotor. The parameter $\epsilon < 1$ depends on the eccentricity $e$ and the masses $m$ and $M$.

With $y = x_3$ as the output, determine the relative degree and the zero dynamics. Provide a physical interpretation of the zero dynamics.



**13.24** Consider the system (13.44)–(13.45), where $A - BK$ is Hurwitz, the origin of $\dot{\eta} = f_0(\eta, 0)$ is asymptotically stable with a Lyapunov function $V_0(\eta)$ such that $[\partial V_0/\partial \eta] f_0(\eta, 0) \le -W(\eta)$ for some positive definite function $W(\eta)$. Suppose $\|\delta\| \le k[\|\xi\| + W(\eta)]$. Using a composite Lyapunov function of the form $V = V_0(\eta) + \lambda\sqrt{\xi^T P \xi}$, where $P$ is the solution of $P(A - BK) + (A - BK)^T P = -I$, show that, for sufficiently small $k$, the origin $z = 0$ is asymptotically stable

**13.25** Consider the system

$$\dot{x}_1 = x_2 + 2x_1^2, \quad \dot{x}_2 = x_3 + u, \quad \dot{x}_3 = x_1 - x_3, \quad y = x_1$$

Design a state feedback control law such that the output $y$ asymptotically tracks the reference signal $r(t) = \sin t$.

**13.26** Repeat the previous exercise for the system

$$\dot{x}_1 = x_2 + x_1 \sin x_1, \quad \dot{x}_2 = x_1 x_2 + u, \quad y = x_1$$

**13.27** The magnetic suspension system of Exercise 1.18 is modeled by

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= g - \frac{k}{m}x_2 - \frac{L_0 a x_3^2}{2m(a + x_1)^2} \\
\dot{x}_3 &= \frac{1}{L(x_1)}\left[-Rx_3 + \frac{L_0 a x_2 x_3}{(a + x_1)^2} + u\right]
\end{aligned}$$

where $x_1 = y$, $x_2 = \dot{y}$, $x_3 = i$, and $u = v$. Use the following numerical data: $m = 0.1$ kg, $k = 0.001$ N/m/sec, $g = 9.81$ m/sec$^2$, $a = 0.05$ m, $L_0 = 0.01$ H, $L_1 = 0.02$ H, and $R = 1\ \Omega$.

**(a)** Show that the system is feedback linearizable.

**(b)** Using feedback linearization, design a state feedback control law to stabilize the ball at $y = 0.05$ m. Repeat parts (d) and (e) of Exercise 12.8 and compare the performance of this controller with the one designed in part (c) of that exercise.

**(c)** Show that, with the ball position $y$ as the output, the system is input–output linearizable.

**(d)** Using feedback linearization, design a state feedback control law so that the output $y$ asymptotically tracks $r(t) = 0.05 + 0.01 \sin t$. Simulate the closed-loop system.

third equation is a torque equation for the shaft, with $J$ as the rotor inertia and $c_3$ as a damping coefficient. The term $c_1 i_f \omega$ is the back e.m.f. induced in the armature circuit, and $c_2 i_f i_a$ is the torque produced by the interaction of the armature current with the field circuit flux.

(a) For a separately excited DC motor, the voltages $v_a$ and $v_f$ are independent control inputs. Choose appropriate state variables and find the state equation.

(b) Specialize the state equation of part(a) to the field controlled DC motor, where $v_f$ is the control input, while $v_a$ is held constant.

(c) Specialize the state equation of part(a) to the armature controlled DC motor, where $v_a$ is the control input, while $v_f$ is held constant. Can you reduce the order of the model in this case?

(d) In a shunt wound DC motor, the field and armature windings are connected in parallel and an external resistance $R_x$ is connected in series with the field winding to limit the field flux; that is, $v = v_a = v_f + R_x i_f$. With $v$ as the control input, write down the state equation.

**1.18** Figure 1.26 shows a schematic diagram of a magnetic suspension system, where a ball of magnetic material is suspended by means of an electromagnet whose current is controlled by feedback from the, optically measured, ball position [211, pp. 192–200]. This system has the basic ingredients of systems constructed to levitate mass, used in gyroscopes, accelerometers, and fast trains. The equation of motion of the ball is

$$m\ddot{y} = -k\dot{y} + mg + F(y, i)$$

where $m$ is the mass of the ball, $y \geq 0$ is the vertical (downward) position of the ball measured from a reference point ($y = 0$ when the ball is next to the coil), $k$ is a viscous friction coefficient, $g$ is the acceleration due to gravity, $F(y, i)$ is the force generated by the electromagnet, and $i$ is its electric current. The inductance of the electromagnet depends on the position of the ball and can be modeled as

$$L(y) = L_1 + \frac{L_0}{1 + y/a}$$

where $L_1$, $L_0$, and $a$ are positive constants. This model represents the case that the inductance has its highest value when the ball is next to the coil and decreases to a constant value as the ball is removed to $y = \infty$. With $E(y, i) = \frac{1}{2}L(y)i^2$ as the energy stored in the electromagnet, the force $F(y, i)$ is given by

$$F(y, i) = \frac{\partial E}{\partial y} = -\frac{L_0 i^2}{2a(1 + y/a)^2}$$

When the electric circuit of the coil is driven by a voltage source with voltage $v$, Kirchhoff's voltage law gives the relationship $v = \dot{\phi} + Ri$, where $R$ is the series resistance of the circuit and $\phi = L(y)i$ is the magnetic flux linkage.

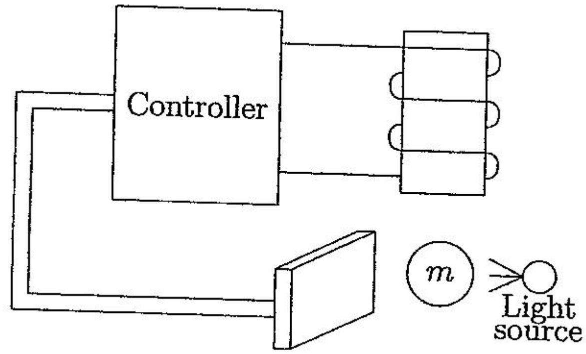



Figure 1.26: Magnetic suspension system of Exercise 1.18.

(a) Using $x_1 = y$, $x_2 = \dot{y}$, and $x_3 = i$ as state variables and $u = v$ as control input, find the state equation.

(b) Suppose it is desired to balance the ball at a certain position $r > 0$. Find the steady-state values $I_{ss}$ and $V_{ss}$ of $i$ and $v$, respectively, which are necessary to maintain such balance.

The next three exercises give examples of hydraulic systems [41].

**1.19** Figure 1.27 shows a hydraulic system where liquid is stored in an open tank. The cross-sectional area of the tank, $A(h)$, is a function of $h$, the height of the liquid level above the bottom of the tank. The liquid volume $v$ is given by $v = \int_0^h A(\lambda)\, d\lambda$. For a liquid of density $\rho$, the absolute pressure $p$ is given by $p = \rho g h + p_a$, where $p_a$ is the atmospheric pressure (assumed constant) and $g$ is the acceleration due to gravity. The tank receives liquid at a flow rate $w_i$ and loses liquid through a valve that obeys the flow-pressure relationship $w_o = k\sqrt{\Delta p}$. In the current case, $\Delta p = p - p_a$. Take $u = w_i$ to be the control input and $y = h$ to be the output.

(a) Using $h$ as the state variable, determine the state model.

(b) Using $p - p_a$ as the state variable, determine the state model.

(c) Find $u_{ss}$ that is needed to maintain the output at a constant value $r$.

**1.20** The hydraulic system shown in Figure 1.28 consists of a constant speed centrifugal pump feeding a tank from which liquid flows through a pipe and a valve that obeys the relationship $w_o = k\sqrt{p - p_a}$. The pump characteristic for the specified pump speed is shown in Figure 1.29. Let us denote this relationship by $\Delta p = \phi(w_i)$ and denote its inverse, whenever defined, by $w_i = \phi^{-1}(\Delta p)$. For the current pump, $\Delta p = p - p_a$. The cross-sectional area of the tank is uniform; therefore, $v = Ah$ and $p = p_a + \rho g v / A$, where the variables are defined in the previous exercise.